Order of Operations and Evaluating Expressions

PEMDAS

1) Parentheses (Grouping Symbols) → [ ], ( ), { }
2) Exponents
3) Multiply / Divide from Left to Right
4) Add / Subtract from Left to Right

Simplify each expression.

1) $3[10^2-(50-4+20)]$

$3[10^2-(46+20)]$

$3(10^2-66)$

$3(100-66)$

$3(34)$

$\boxed{102}$

2) $\dfrac{3^2+5^3}{(10-8)^2}$

$\dfrac{9+125}{2^2}$

$\dfrac{134}{4}=\boxed{33.5}$

Evaluate each expression for $a=5$, $b=10$, and $c=3$.

3) $3a^2-7b$

$3\cdot5^2-7\cdot10$

$3\cdot25-7\cdot10$

$75-70$

$\boxed{5}$

4) $\dfrac{5c+7a}{2b}$

$\dfrac{5\cdot3+7\cdot5}{2\cdot10}$

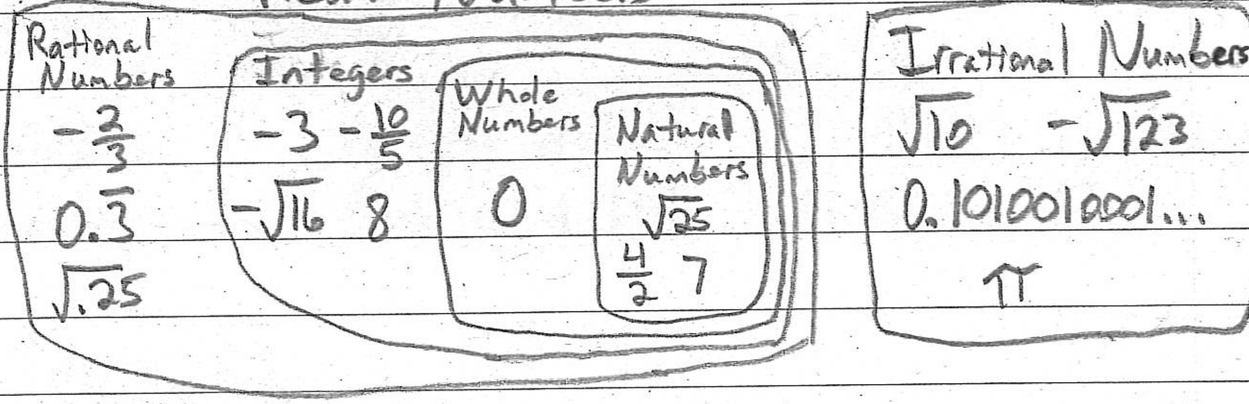
$\dfrac{15+35}{20}=\dfrac{5\cancel{0}}{2\cancel{0}}=\boxed{2\frac{1}{2}}$

Real Numbers

Simplify each expression.

1) $\sqrt{144} = \boxed{12}$ 2) $\sqrt{\frac{16}{225}} = \frac{\sqrt{16}}{\sqrt{225}} = \boxed{\frac{4}{15}}$

Real Numbers



Inequality Symbols

$<$, less than $\leq$, less than or equal to

$>$, greater than $\geq$, greater than or equal to

Compare the numbers using an inequality symbol.

3) $\sqrt{52}$ and $7\frac{1}{4}$ $\sqrt{52} = 7.211102551...$

$7\frac{1}{4} = 7.25$

$\boxed{\sqrt{52} < 7\frac{1}{4}}$

Properties of Real Numbers

Commutative Property

	Algebra	Example
Addition	$a+b=b+a$	$18+54=54+18$
Multiplication	$a\cdot b=b\cdot a$	$12\cdot\frac{1}{2}=\frac{1}{2}\cdot 12$

Associative Property

	Algebra	Example
Addition	$(a+b)+c=a+(b+c)$	$(23+9)+11=23+(9+11)$
Multiplication	$(a\cdot b)\cdot c=a\cdot(b\cdot c)$	$(17\cdot 5)\cdot 2=17\cdot(5\cdot 2)$

Identity Property

	Algebra	Example
Addition	$a+0=a$	$7\frac{1}{3}+0=7\frac{1}{3}$
Multiplication	$a\cdot 1=a$	$23\cdot 1=23$

Zero Property of Multiplication

Algebra	Example
$a\cdot 0=0$	$21\cdot 0=0$

Multiplication Property of -1

Algebra	Example
$-1\cdot a=-a$	$-1\cdot 12=-12$

Adding and Subtracting Real Numbers

Absolute Value

$|8| = 8$, $|-8| = 8$

Same Sign (SS) → Add, Keep Sign

Different Sign (DS) → Subtract, Original Sign of Larger Absolute Value

Add or subtract.

1) $-10 + (-15)$ SS
-25

2) $-21 + 10$ DS
-11

3) $18 + (-9)$ DS
9

4) $-3 - 8$ SS
-11

5) $12 - 15$ DS
-3

6) $-5 \overset{+}{-} (-8)$ DS
3

7) $9 \overset{+}{-} (-6)$
15

Multiplying and Dividing Real Numbers

Even Amount of (-), Answer is (+)
Odd Amount of (-), Answer is (-)

Inverse Property of Multiplication
(Multiplicative Inverse or Reciprocal)

Algebra	Example
$a \cdot \left(\frac{1}{a}\right) = 1$	$-7\left(-\frac{1}{7}\right) = 1$

Simplify each expression.

1) $-3(-8)$ Even
24

2) $-24 \div 4$ Odd
-6

3) $7(-2)$ Odd
-14

4) $\pm\sqrt{81} = \boxed{\pm 9}$

5) $-\sqrt{\frac{9}{64}} = \boxed{-\frac{3}{8}}$

6) $-8 \div \left(-\frac{16}{5}\right)$

$-\frac{\cancel{8}^{1}}{1} \cdot \left(-\frac{5}{\cancel{16}_{2}}\right)$

$\boxed{\frac{5}{2} \text{ or } 2\frac{1}{2}}$

Division Involving 0

Algebra	Example
$0 \div a = 0$	$0 \div 9 = 0$
$a \div 0 =$ Undefined	$9 \div 0 =$ Undefined

The Distributive Property

Simplify.

① $5(y-2)$
$5(y)-5(2)$
$\boxed{5y-10}$

② $(3m-8)(-4)$
$(3m)(-4)-8(-4)$
$\boxed{-12m+32}$

③ $-(2c-12)$
$(-1)(2c-12)$
$\boxed{-2c+12}$

What sum or difference is equivalent to $\frac{8m-5}{13}$?

④ $\frac{8m-5}{13} = \frac{1}{13}(8m-5)$ Write division as multiplication

$= \left(\frac{1}{13}\right)(8m) - \left(\frac{1}{13}\right)(5)$ Distributive Property

$= \boxed{\frac{8}{13}m - \frac{5}{13}}$ Simplify

Simplify. (Combine Like Terms)

⑤ $5m-2n-7n-12-8m$

$\boxed{-3m-9n-12}$

⑥ $7x^2-8x^2-3x+8$

$\boxed{-x^2-3x+8}$

Solving One-Step Equations

Solve each equation.

1) $$x + 12 = -5$$
$$\underline{\quad -12 \quad -12 \quad}$$
$$x = -17$$

2) $$-8 = y - 4$$
$$\underline{+4 \qquad +4}$$
$$-4 = y$$

3) $$\frac{\cancel{3}c}{\cancel{3}} = \frac{-13.5}{3}$$
$$c = -4.5$$

4) $$-7 = \frac{m}{3}$$
$$3(-7) = \frac{m}{\cancel{3}} \cdot \cancel{3}$$
$$-21 = m$$

5) $$\frac{2}{3}n = -\frac{8}{9}$$
$$\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} n = -\frac{\cancel{8}^{4}}{{}_{3}\cancel{9}} \cdot \frac{\cancel{3}^{1}}{\cancel{2}_{1}}$$
$$n = -\frac{4}{3} = \boxed{-1\frac{1}{3}}$$

Solving Two-Step Equations

Solve each equation.

1) $12 - 3x = -15$

$\underline{-12 \qquad\quad -12}$

$\frac{-\cancel{3}x}{-\cancel{3}} = \frac{-27}{-3}$

$x = 9$

2) $-13 = \frac{m}{5} - 3$

$\underline{+3 \qquad\qquad +3}$

$(5)(-10) = \frac{m}{\cancel{5}}\cancel{(5)}$

$-50 = m$

3) $\frac{c-8}{4} = -5$

$\cancel{4}\left(\frac{c-8}{\cancel{4}}\right) = -5(4)$

$c - 8 = -20$

$\underline{+8 \qquad +8}$

$c = -12$

4) $-y + 3 = -8$

$\underline{-3 \quad -3}$

$-y = -11$

$\frac{-\cancel{1}y}{-\cancel{1}} = \frac{-11}{-1}$

$y = 11$

Solving Multi-Step Equations

Solve each equation.

1) $-12-2=4c+6+c$

$-14=5c+6$

$-6 \quad -6$

$\dfrac{-20}{5}=\dfrac{\cancel{5}c}{\cancel{5}}$

$\boxed{-4=c}$

2) $-7(3m-7)=7$

$-21m+49=7$

$-49 \quad -49$

$\dfrac{\cancel{-21}m}{\cancel{-21}}=\dfrac{-42}{-21}$

$\boxed{m=2}$

3) $\dfrac{3y}{4}-\dfrac{2y}{5}=-7$

$20\left(\dfrac{3y}{4}-\dfrac{2y}{5}\right)=(-7)(20)$

$15y-8y=-140$

$\dfrac{\cancel{7}y}{\cancel{7}}=\dfrac{-140}{7}$

$\boxed{y=-20}$

Solving Equations with Variables on Both Sides

Solve each equation.

1). $6b - 4 = -2b - 28$
$+2b \quad\quad +2b$
$8b - 4 = -28$
$+4 \quad +4$
$8b = -24$
$\frac{8b}{8} = \frac{-24}{8}$
$b = -3$

2) $-3(2y - 8) = 4(y - 4)$
$-6y + 24 = 4y - 16$
$+6y \quad\quad +6y$
$24 = 10y - 16$
$+16 \quad\quad +16$
$40 = 10y$
$\frac{40}{10} = \frac{10y}{10}$
$\boxed{4 = y}$

3) $-3m + 18 = -3(m - 6)$
$-3m + 18 = -3m + 18$
Answer: All Real Numbers

4) $5(x - 2) = 8x + 2 - 3x$
$5x - 10 = 5x + 2$
$-5x \quad\quad -5x$
$-10 = 2$ X
Since $-10 \neq 2$, $\boxed{\text{No Solution}}$

Literal Equations and Formulas

Solve each equation for y. Then find the value of each value of x.

1) $10x + 5y = 80;\ x = 3, 5$

$-10x \qquad -10x$

$\frac{\cancel{5}y}{\cancel{5}} = \frac{80 - 10x}{5}$

$\boxed{y = 16 - 2x}$

$y = 16 - 2x$
$y = 16 - 2(3)$
$y = 16 - 6$
$y = 10$

$y = 16 - 2x$
$y = 16 - 2(5)$
$y = 16 - 10$
$y = 6$

What equation do you get when you solve $ax - bx = c$ for x?

2). $ax - bx = c$

$\frac{x\cancel{(a-b)}}{\cancel{a-b}} = \frac{c}{a-b}$

$\boxed{x = \frac{c}{a-b}}$

Rewrite each formula.

3) $V = \frac{1}{3}\pi r^2 h$ for h

$3 \cdot V = \cancel{3} \cdot \frac{1}{3} \pi r^2 h$

$\frac{3V}{\pi r^2} = \frac{\cancel{\pi r^2} h}{\cancel{\pi r^2}}$

$\frac{3V}{\pi r^2} = h$

4) $2(x + a) = 4b$ for a

$2x + 2a = 4b$

$-2x \qquad -2x$

$\frac{\cancel{2}a}{\cancel{2}} = \frac{4b - 2x}{2}$

$a = 2b - x$

Ratios, Rates, and Conversions

① Which store offers the best deal?

