

## Division Properties of Exponents

$$\frac{a^m}{a^n} = a^{m-n} \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

What is the simplified form of each expression?

$$\textcircled{1} \frac{x^5}{x^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = x^{5-2} = \boxed{x^3} \quad \textcircled{2} \frac{y^{\frac{3}{4}}}{y^{\frac{1}{2}}} = y^{\frac{3}{4} - \frac{1}{2}} = \boxed{y^{\frac{1}{4}}}$$

$$\textcircled{3} \frac{k^6 j^2}{k j^5} = k^{6-1} j^{2-5} = k^5 j^{-3} = \boxed{\frac{k^5}{j^3}} \quad \textcircled{4} \frac{x^4 y^{-1} z^8}{x^4 y^{-5} z} = \frac{\cancel{x^4} y^{-1} z^8}{\cancel{x^4} y^{-5} z} = \boxed{y^4 z^7}$$

⑤ What is  $1.21 \times 10^7 \div 4.81 \times 10^5$  in standard notation?

$$\frac{1.21 \times 10^7}{4.81 \times 10^5} = \frac{1.21}{4.81} \times 10^{7-5} = .252 \times 10^2 = \boxed{25.2}$$

What is the simplified form?

$$\textcircled{6} \left(\frac{z^{\frac{2}{3}}}{5}\right)^3 = \boxed{\frac{z^2}{125}} \quad \textcircled{7} \left(\frac{2x^6}{y^4}\right)^{-3} = \frac{(y^4)^3}{(2x^6)^3} = \boxed{\frac{y^{12}}{8x^{18}}}$$

# Rational Exponents and Radicals

$$a^{\frac{1}{n}} = \sqrt[n]{a} \text{ and } a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

What is the simplified form of each expression?

$$\begin{aligned} \textcircled{1} \sqrt[3]{64} &= \sqrt[3]{4 \cdot 4 \cdot 4} \\ &= (4 \cdot 4 \cdot 4)^{\frac{1}{3}} \\ &= \boxed{4} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sqrt[5]{243} &= \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} \\ &= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)^{\frac{1}{5}} \\ &= \boxed{3} \end{aligned}$$

Write each expression in radical form.

$$\textcircled{3} 5x^{\frac{1}{5}} = \boxed{5\sqrt[5]{x}}$$

$$\begin{aligned} \textcircled{4} (54y)^{\frac{2}{3}} &= (27^{\frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= (3^{3 \cdot \frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= (3^2)(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= 9\sqrt[3]{2^2 \cdot y^2} = \boxed{9\sqrt[3]{4y^2}} \end{aligned}$$

Write each expression in exponential form.

$$\textcircled{5} \sqrt[3]{d^2} = \boxed{d^{\frac{2}{3}}}$$

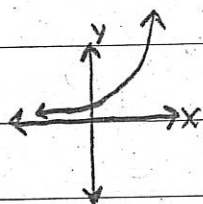
$$\begin{aligned} \textcircled{6} \sqrt{(4y)^5} &= (4y)^{\frac{5}{2}} \\ &= 2^{2 \cdot \frac{5}{2}} y^{\frac{5}{2}} \\ &= 2^5 y^{\frac{5}{2}} \\ &= \boxed{32y^{\frac{5}{2}}} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \sqrt[4]{(32m)^3} &= (32^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= (16^{\frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= (2^{4 \cdot \frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= 2^3 \cdot 2^{\frac{3}{4}} \cdot m^{\frac{3}{4}} \\ &= \boxed{8(2m)^{\frac{3}{4}}} \end{aligned}$$

# Exponential Functions

$y = a \cdot b^x$ , where  $a \neq 0$ ,  $b > 0$ ,  $b \neq 1$ ,  $x$  is a real number

Examples:



x	1	2	3	4
y	3	12	48	192

Arrows above the table indicate multiplication by 4 from x=1 to 2, 2 to 3, and 3 to 4. Arrows below the table indicate multiplication by 4 from y=3 to 12, 12 to 48, and 48 to 192.

Evaluate each function for the given value.