Store A: $\frac{\$25}{2\text{ shirts}} \div 2$ = $\frac{\$12.50}{1\text{ shirt}}$

Store B: $\frac{\$45}{4\text{ shirts}} \div 4 = \frac{\$11.25}{1\text{ shirt}}$

Store C: $\frac{\$30}{3\text{ shirts}} \div 3 = \frac{\$10}{1\text{ shirt}}$

Best Deal

$\boxed{\text{Unit Rate}}$

② Convert 3.2ft to cm.

$$\frac{3.2\cancel{ft}}{1} \cdot \frac{12\cancel{in}}{1\cancel{ft}} \cdot \frac{2.54cm}{1\cancel{in}} = 97.5\,cm \text{ (Round - Tenths)}$$

③ Convert 75mph to m/s.

$$\frac{75\cancel{mi}}{1\cancel{hr}} \cdot \frac{1\cancel{hr}}{60\cancel{min}} \cdot \frac{1\cancel{min}}{60sec} \cdot \frac{1.609\cancel{km}}{1\cancel{mi}} \cdot \frac{1{,}000m}{1\cancel{km}} = \boxed{33.5\frac{m}{s}}$$

Rounded (Tenths)

Solving Proportions

Solve each equation.

1) $\frac{7}{8} = \frac{m}{12}$

$8 \cdot m = 7 \cdot 12$

$\frac{8m}{8} = \frac{84}{8}$

$m = 10.5$

2) $\frac{b-8}{5} = \frac{b+3}{4}$

$4(b-8) = 5(b+3)$

$4b - 32 = 5b + 15$

$-4b \quad -4b$

$-32 = b + 15$

$-15 \quad -15$

$-47 = b$

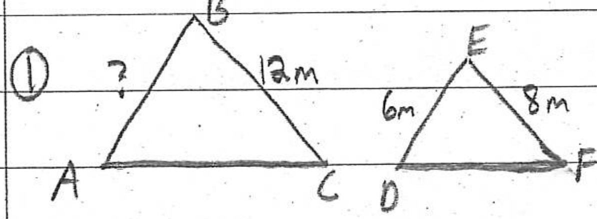
3) 8oz of orange juice contains 97 mg of vitamin C.
How many mg of vitamin C in 12 oz?

$\frac{8oz}{97mg} = \frac{12oz}{x\ mg}$

$\frac{8x}{8} = \frac{1{,}164}{8}$

$x = 145.5mg$

Proportions and Similar Figures

① 

$\triangle ABC \sim \triangle DEF$

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{AB}{6} = \frac{12}{8}$$

$$\frac{8(AB)}{8} = \frac{72}{8}$$

$AB = 9\text{m}$

② The distance from Jacksonville to Gainesville on a map is about 0.6in. What is the actual distance?

Scale
1in : 110mi

$$\text{Map scale} = \frac{\text{map distance}}{\text{actual distance}}$$

$$\frac{1\text{in}}{110\text{mi}} = \frac{.6\text{in}}{X}$$

$X = 66 \text{ miles}$

Percents

① What percent of 90 is 54?

$\frac{a}{b} = \frac{p}{100}$

$\frac{54}{90} = \frac{p}{100}$

$\frac{\cancel{90}p}{\cancel{90}} = \frac{5{,}400}{90}$

$\boxed{p = 60\%}$

$p \cdot 90 = 54$

$\frac{90p}{\cancel{90}} = \frac{54}{90}$

$p = 0.6$

$\boxed{p = 60\%}$

② What is 35% of 120?

$\frac{a}{b} = \frac{p}{100}$

$\frac{a}{120} = \frac{35}{100}$

$\frac{\cancel{100}a}{\cancel{100}} = \frac{4{,}200}{100}$

$\boxed{a = 42}$

$n = (.35)(120)$

$\boxed{n = 42}$

③ 85% of what number is 127.5?

$\frac{a}{b} = \frac{p}{100}$

$\frac{127.5}{b} = \frac{85}{100}$

$\frac{\cancel{85}b}{\cancel{85}} = \frac{12{,}750}{85}$

$\boxed{b = 150}$

$.85 \times n = 127.5$

$\frac{\cancel{.85}n}{\cancel{.85}} = \frac{127.5}{.85}$

$\boxed{n = 150}$

④ You deposit: \$1,200

Interest Rate: 4.2%

Time: 5 years

How much interest?

$I = PRT$

$I = (1{,}200)(.042)(5)$

$\boxed{I = \$252}$

Percent of Change

$$\text{Percent of Change} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

Students at SK Last Year : 625
Students at SK This Year : 652

Find the percent of change.

Ex) $\%\text{ of Inc} = \frac{\text{Amt. of Ch.}}{\text{Orig. Amt.}}$

$$= \frac{652-625}{625}$$

$$= \frac{27}{625}$$

$$= .0432 = \boxed{4.32\% \text{ Increase}}$$

Inequalities and Their Graphs

Determine whether each number is a solution of the inequality

1) $3x + 1 \geq 10$

a) 5

$3(5)+1 \geq 10$

$15+1 \geq 10$

$16 \geq 10$ ✓

b) 3

$3(3)+1 \geq 10$

$9+1 \geq 10$

$10 \geq 10$ ✓

c) 0

$3(0)+1 \geq 10$

$0+1 \geq 10$

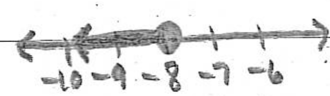
$1 \ngeq 10$ X

Therefore 5 and 3 are solutions and 0 is not a solution.

Graph each inequality.

2) $x > -2$

3) $-8 \geq m$



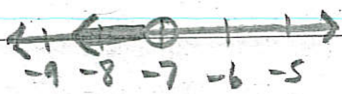
4) $y \geq 10$

5) $n \leq -25$

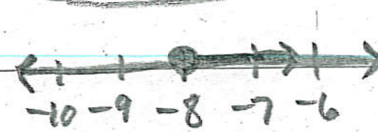
Solving Inequalities Using Addition or Subtraction

Solve and graph each inequality.

1)
$$f - 5 < -12$$
$$\underline{\quad +5 \qquad +5 \quad}$$
$$f < -7$$



2)
$$-3 \leq C + 5$$
$$\underline{-5 \qquad\quad -5}$$
$$-8 \leq C$$



3) Your cell phone has 16 GB of storage capacity. You have used 12.5 GB. Write and solve an inequality to find the possible amount storage that can be used.

C = storage capacity that can be used

$$C + 12.5 \leq 16$$
$$\underline{-12.5 \quad -12.5}$$
$$C \leq 3.5 \text{ GB}$$

Solving Inequalities Using Multiplication or Division

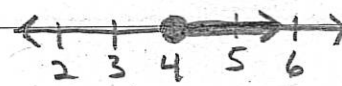
Solve and graph each inequality

① $\cancel{8} \cdot \frac{y}{\cancel{8}} < -2 \cdot 8$

$y < -16$

② $\frac{24}{6} \le \frac{\cancel{6}m}{\cancel{6}}$

$4 \le m$



③ $\frac{-18}{-2} < \frac{\cancel{-2}c}{\cancel{-2}}$

$9 > c$

④ $\cancel{(-3)} \frac{n}{\cancel{-3}} \ge 5(-3)$

$n \le -15$

Solving Multi-Step Inequalities

Solve each inequality:

① $-3(y+1)+4y \geq 4(y-6)$

$-3y-3+4y \geq 4y-24$

$y-3 \geq 4y-24$

$-4y \qquad -4y$

$-3y-3 \geq -24$

$+3 \qquad +3$

$\frac{-3y}{-3} \geq \frac{-21}{-3}$

$y \leq 7$

② $10-8a \geq 2(5-4a)$

$10-8a \geq 10-8a$

$+8a \qquad +8a$

$10 \geq 10$

All Real Numbers

③ $6m-5 > 7m+7-m$

$6m-5 > 6m+7$

$-6m \qquad -6m$

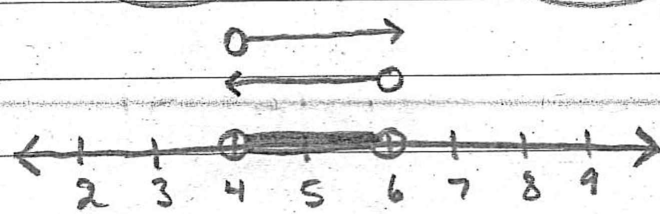
$-5 \not> 7$

No Solution

Compound Inequalities

① What are the solutions of $2 < \frac{3x-8}{2} < 5$? Graph the solutions.

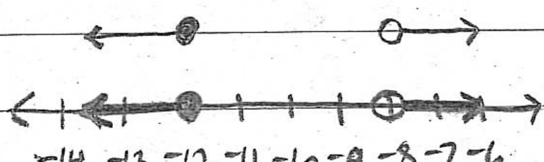
$\cancel{2} \cdot \frac{3x-8}{\cancel{2}} > 2 \cdot 2$	$\cancel{2} \cdot \frac{3x-8}{\cancel{2}} < 5 \cdot 2$
$3x-8 > 4$	$3x-8 < 10$
$+8 \quad +8$	$+8 \quad +8$
$\frac{\cancel{3}x}{\cancel{3}} > \frac{12}{3}$	$\frac{\cancel{3}x}{\cancel{3}} < \frac{18}{3}$
$x > 4$	$x < 6$



Interval Notation

$(4, 6)$

② Graph the solutions of $x \leq -12$ or $x > -8$.



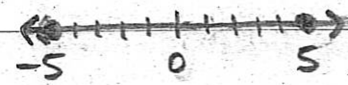
Interval Notation

$(-\infty, -12]$ or $(-8, \infty)$

Absolute Value Equations and Inequalities

① What are the solutions of $|x|+3=8$? Graph.

$$|x|+3=8$$
$$-3 \quad -3$$
$$|x|=5$$



$$x=5 \qquad x=-5$$

② $3|5y-7|-6>24$

$$+6 \quad +6$$
$$\frac{\cancel{3}|5y-7|}{\cancel{3}}>\frac{30}{3}$$
$$|5y-7|>10$$

$$5y-7>10 \qquad 5y-7<-10$$
$$+7 \quad +7 \qquad +7 \quad +7$$
$$\frac{\cancel{5}y}{\cancel{5}}<\frac{17}{5} \qquad \frac{\cancel{5}y}{\cancel{5}}<\frac{-3}{5}$$

$$y>\frac{17}{5} \text{ or } y<-\frac{3}{5}$$

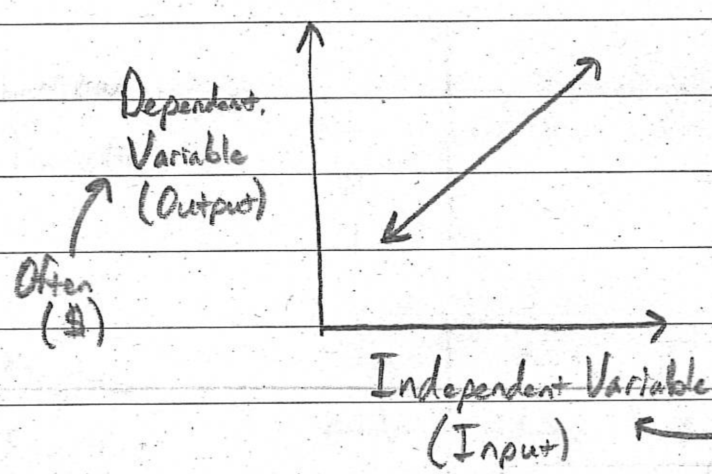
③ $10+|3m-8|<8$

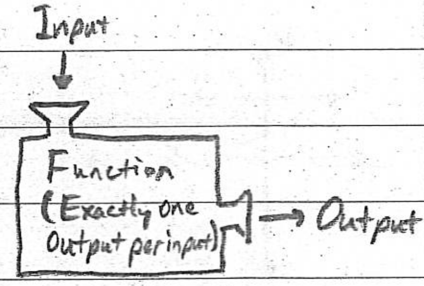
$$-10 \qquad -10$$
$$|3m-8|<-2$$

The absolute value of an expression cannot be negative.

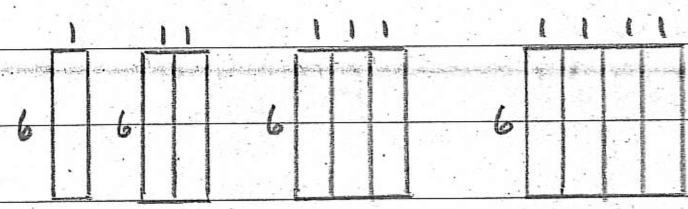
No Solution

Patterns and Linear Functions

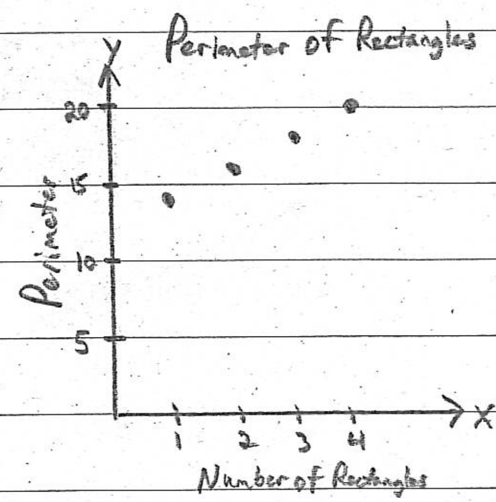




In the diagram below, what is the relationship between the number of rectangles and the perimeter? Represent this relationship using a table, equation, and a graph.



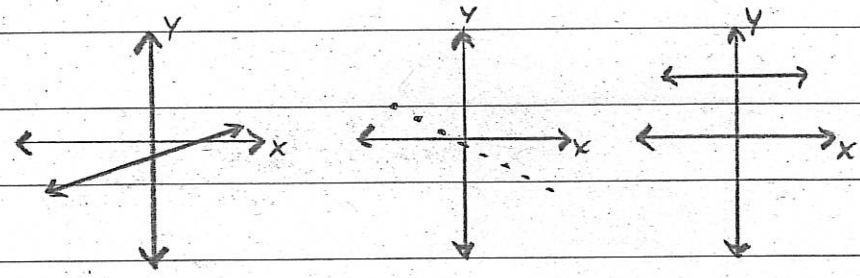
Number of Rectangles (x)	Perimeter (y)	Ordered Pair (x,y)
1	14	(1,14)
2	16	(2,16)
3	18	(3,18)
4	20	(4,20)



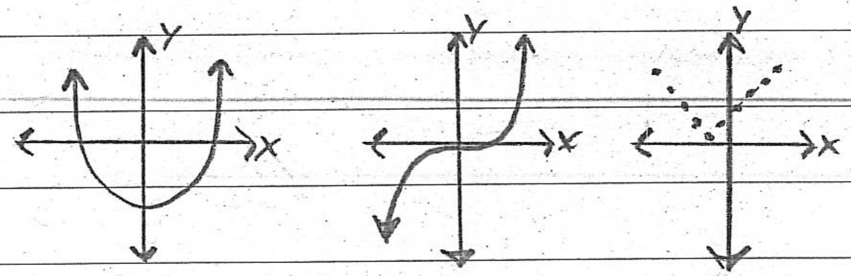
Equation: $y = 2x + 12$

Patterns and Nonlinear Functions

Linear Functions →



Nonlinear Functions →



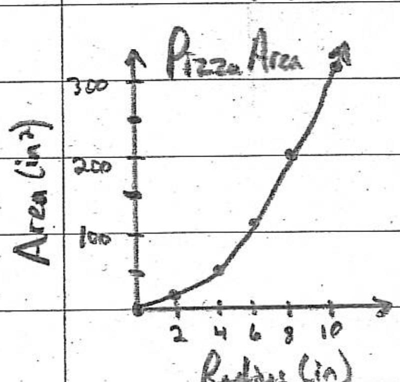
Graph these functions. Linear or nonlinear?

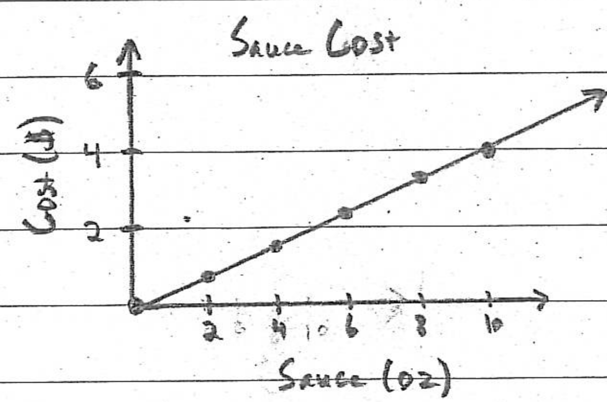
Pizza Area

r (in)	Area (in^2)
2	12.57
4	50.27
6	113.10
8	201.06
10	314.16

Sauce Cost

Weight (oz)	Cost
2	$.80
4	$1.60
6	$2.40
8	$3.20
10	$4.00





Nonlinear

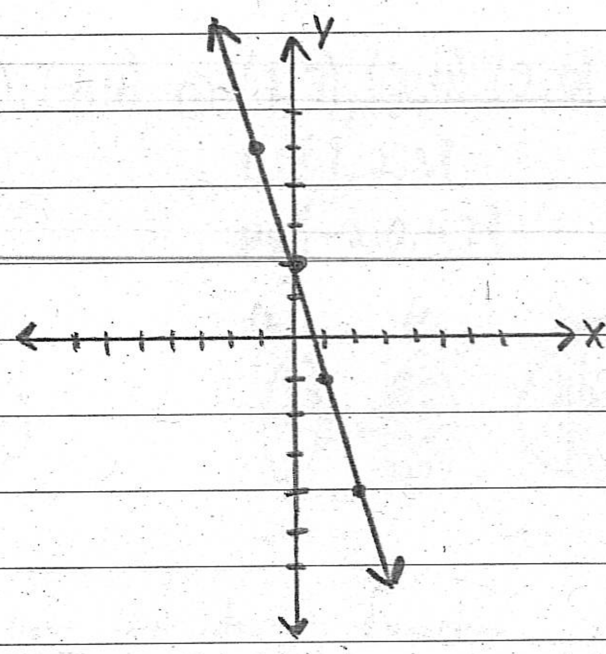
Linear

Graphing a Function Rule

What is the graph of each function rule?

① $y = -3x + 2$

x	y	(x, y)
0	2	(0,2)
1	-1	(1,-1)
-1	5	(-1,5)
2	-4	(2,-4)
-2	8	(-2,8)



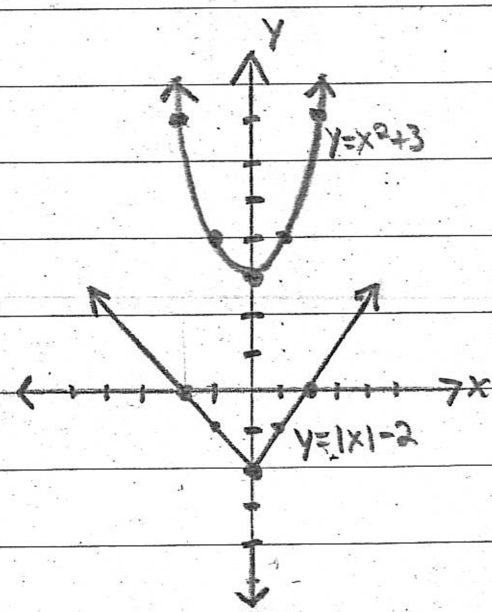
② $y = |x| - 2$ ③ $y = x^2 + 3$

②

x	y
0	-2
1	-1
-1	-1
2	0
-2	0

③

x	y
0	3
1	4
-1	4
2	7
-2	7
3	12
-3	12



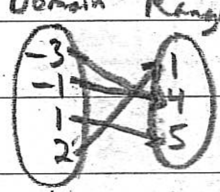
Functions, Domain, and Range

Identify the domain and range. Is the relation a function.

① $(2,1),(-3,4),(1,5),(-1,4)$

$D: \{-3,-1,1,2\}$

$R: \{1,4,5\}$

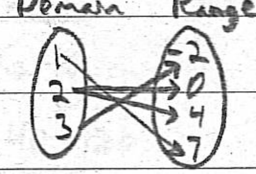


Function

② $(1,7),(2,4),(2,0),(3,-2)$

$D: \{1,2,3\}$

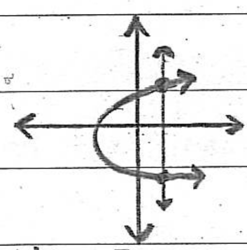
$R: \{-2,0,4,7\}$



Not a Function

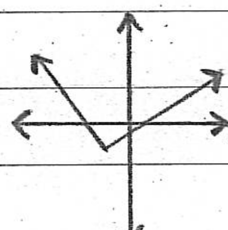
Use the vertical line test to identify functions

③



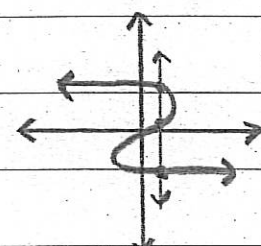
Not a Function

④



Function

⑤



Not a Function

⑥ The domain $f(x)=-2x+5$ is $\{-1,0,1\}$.

What is the range?

x	$-2x+5$	$f(x)$
-1	$-2(-1)+5$	7
0	$-2(0)+5$	5
1	$-2(1)+5$	3

The range is $\{3,5,7\}$.

Function Notation

$f(x)=-2x+5$

Replaces y

Rate of Change and Slope

$$\text{Rate of Change} = \text{Slope} = m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the rate of change.

①

Time (min.)	Distance (ft)
1	120
2	240
3	360
4	480

$$\text{Rate of Change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{240 - 120}{2 - 1} = \frac{120}{1} = \boxed{120 \, \frac{ft}{min}}$$

Find the slope.

② $(-1, 0)$ and $(3, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-2)}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2}$$

③ $(1, 3)$ and $(4, -1)$

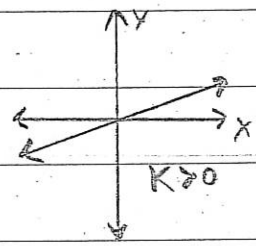
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{1 - 4} = \frac{4}{-3} = -\frac{4}{3}$$

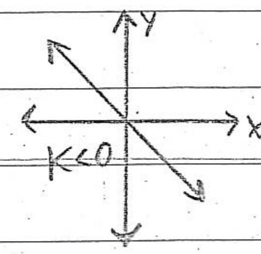
Direct Variation

$y = kx$

↑ Constant of Variation (slope)

Graphs of Direct Variations





Tell whether y varies directly with x. If it does, write an equation.

①

x	y	
4	6	$6 \div 4 = 1.5$
8	12	$12 \div 8 = 1.5$
10	15	$15 \div 10 = 1.5$

Yes, y varies directly with x

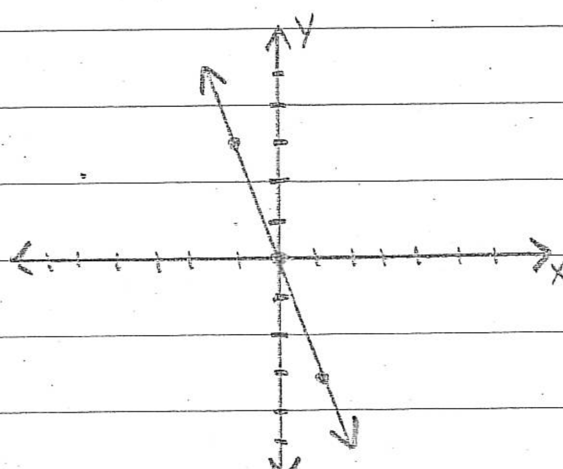
$\boxed{y = 1.5x}$

②

x	y	
-2	3.2	$3.2 \div (-2) = -1.6$
1	2.4	$2.4 \div 1 = 2.4$
4	1.6	$1.6 \div 4 = .4$

$\boxed{\text{No, y does not vary directly with x}}$

③ Graph $y = -3x$



Slope-Intercept Form

$$y = mx + b$$

m ↑ Slope b ↑ y-Intercept

y-Intercept is the y-coordinate of a point where the graph crosses the y-axis.

① What is an equation of the line with slope $-\frac{2}{3}$ and y-Int. 10?

$$\boxed{y = -\frac{2}{3}x + 10}$$

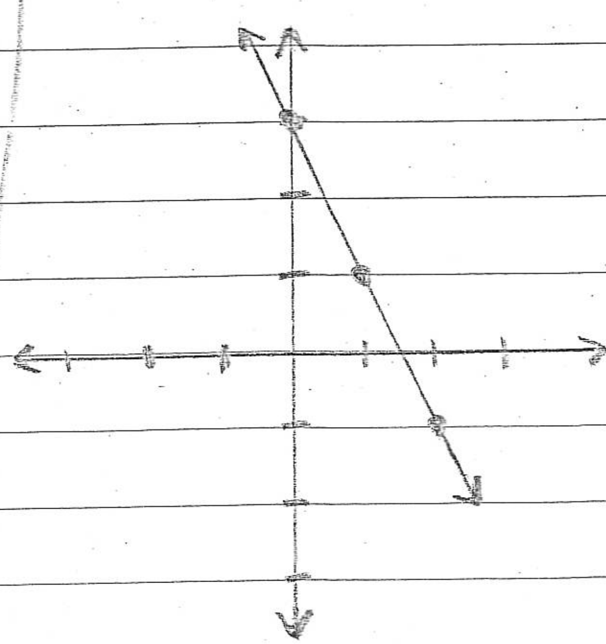
② What equation in slope-intercept form represents the line that passes through $(3,-2)$ and $(1,-3)$?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-2)}{1-3} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

$$y = mx + b$$
$$y = \frac{1}{2}x + b$$
$$-2 = \frac{1}{2}(3) + b$$
$$-2 = 1\tfrac{1}{2} + b$$
$$-1\tfrac{1}{2} \quad -1\tfrac{1}{2}$$
$$-3\tfrac{1}{2} = b$$
$$-\frac{7}{2} = b$$

$$y = \frac{1}{2}x - \frac{7}{2}$$

③ Graph $y = -2x + 3$



Point-Slope Form

$y - y_1 = m(x - x_1)$

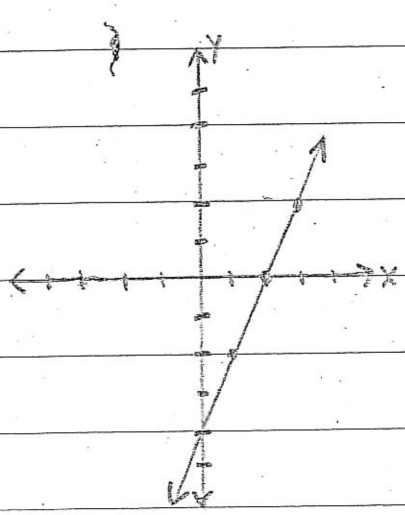
y-coordinate, slope, x-coordinate

① A line passes through (8, −4) and has a slope of $\frac{2}{3}$. What is an equation in point-slope form of the line?

$y - y_1 = m(x - x_1)$

$y + 4 = \frac{2}{3}(x - 8)$

② What is the graph of the equation $y + 2 = 2(x - 1)$



③ Given: (−2, −3) and (1, 4)

Write an equation.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4 - (-3)}{1 - (-2)}$

$= \frac{7}{3}$

$y + 3 = \frac{7}{3}(x + 2)$

or

$y - 4 = \frac{7}{3}(x - 1)$

Standard Form

$Ax + By = C$

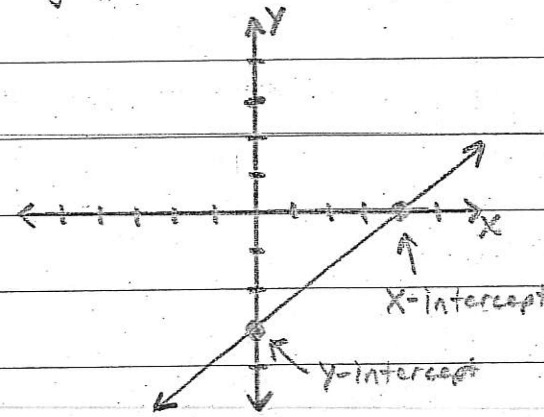
① What are the x- and y-intercepts and graph of $3x - 4y = 12$?

$3x - 4y = 12$

$3x - 4(0) = 12$ $\qquad$ $3(0) - 4y = 12$

$\frac{3x}{3} = \frac{12}{3}$ $\qquad$ $\frac{-4y}{-4} = \frac{12}{-4}$

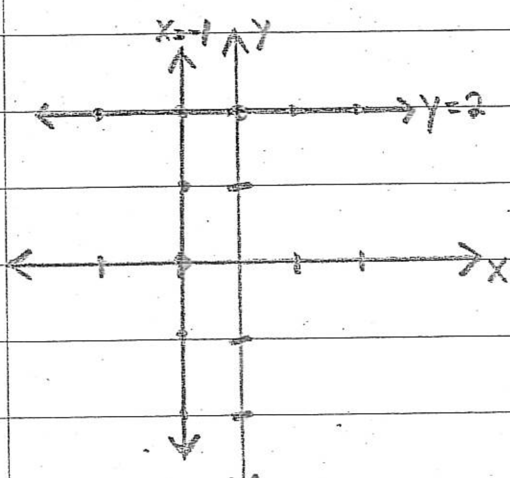
$x = 4$ (x-intercept) $\qquad$ $y = -3$ (y-intercept)



② What is the graph?

$\boxed{A}$ $x = -1$ $\qquad$ $\boxed{B}$ $y = 2$

$x + 0y = -1$ $\qquad$ $0x + y = 2$



③ Write in standard form

$\boxed{A}$ $y = -\frac{3}{7}x + 5$

$(y = -\frac{3}{7}x + 5)(7)$

$7y = -3x + 35$

$+3x \quad +3x$

$3x + 7y = 35$

$\boxed{B}$ $y - 2 = -\frac{1}{3}(x + 6)$

$(y - 2 = -\frac{1}{3}x - 2)(3)$

$3y - 6 = -x - 6$

$+x \quad +x$

$x + 3y - 6 = -6$

$+6 \quad +6$

$x + 3y = 0$

Parallel and Perpendicular Lines

Parallel Lines $\Rightarrow$ Slopes are Equal	Perpendicular Lines $\Rightarrow$ Slopes are Opposite Reciprocals
Ex. $y=-3x+8$	Ex. $y=\frac{2}{3}x+9$
$y=-3x-7$	$y=-\frac{3}{2}x-2$
Slope is equal, lines are parallel.	Since $\frac{2}{3}\cdot\left(-\frac{3}{2}\right)=-1$, lines are perpendicular

① A line passes through $(-3,-1)$ and is parallel to $y=2x+3$. What equation represents the line in slope-intercept form?

$y=mx+b$

$y=2x+b$

$-1=2(-3)+b$

$-1=-6+b$

$+6 \quad +6$

$5=b$

$\boxed{y=2x+5}$

② A line passes through $(5,-2)$ and is perpendicular to $y=5x+4$. Write an equation (Slope-Intercept).

$y=mx+b$

$y=-\frac{1}{5}x+b$

$-2=-\frac{1}{5}(5)+b$

$-2=-1+b$

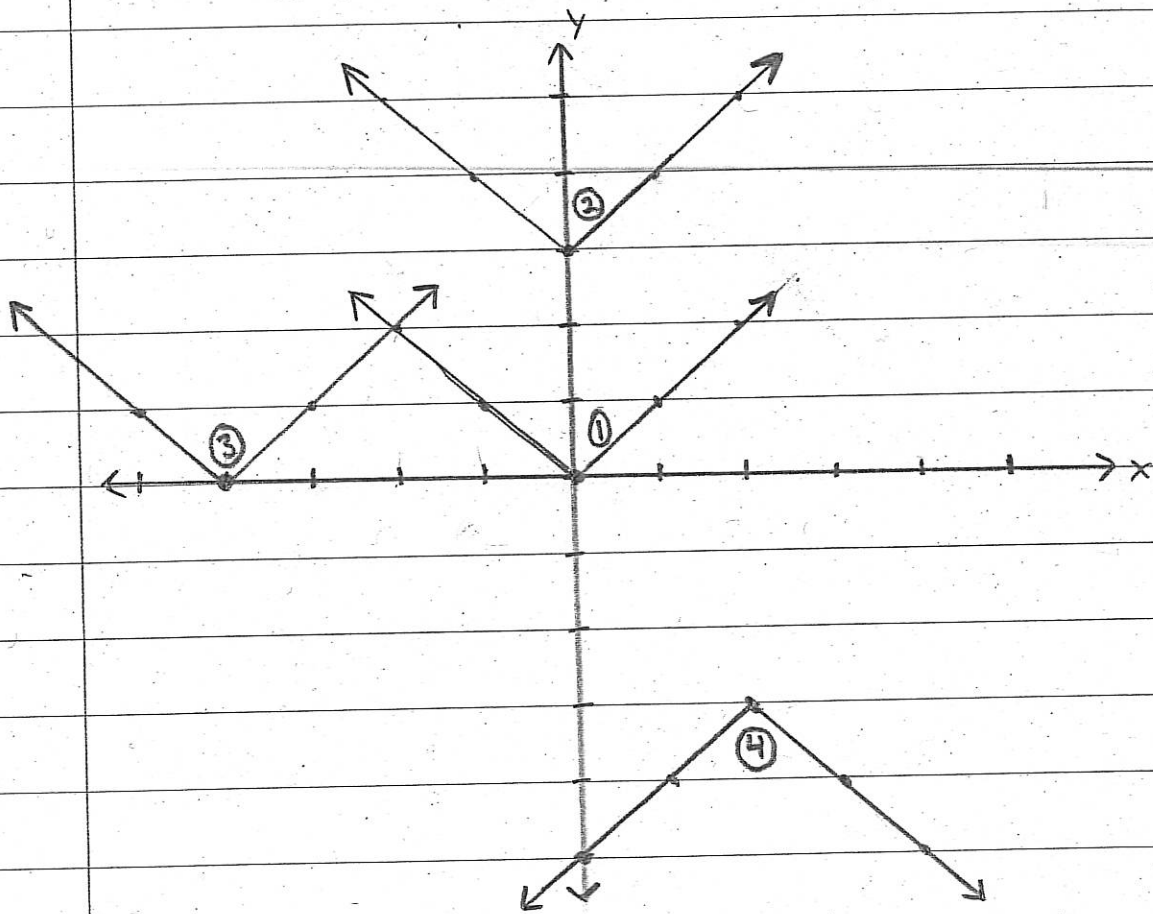
$+1 \quad +1$

$-1=b$

$\boxed{y=-\frac{1}{5}x-1}$

Graphing Absolute Value Functions

Graph (1) $y=|x|$, (2) $y=|x|+3$, (3) $y=|x+4|$, and (4) $y=-|x-2|-3$.



(1)

x	y
1	1
0	0
-1	1
2	2
-2	2

(2)

x	y
0	3
-1	4
1	4
2	5
-2	5

(3)

x	y
-4	0
-3	1
-5	1
-6	2
-2	2

(4)

x	y
2	-3
1	-4
3	-4
0	-5
4	-5

Solving Systems of Equations by Substitution

What is the solution of the system?

① $\begin{cases} 2x+y=-7 \\ y=4x+5 \end{cases}$

$$2x+(4x+5)=-7$$
$$2x+4x+5=-7$$
$$6x+5=-7$$
$$-5 \quad -5$$
$$\frac{\cancel{6}x}{\cancel{6}}=\frac{-12}{6}$$
$$\boxed{x=-2}$$

$$y=4x+5$$
$$y=4(-2)+5$$
$$y=-8+5$$
$$\boxed{y=-3}$$

$$\boxed{(-2,-3)}$$

② $\begin{cases} y=14x \\ y=\frac{1}{2}(28x+15) \end{cases}$

$$14x=\frac{1}{2}(28x+15)$$
$$14x=\frac{1}{2}(28x)+\frac{1}{2}(15)$$
$$14x=14x+\frac{15}{2}$$
$$-14x \quad -14x$$
$$0\neq\frac{15}{2}$$

No Solution

③ $\begin{cases} 4x+2y=20 \\ y=-2x+10 \end{cases}$

$$4x+2(-2x+10)=20$$
$$4x-4x+20=20$$
$$20=20$$

Infinitely Many Solutions

Solving Systems of Equations by Elimination

Solve each system using elimination.

① $\left.\begin{array}{l} 3x-2y=0 \\ 4x+2y=14 \end{array}\right\}$ Add the equations

$$\frac{\cancel{7}x}{\cancel{7}}=\frac{14}{7}$$

$x=2$

$$4x+2y=14$$
$$4(2)+2y=14$$
$$8+2y=14$$
$$-8 \quad -8$$
$$\frac{\cancel{2}y}{\cancel{2}}=\frac{6}{2}$$

$y=3$

$\boxed{(2,3)}$

② $(6x-3y=15)(4)$

$(7x+4y=10)(3)$

$$24x-12y=60$$
$$21x+12y=30$$
$$\frac{\cancel{45}x}{\cancel{45}}=\frac{90}{45}$$

$x=2$

$$6x-3y=15$$
$$6(2)-3y=15$$
$$12-3y=15$$
$$-12 \quad -12$$
$$\frac{\cancel{-3}y}{\cancel{-3}}=\frac{3}{-3}$$

$y=-1$

$\boxed{(2,-1)}$

Linear Inequalities

Is the ordered pair a solution of $y < x-2$?

a) $(1,5)$

$y < x-2$

$5 < 1-2$

$5 < -1$ X

Not a Solution

b) $(6,-2)$

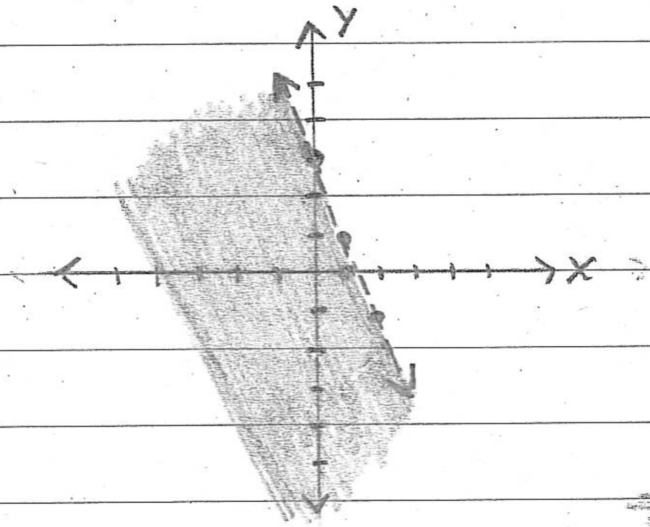
$y < x-2$

$-2 < 6-2$

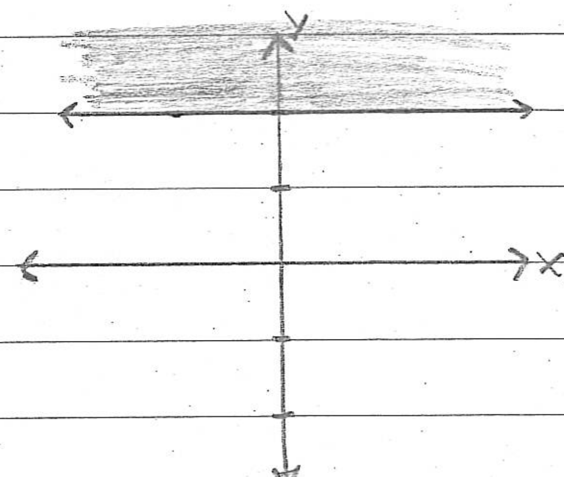
$-2 < 4$ ✓

$(6,-2)$ is a solution

What is the graph of $y < -2x+3$?



What is the graph of $y \geq 2$?



Systems of Linear Inequalities

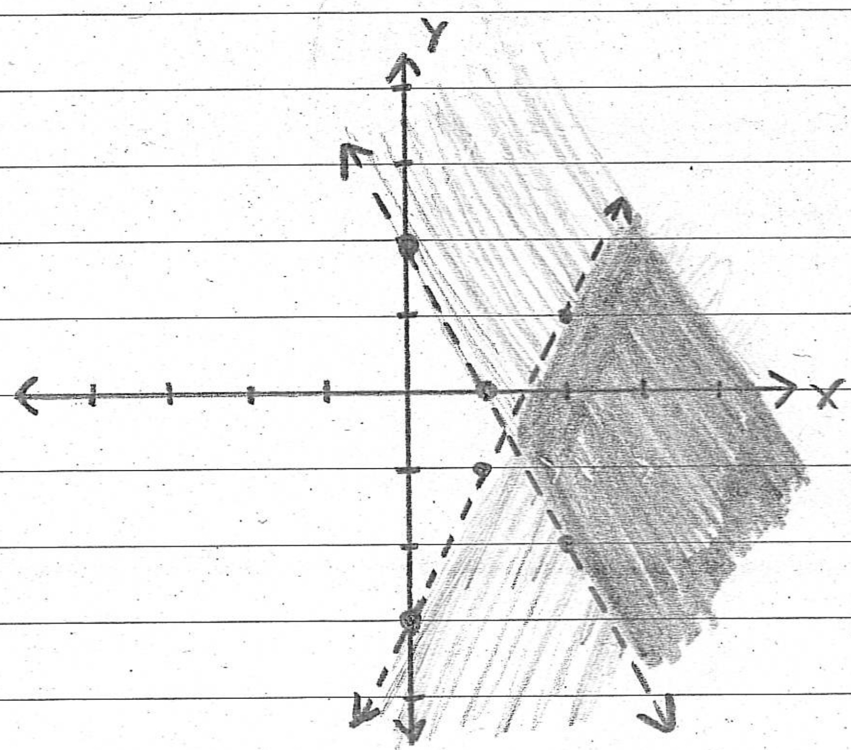
What is the graph of the system?

$y < 2x - 3$

$2x + y > 2$

$\underline{-2x \qquad -2x}$

$y > -2x + 2$



Zero and Negative Exponents

Zero as an exponent.

a) $4^0 = 1$ b) $(-3)^0 = 1$ c) $(5.14)^0 = 1$ d) $x^0 = 1$

Negative exponents

a) $7^{-3} = \frac{1}{7^3}$ b) $(-5)^{-2} = \frac{1}{(-5)^2}$ c) $x^4y^{-3} = \frac{x^4}{y^3}$

Simplify.

① $4^{-3} = \frac{1}{4^3} = \boxed{\frac{1}{64}}$

② $\frac{6x^{-3}z^{-5}}{y^{-4}} = \boxed{\frac{6y^4}{x^3z^5}}$

③ What is the value of $5x^2y^{-3}$ for $x=6$ and $y=3$?

$$5x^2y^{-3} = \frac{5x^2}{y^3}$$

$$= \frac{5(6^2)}{3^3}$$

$$= \frac{5(\cancel{36}\,4)}{\cancel{27}\,3}$$

$$= \frac{20}{3} = \boxed{6\frac{2}{3}}$$

Multiplying Powers with the Same Base

$a^m \cdot a^n = a^{m+n}$

What is each expression written using each base once?

① $12^3 \cdot 12^5 = (12 \cdot 12 \cdot 12) \cdot (12 \cdot 12 \cdot 12 \cdot 12 \cdot 12) = 12^{3+5} = \boxed{12^8}$

② $(7)^{-3}(7)^9 = \dfrac{\cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{\cancel{7} \cdot \cancel{7} \cdot \cancel{7}} = 7^{9-3} = \boxed{7^6}$

What is the simplified form of each expression?

③ $4y^5 \cdot 9y^{-12} = (4 \cdot 9)(y^{5-12})$

$= 36y^{-7}$

$= \boxed{\dfrac{36}{y^7}}$

④ $2a \cdot 9b^4 \cdot 3a^2 = (2 \cdot 9 \cdot 3)(a \cdot a^2)(b^4)$

$= \boxed{54a^3b^4}$

Multiplying with Scientific Notation

⑤ $(1.13 \times 10^{-7})(9.98 \times 10^5)(3.34 \times 10^{22})$

$(1.13 \times 9.98 \times 3.34)(10^{-7} \cdot 10^5 \cdot 10^{22})$

$\approx 37.7 \times 10^{20} = \boxed{3.77 \times 10^{21}}$

More Multiplication Properties of Exponents

ⓐ $(a^m)^n = a^{mn}$ ⓑ $(ab)^n = a^n b^n$

Simplify.

① $(x^3)^7 = \boxed{x^{21}}$ ② $(m^{\frac{2}{3}})^{\frac{1}{3}} = \boxed{m^{\frac{2}{9}}}$

③ $x^2(x^6)^{-4} = x^2(x^{-24}) = x^{-22} = \boxed{\frac{1}{x^{22}}}$

④ $(2b)^{-4} = 2^{-4}b^{-4} = \boxed{\frac{1}{16b^4}}$

⑤ $(n^{\frac{1}{2}})^{10}(4mn^{-\frac{2}{3}})^3 = (n^5)(4^3m^3n^{-2})$
$= \boxed{64m^3n^3}$

Scientific Notation Example

Kinetic energy

⑥ $\frac{1}{2}mv^2$

$m = 1.3 \times 10^3$ kg
$v = 3.1 \times 10^3$ m/s

$\frac{1}{2}(1.3\times10^3)(3.1\times10^3)^2$

$\frac{1}{2}(1.3)(10^3)(3.1^2)(10^6)$

$(\frac{1}{2}\times1.3\times3.1\times3.1)(10^3\cdot10^6)$

$\boxed{6.2465\times10^9 \text{ joules}}$

Division Properties of Exponents

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

What is the simplified form of each expression?

① $\frac{x^5}{x^2} = \frac{\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}} = x^{5-2} = \boxed{x^3}$ ② $\frac{y^{\frac{3}{4}}}{y^{\frac{1}{2}}} = y^{\frac{3}{4}-\frac{1}{2}} = \boxed{y^{\frac{1}{4}}}$

③ $\frac{k^6 j^2}{k j^5} = k^{6-1} j^{2-5} = k^5 j^{-3} = \boxed{\frac{k^5}{j^3}}$ ④ $\frac{x^4 y^{-1} z^8}{x^4 y^{-5} z} = \frac{\cancel{x^4} y^{-1} z^8}{\cancel{x^4} y^{-5} z}$

$= \boxed{y^4 z^7}$

⑤ What is $1.21 \times 10^7 \div 4.81 \times 10^5$ in standard notation?

$$\frac{1.21 \times 10^7}{4.81 \times 10^5} = \frac{1.21}{4.81} \times 10^{7-5} = .252 \times 10^2 = \boxed{25.2}$$

What is the simplified form?

⑥ $\left(\frac{z^{\frac{2}{3}}}{5}\right)^3 = \boxed{\frac{z^2}{125}}$ ⑦ $\left(\frac{2x^6}{y^4}\right)^{-3} = \frac{(y^4)^3}{(2x^6)^3} = \boxed{\frac{y^{12}}{8x^{18}}}$

Rational Exponents and Radicals

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

What is the simplified form of each expression?

① $\sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4}$
$= (4 \cdot 4 \cdot 4)^{\frac{1}{3}}$
$= \boxed{4}$

② $\sqrt[5]{243} = \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$
$= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)^{\frac{1}{5}}$
$= \boxed{3}$

Write each expression in radical form.

③ $5x^{\frac{1}{3}} = \boxed{5\sqrt[3]{x}}$

④ $(54y)^{\frac{2}{3}} = (27^{\frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}})$
$= (3^{3 \cdot \frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}})$
$= (3^2)(2^{\frac{2}{3}})(y^{\frac{2}{3}})$
$= 9\sqrt[3]{2^2 \cdot y^2} = \boxed{9\sqrt[3]{4y^2}}$

Write each expression in exponential form.

⑤ $\sqrt[3]{d^2} = \boxed{d^{\frac{2}{3}}}$

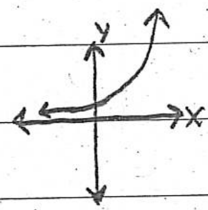
⑥ $\sqrt{(4y)^5} = (4y)^{\frac{5}{2}}$
$= 2^{2 \cdot \frac{5}{2}} y^{\frac{5}{2}}$
$= 2^5 y^{\frac{5}{2}}$
$= \boxed{32y^{\frac{5}{2}}}$

⑦ $\sqrt[4]{(32m)^3} = (32^{\frac{3}{4}})(m^{\frac{3}{4}})$
$= (16^{\frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}})$
$= (2^{4 \cdot \frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}})$
$= 2^3 \cdot 2^{\frac{3}{4}} \cdot m^{\frac{3}{4}}$
$= \boxed{8(2m)^{\frac{3}{4}}}$

Exponential Functions

$y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$, x is a real number

Examples:



+1 +1 +1

x	1	2	3	4
y	3	12	48	192

×4 ×4 ×4

Evaluate each function for the given value.

① $f(x) = 3 \cdot 2^x$, where $x = 5$

$= 3 \cdot 2^5$

$= 3 \cdot 32$

$= \boxed{64}$

② $y = 18(.015^x)$, $x = 3$

$y = 18(.015^3)$

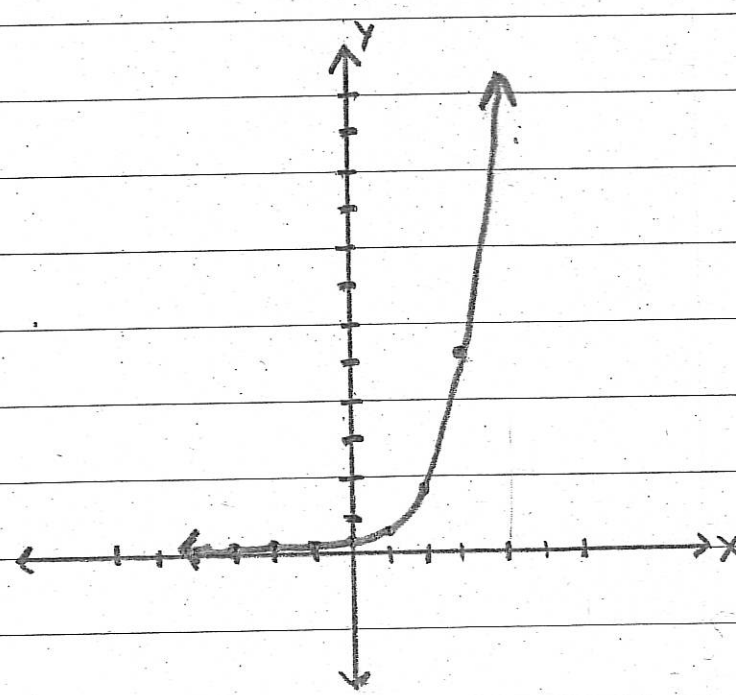
$y = 18(.000003375)$

$y = .00006075$

$\boxed{y = 6.075 \times 10^{-5}}$

Graph $y = .2(3^x)$

x	y	x	y
−3	.0074	2	1.8
−2	.0222	3	5.4
−1	.0667	4	16.2
0	.2	5	48.6
1	.6	6	145.8



Exponential Growth and Decay

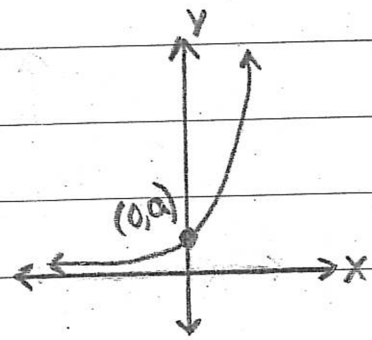
Exponential Growth

initial amount $(x=0)$

$$y = a \cdot b^x$$

base (>1)

b = growth factor



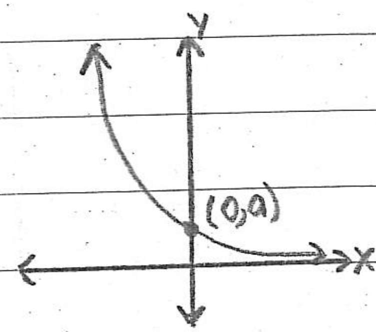
If increase by 6%, then $b = 1.06$

Exponential Decay

$$y = a \cdot b^x$$

base (<1)

b = decay factor



If decrease by 12%, then $b = .88$

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = the balance
P = principal
r = rate (decimal)
n = number of times inerest is compounded per year
t = time in years

Adding and Subtracting Polynomials

Standard Form of Polynomials

Ex. $3x^5 + 2x^3 - 8x^2 - 4x + 8$

Simplify. Write each answer in standard form.

① $m^2 - 8m^2$
$-7m^2$

② $13b^4c + 8b^4c$
$21b^4c$

③ $7n^5 - 12n^5$
$-5n^5$

④ $(3x^2 - 2x + 8) + (2x^2 - 7x - 18)$
$(3x^2 + 2x^2) + (-2x - 7x) + (8 - 18)$
$5x^2 - 9x - 10$

⑤ $(-8a^2 - 6a + 12) - (3a^2 + 8a + 4)$
$-8a^2 - 6a + 12 - 3a^2 - 8a - 4$
$-11a^2 - 14a + 8$

Multiplying and Factoring Polynomials

Multiplying a Monomial and a Trinomial

① $-x^3(8x^2-4x+7)$

$-8x^5+4x^4-7x^3$

Finding the GCF (Greatest Common Factor)

② $25x^4+45x^3+15x^2$

$25x^4 = 5\cdot 5\cdot x\cdot x\cdot x\cdot x$

$45x^3 = 3\cdot 3\cdot 5\cdot x\cdot x\cdot x$

$15x^2 = 3\cdot 5\cdot x\cdot x$

$GCF = 5\cdot x\cdot x = \boxed{5x^2}$

Factoring Out a Monomial

③ $9x^6+15x^4-12x^2$ $\quad GCF = 3x^2$

$3x^2(3x^4+5x^2-4)$

Multiplying Binomials (FOIL)

Simplify each product using FOIL.

$$\text{(1)}\quad (3x-4)(x+2) = \underset{\text{First}}{(3x)(x)} + \underset{\text{Outer}}{(3x)(2)} + \underset{\text{Inner}}{(-4)(x)} + \underset{\text{Last}}{(-4)(2)}$$
$$= 3x^2 + 6x - 4x - 8$$
$$= 3x^2 + 2x - 8$$

$$\text{(2)}\quad (2n-8)(5n-6) = (2n)(5n) + (2n)(-6) + (-8)(5n) + (-8)(-6)$$
$$= 10n^2 - 12n - 40n + 48$$
$$= 10n^2 - 52n + 48$$

$$\text{(3)}\quad (3y^2+4)(2y-5) = (3y^2)(2y) + (3y^2)(-5) + (4)(2y) + (4)(-5)$$
$$= 6y^3 - 15y^2 + 8y - 20$$

Squaring Binomials and the Product of a Sum and Difference

① The Square of a Binomial

Algebra

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

Examples

$(x+8)^2 = x^2+16x+64$

$(x-5)^2 = x^2-10x+25$

② The Product of a Sum and Difference

Algebra

$(a+b)(a-b) = a^2-b^2$

Example

$(x+7)(x-7) = x^2-49$

Factoring (x^2+bx+c)

① $x^2+11x+28 = (x+\square)(x+\square)$
$= \boxed{(x+4)(x+7)}$

Factors of 28	Sum
1, 28	29 x
2, 14	16 x
4, 7	11 ✓

② $x^2-10x+24 = (x-\square)(x-\square)$
$= \boxed{(x-4)(x-6)}$

Factors of 24	Sum
1, 24	25 x
2, 12	14 x
3, 8	11 x
4, 6	10 ✓

③ $x^2+2x-15 = (x+\square)(x-\square)$
$= \boxed{(x+5)(x-3)}$

$x^2-2x-15 = (x+\square)(x-\square)$
$= \boxed{(x+3)(x-5)}$

Factors of -15	Sum
1, -15	-14
-1, 15	14
3, -5	-2
-3, 5	2

Factoring (ax^2+bx+c)

① $5x^2+11x+2 = (5x+\square)(x+\square)$

$= (5x+2)(x+1)$ or $\boxed{(5x+1)(x+2)}$

② $3x^2+4x-15 = (3x+\square)(x-\square)$ or $(3x-\square)(x+\square)$

$\downarrow$

$\boxed{(3x-5)(x+3)}$

③ $18x^2-33x+12 = 3(6x^2-11x+4)$

$= 3(6x-\square)(x-\square)$ or $3(3x-\square)(2x-\square)$

$\downarrow$

$\boxed{3(3x-4)(2x-1)}$

Factoring Special Cases

Factoring Perfect-Square Trinomials

$a^2+2ab+b^2=(a+b)(a+b)=(a+b)^2$

$a^2-2ab+b^2=(a-b)(a-b)=(a-b)^2$

Examples:

$x^2+8x+16=(x+4)^2$

$4n^2-12n+9=(2n-3)^2$

Factoring Difference of Squares

$a^2-b^2=(a+b)(a-b)$

Examples:

$x^2-64=(x+8)(x-8)$

$25y^2-36=(5y+6)(5y-6)$

Factor each polynomial.

① $x^2-14x+49$

$(x-7)(x-7)$

$(x-7)^2$

② $12t^2-48$

$12(t^2-4)$

$12(t+2)(t-2)$

Factoring by Grouping

Factor each expression.

① $3n^3 - 12n^2 + 2n - 8 = 3n^2(n-4) + 2(n-4)$

$= (3n^2+2)(n-4)$

② $8t^3 + 14t^2 + 20t + 35 = 2t^2(4t+7) + 5(4t+7)$

$= (2t^2+5)(4t+7)$

③ $4q^4 - 8q^3 + 12q^2 - 24q = 4q(q^3 - 2q^2 + 3q - 6)$

$= 4q[q^2(q-2) + 3(q-2)]$

$= \boxed{4q(q^2+3)(q-2)}$

④ $6h^4 + 9h^3 + 12h^2 + 18h = 3h(2h^3 + 3h^2 + 4h + 6)$

$= 3h[h^2(2h+3) + 2(2h+3)]$

$= \boxed{3h(h^2+2)(2h+3)}$

Quadratic Graphs and Their Properties

Standard Form of a Quadratic Function

$y = ax^2 + bx + c, \ a \neq 0$

Examples: $y = 3x^2$ $\quad y = x^2 + 4$ $\quad y = 2x^2 - 3x + 1$

Quadratic Parent Function: $f(x) = x^2$ or $y = x^2$

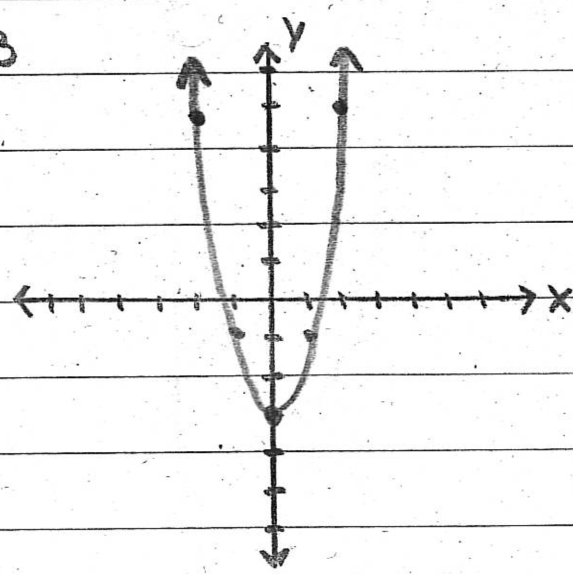
Simplest quadratic function

Parabola: U-shaped curve of a quadratic function (parabolas will have an axis of symmetry)

Graph the function. Then identify the domain and range.

$y = 2x^2 - 3$

x	y
0	-3
1	-1
-1	-1
2	5
-2	5



Domain: All Real Numbers

Range: $y \geq -3$

Quadratic Functions

Graph of a Quadratic Function

$y = ax^2 + bx + c, \ a \neq 0$

Axis of Symmetry $\rightarrow$ Line $\boxed{x = \frac{-b}{2a}}$

The x-coordinate of the vertex is $\frac{-b}{2a}$

Graph $y = x^2 - 6x + 4$

$x = \frac{-b}{2a}$

$x = \frac{-(-6)}{2(1)}$

$x = 3$

Vertex (x, y)

$(3, y)$

$(3, -5)$

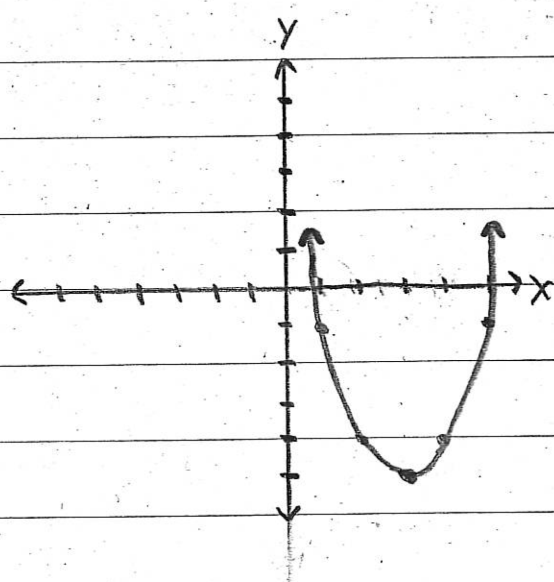
To find y, substitute 3 in for x

$y = x^2 - 6x + 4$

$y = 3^2 - 6(3) + 4$

$y = 9 - 18 + 4$

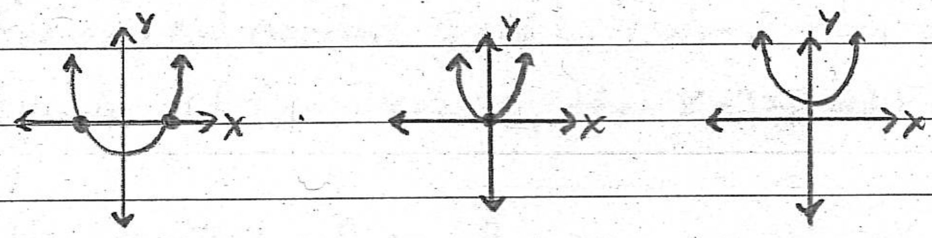
$y = -5$



X	Y
3	-5
2	-4
4	-4
1	-1
5	-1

Solving Quadratic Equations

Roots of the Equation or Zeros of the Function



Two Solutions — One Solution — No Real Number Solutions

Solve each equation by finding square roots.

① $3x^2 - 75 = 0$

$ +75 \quad +75$

$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{75}{3}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

② $2x^2 + 8 = -64$

$ -8 \quad -8$

$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{-72}{2}$

$\sqrt{x^2} = \sqrt{-36}$

No Real Number Solutions

Factoring to Solve Quadratic Equations

Zero-Product Property

For any real numbers a and b, if $ab=0$, then $a=0$ or $b=0$

Example: If $(x+3)(x+2)=0$, then $x+3=0$ and $x+2=0$

Use the Zero-Product Property to solve.

① $(x+8)(x-3)=0$

$x+8=0$ $\quad$ $x-3=0$

$-8 \; -8$ $\quad$ $+3 \; +3$

$x=-8$ and $x=3$

② $-4m(3m+4)=0$

$\frac{-4m}{-4}=\frac{0}{-4}$ $\quad$ $3m+4=0$

$-4 \; -4$

$\frac{3m}{3}=\frac{-4}{3}$

$m=0$ and $m=-\frac{4}{3}$

Solve by factoring.

③ $x^2-5x-14=0$

$(x+2)(x-7)=0$

$x+2=0$ $\quad$ $x-7=0$

$-2 \; -2$ $\quad$ $+7 \; +7$

$x=-2$ or $x=7$

④ $y^2+14y=-49$

$+49 \quad +49$

$y^2+14y+49=0$

$(y+7)(y+7)=0$

$y+7=0$

$-7 \; -7$

$y=-7$

⑤ $5b^2=20b$

$-20b \quad -20b$

$5b^2-20b=0$

$5b(b-4)=0$

$\frac{5b}{5}=\frac{0}{5}$ $\quad$ $b-4=0$

$+4 \; +4$

$b=0$ and $b=4$

Completing the Square

Find c to complete the square. $\boxed{c=\left(\frac{b}{2}\right)^2}$

① $x^2-16x+c$

$c=\left(\frac{b}{2}\right)^2=\left(\frac{-16}{2}\right)^2=(-8)^2=64$

$c=64$

What are the solutions of the equation?

② $x^2-14x+16=0$

$\quad\quad -16 \quad -16$

$x^2-14x\boxed{+49}=-16\boxed{+49}$

$\sqrt{(x-7)^2}=\sqrt{33}$

$x-7=\pm\sqrt{33}$

$x-7=\sqrt{33}$ $\quad$ $x-7=-\sqrt{33}$

$+7 \quad +7$ $\quad$ $+7 \quad +7$

$x=12.74$ $\quad$ $x=1.26$

$c=\left(\frac{b}{2}\right)^2$

$c=\left(\frac{-14}{2}\right)^2$

$c=(-7)^2$

$c=49$

Find the vertex by completing the square.

③ $y=x^2+6x+8$

$-8 \quad\quad -8$

$y-8\boxed{+9}=x^2+6x\boxed{+9}$

$y+1=(x+3)^2$

$-1 \quad\quad -1$

$y=(x+3)^2-1$

Vertex: $\boxed{(-3,-1)}$

$c=\left(\frac{b}{2}\right)^2$

$c=\left(\frac{6}{2}\right)^2$

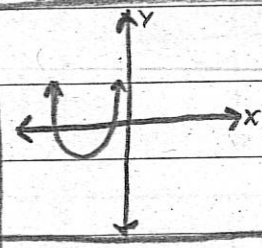
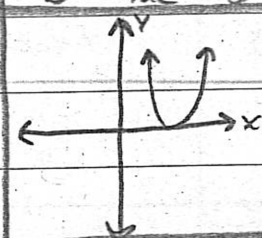
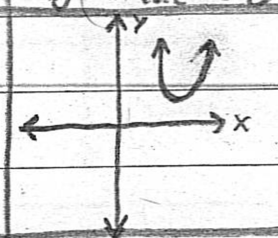
$c=3^2$

$c=9$

The Quadratic Formula and the Discriminant

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac

Discriminant	$b^2-4ac > 0$	$b^2-4ac = 0$	$b^2-4ac < 0$
Example Graph			
Number of Solutions	2 Solutions	1 Solution	No Real Number Solutions

What are the solutions?

Ex. $x^2-4x=21$

$\quad\;\; -21 \quad -21$

$x^2-4x-21=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{4 \pm \sqrt{16-(-84)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{100}}{2}$

$x = \dfrac{4 \pm 10}{2}$

$x = \dfrac{4+10}{2} = \dfrac{14}{2} = 7$

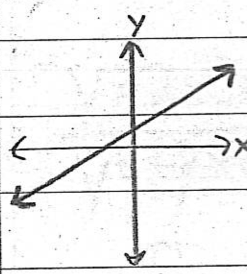
$x = \dfrac{4-10}{2} = \dfrac{-6}{2} = -3$

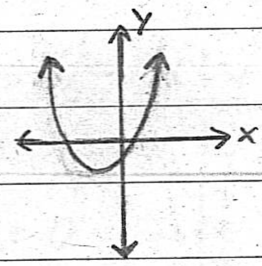
$\boxed{\begin{array}{l} x=7 \\ x=-3 \end{array}}$

Linear, Quadratic, and Exponential Models

Linear	Quadratic	Exponential
$y=mx+b$	$y=ax^2+bx+c$	$y=a\cdot b^x$

Graph Examples





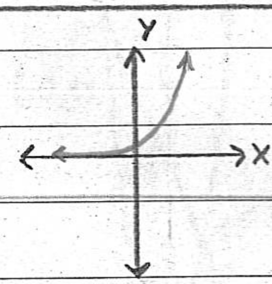


Table Examples

X	Y	
−2	−1	
−1	2	+3
0	5	+3
1	8	+3

X	Y		
−1	1		
0	−1	−2	
1	1	+2	+4
2	7	+6	+4

X	Y	
−2	.25	
−1	.5	×2
0	1	×2
1	2	×2

Systems of Linear and Quadratic Equations

What are the solutions of the system?

① Solve by graphing.

$y = 2x^2 + 1$

$y = -2x + 5$

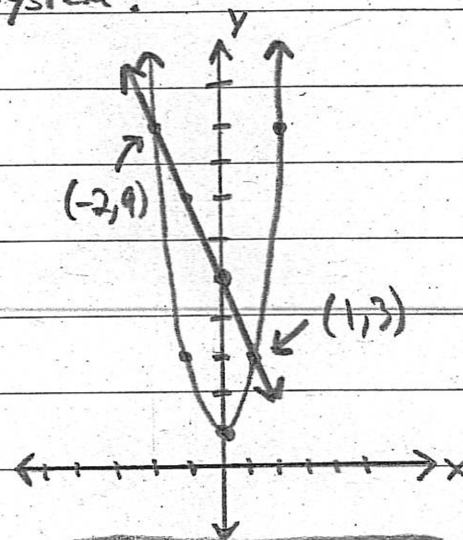
↑ m ↑ y-Int

$x = \frac{-b}{2a}$

$x = \frac{-0}{2(2)}$

$x = 0$

X	Y
0	1
-1	3
1	3
-2	9
2	9



Solution: (-2,9) and (1,3)

② Solve by substitution.

$y = 2x^2 + 1$

$y = -2x + 5$

$2x^2 + 1 = -2x + 5$

$\quad +2x \quad +2x$

$2x^2 + 2x + 1 = 5$

$\quad -5 \quad -5$

$\frac{2x^2 + 2x - 4}{2} = \frac{0}{2}$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, x = 1$

$y = -2x + 5$

$y = -2(-2) + 5$

$y = 4 + 5$

$y = 9$ (-2,9)

$y = -2x + 5$

$y = -2(1) + 5$

$y = -2 + 5$

$y = 3$ (1,3)

Solution: (-2,9), (1,3)

③ Solve by elimination.

$(y = 2x^2 + 1)(-1)$

$y = -2x + 5$

$-y = -2x^2 - 1$

$\frac{0}{-2} = \frac{-2x^2 - 2x + 4}{-2}$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2, x = 1$

$y = -2x + 5$

$y = -2(-2) + 5$

$y = 9$ (-2,9)

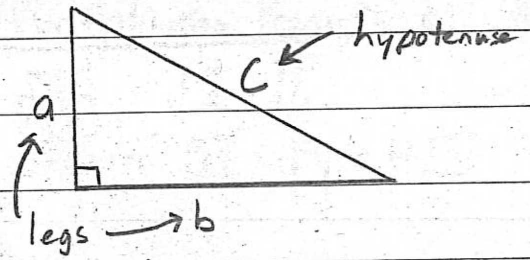
$y = -2x + 5$

$y = -2(1) + 5$

$y = 3$ (1,3)

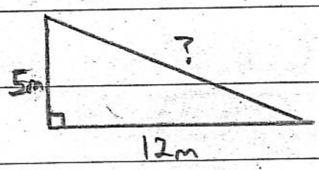
Solution: (-2,9), (1,3)

The Pythagorean Theorem



$c^2 = a^2 + b^2$

Find the missing side length.

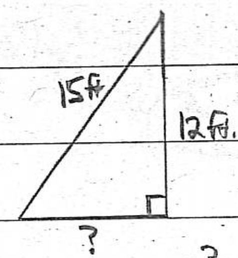
① 

$c^2 = a^2 + b^2$

$c^2 = 5^2 + 12^2$

$\sqrt{c^2} = \sqrt{169}$

$c = 13m$

② 

$c^2 = a^2 + b^2$

$15^2 = a^2 + 12^2$

$225 = a^2 + 144$

$-144 \quad -144$

$\sqrt{81} = \sqrt{a^2}$

$9ft = a$

The Converse of the Pythagorean Theorem

If $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Determine if the side lengths result in a right triangle.

③ 4, 8, 10

$10^2 \overset{?}{=} 4^2 + 8^2$

$100 \overset{?}{=} 16 + 64$

$100 \neq 80$ No

④ 10, 24, 26

$26^2 \overset{?}{=} 10^2 + 24^2$

$676 \overset{?}{=} 100 + 576$

$676 = 676$ Yes, it is a right triangle.

Simplifying Radicals

Multiplication Property of Square Roots

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

Simplify each radical expression.

① $\sqrt{72} = \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{2}$
$= 3 \cdot 2 \cdot \sqrt{2}$
$= 6\sqrt{2}$

② $\sqrt{54y^9} = \sqrt{9y^8} \cdot \sqrt{6y}$
$= 3y^4\sqrt{6y}$

③ $2\sqrt{7b} \cdot 3\sqrt{14b^2} = 6\sqrt{98b^3}$
$= 6 \cdot \sqrt{49b^2} \cdot \sqrt{2b}$
$= 6 \cdot 7b \cdot \sqrt{2b}$
$= 42b\sqrt{2b}$

④ $\sqrt{\frac{8x^3}{50x}} = \sqrt{\frac{4x^2}{25}}$
$= \frac{\sqrt{4x^2}}{\sqrt{25}} = \frac{2x}{5}$

Division Property of Square Roots

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$

Rationalize the denominator.

⑤ $\frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$

Operations with Radical Expressions

What is the simplified form of each expression?

① $\sqrt{5} - 8\sqrt{5}$

$-7\sqrt{5}$

② $4\sqrt{7} + 2\sqrt{28}$

$4\sqrt{7} + 2\sqrt{4}\cdot\sqrt{7}$

$4\sqrt{7} + 4\sqrt{7}$

$8\sqrt{7}$

③ $(\sqrt{6} - 2\sqrt{3})(4\sqrt{3} + 3\sqrt{6}) \longleftarrow$ FOIL

$4\sqrt{18} + 3\sqrt{36} - 8\sqrt{9} - 6\sqrt{18}$

$4\cdot\sqrt{9}\cdot\sqrt{2} + 3\cdot 6 - 8\cdot 3 - 6\sqrt{9}\cdot\sqrt{2}$

$4\cdot 3\cdot\sqrt{2} + 18 - 24 - 6\cdot 3\cdot\sqrt{2}$

$12\sqrt{2} + 18 - 24 - 18\sqrt{2}$

$-6\sqrt{2} - 6$

Rationalize the denominator.

④ $\dfrac{10}{\sqrt{7}-\sqrt{2}} \cdot \dfrac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$ Conjugate

$\dfrac{10(\sqrt{7}+\sqrt{2})}{7-2}$

$\dfrac{10(\sqrt{7}+\sqrt{2})}{5}$

$2(\sqrt{7}+\sqrt{2})$

$2\sqrt{7}+2\sqrt{2}$

Find the solution. Round-Tenths

⑤ $\dfrac{1+\sqrt{5}}{2} = \dfrac{4}{w}$

$\dfrac{w\cancel{(1+\sqrt{5})}}{\cancel{1+\sqrt{5}}} = \dfrac{8}{1+\sqrt{5}} \cdot \dfrac{1-\sqrt{5}}{1-\sqrt{5}}$

$= \dfrac{8(1-\sqrt{5})}{1-5}$

$= \dfrac{8(1-\sqrt{5})}{-4}$

$= -2(1-\sqrt{5})$

$= -2+2\sqrt{5} \approx 2.5$

Solving Radical Equations

Solve each equation.

① $\sqrt{x} - 5 = -2$

$\quad +5 \quad +5$

$\sqrt{x}^2 = 3^2$

$x = 9$

② $\sqrt{7x-4}^2 = \sqrt{5x+10}^2$

$7x - 4 = 5x + 10$

$-5x \quad -5x$

$2x - 4 = 10$

$+4 \quad +4$

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

③ $(-y)^2 = \sqrt{y+6}^2$

$y^2 = y + 6$

$-y-6 \quad -y-6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, \; y = -2$

$-y = \sqrt{y+6}$	$-y = \sqrt{y+6}$
$-3 = \sqrt{3+6}$	$2 = \sqrt{-2+6}$
$-3 = \sqrt{9}$	$2 = \sqrt{4}$
$-3 \neq 3$	$2 = 2$

Extraneous Solution

$y = -2$

④ $\sqrt{3y} + 8 = 2$

$-8 \quad -8$

$\sqrt{3y}^2 = -6^2$

$\frac{3y}{3} = \frac{36}{3}$

$y = 12$

$\sqrt{3y} + 8 = 2$

$\sqrt{3 \cdot 12} + 8 = 2$

$\sqrt{36} + 8 = 2$

$6 + 8 = 2$

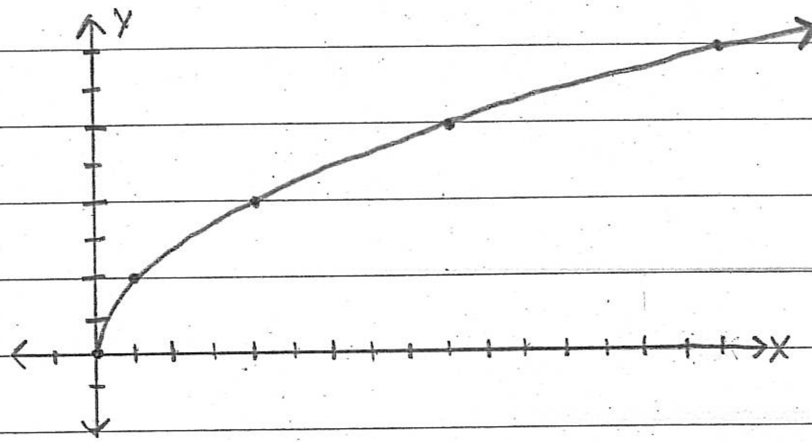
$14 \neq 2$

No Solution

Graphing Square Root Functions

Make a table of values and graph $y = 2\sqrt{x}$

X	Y
0	0
1	2
4	4
9	6
16	8



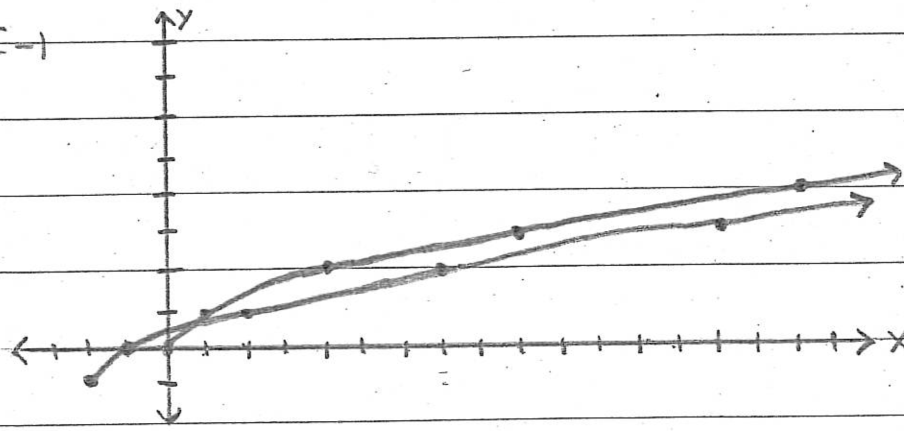
Graph $y = \sqrt{x}$ and $y = \sqrt{x+2} - 1$.

$y = \sqrt{x}$

X	Y
0	0
1	1
4	2
9	3
16	4

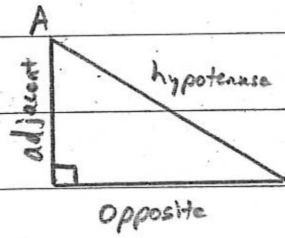
$y = \sqrt{x+2} - 1$

X	Y
-2	-1
-1	0
2	1
7	2
14	3



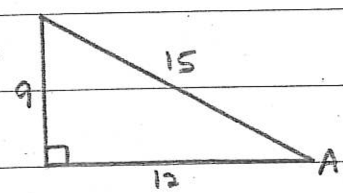
Trigonometric Ratios

Name	Written	Definition
Sine of $\angle A$	$\sin A$	$\frac{\text{opposite}}{\text{hypotenuse}}$
Cosine of $\angle A$	$\cos A$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent of $\angle A$	$\tan A$	$\frac{\text{opposite}}{\text{adjacent}}$



SOHCAHTOA

What are $\sin A$, $\cos A$, and $\tan A$ for the triangle?

① 

$\sin A = \frac{9}{15}$ $\tan A = \frac{9}{12}$

$\cos A = \frac{12}{15}$

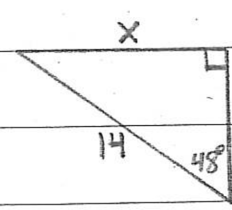
Find the value of each expression.
Round - ten-thousandth

② $\sin 23° = .3907$

③ $\cos 84° = .1045$

④ $\tan 56° = 1.4826$

Find the value of x to the nearest tenth.



$\sin 48° = \frac{x}{14}$

$x = 14(\sin 48°)$

$x \approx 10.4$