①  $f(x) = 3 \cdot 2^x$ , where  $x=5$

$$= 3 \cdot 2^5$$

$$= 3 \cdot 32$$

$$= \boxed{64}$$

②  $y = 18(.015^x)$ ,  $x=3$

$$y = 18(.015^3)$$

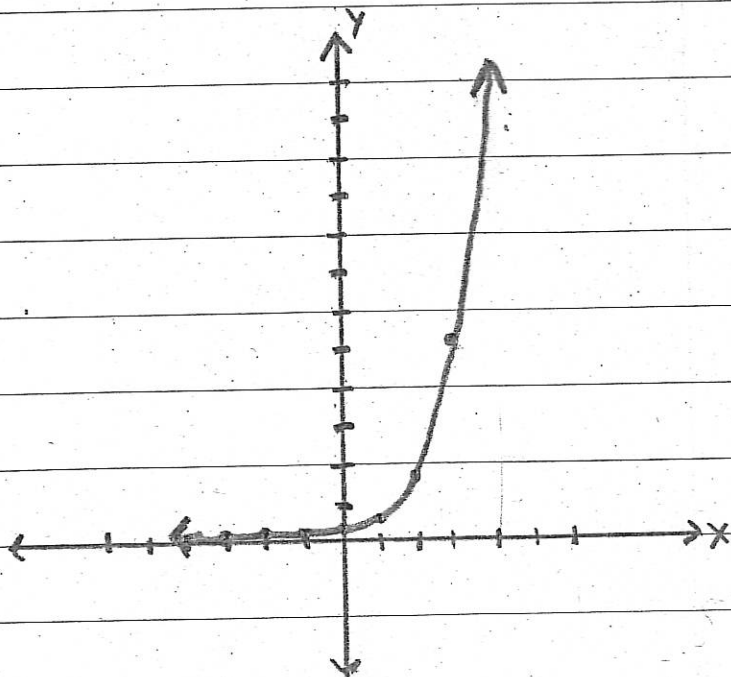
$$y = 18(.00003375)$$

$$y = .00006075$$

$$y = \boxed{6.075 \times 10^{-5}}$$

Graph  $y = .2(3^x)$

X	Y	X	Y
-3	.0074	2	1.8
-2	.0222	3	5.4
-1	.0667	4	16.2
0	.2	5	48.6
1	.6	6	145.8



# Exponential Growth and Decay

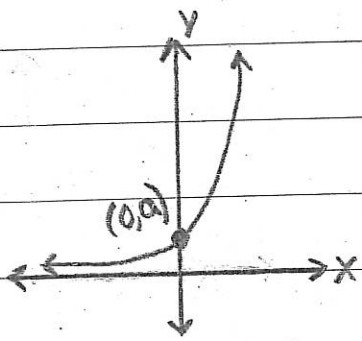
## Exponential Growth

$$y = a \cdot b^x$$

initial amount ( $x=0$ )

base ( $>1$ )

$b$  = growth factor



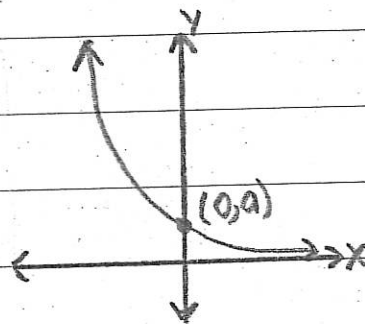
If increase by 6%,  
then  $b = 1.06$

## Exponential Decay

$$y = a \cdot b^x$$

base ( $<1$ )

$b$  = decay factor



If decrease by 12%,  
then  $b = .88$

## Compound Interest

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$A$  = the balance

$P$  = principal

$r$  = rate (decimal)

$n$  = number of times interest is

compounded per year

$t$  = time in years

# Adding and Subtracting Polynomials

## Standard Form of Polynomials

Ex.  $3x^5 + 2x^3 - 8x^2 - 4x + 8$

Simplify. Write each answer in standard form.

①  $m^2 - 8m^2$       ②  $13b^4c + 8b^4c$       ③  $7n^5 - 12n^5$   
 $(-7m^2)$                        $(21b^4c)$                        $(-5n^5)$

④  $(3x^2 - 2x + 8) + (2x^2 - 7x - 18)$   
 $(3x^2 + 2x^2) + (-2x - 7x) + (8 - 18)$   
 $(5x^2 - 9x - 10)$

⑤  $(-8a^2 - 6a + 12) - (3a^2 + 8a + 4)$   
 $\underline{-8a^2} - \underline{6a} + \underline{12} - \underline{3a^2} - \underline{8a} - \underline{4}$   
 $(-11a^2 - 14a + 8)$

# Multiplying and Factoring Polynomials

## Multiplying a Monomial and a Trinomial

$$\textcircled{1} -x^3(8x^2 - 4x + 7)$$
$$\underline{-8x^5 + 4x^4 - 7x^3}$$

## Finding the GCF (Greatest Common Factor)

$$\textcircled{2} 25x^4 + 45x^3 + 15x^2$$

$$25x^4 = 5 \cdot 5 \cdot x \cdot x \cdot x \cdot x$$

$$45x^3 = 3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x$$

$$15x^2 = 3 \cdot 5 \cdot x \cdot x$$

$$\text{GCF} = 5 \cdot x \cdot x = \boxed{5x^2}$$

## Factoring Out a Monomial

$$\textcircled{3} 9x^6 + 15x^4 - 12x^2$$

$$\text{GCF} = \boxed{3x^2}$$

$$\underline{3x^2(3x^4 + 5x^2 - 4)}$$

# Multiplying Binomials (FOIL)

Simplify each product using FOIL.

$$\begin{aligned} \textcircled{1} \quad (3x-4)(x+2) &= \overset{\text{First}}{(3x)}(\overset{\text{First}}{x}) + \overset{\text{Outer}}{(3x)}(\overset{\text{Outer}}{2}) + \overset{\text{Inner}}{(-4)}(\overset{\text{Inner}}{x}) + \overset{\text{Last}}{(-4)}(\overset{\text{Last}}{2}) \\ &= 3x^2 + 6x - 4x - 8 \\ &= \boxed{3x^2 + 2x - 8} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad (2n-8)(5n-6) &= (2n)(5n) + (2n)(-6) + (-8)(5n) + (-8)(-6) \\ &= 10n^2 - 12n - 40n + 48 \\ &= \boxed{10n^2 - 52n + 48} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad (3y^2+4)(2y-5) &= (3y^2)(2y) + (3y^2)(-5) + (4)(2y) + (4)(-5) \\ &= \boxed{6y^3 - 15y^2 + 8y - 20} \end{aligned}$$

## Squaring Binomials and the Product of a Sum and Difference

### ① The Square of a Binomial

Algebra

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Examples

$$(x+8)^2 = x^2 + 16x + 64$$

$$(x-5)^2 = x^2 - 10x + 25$$

### ② The Product of a Sum and Difference

Algebra

$$(a+b)(a-b) = a^2 - b^2$$

Example

$$(x+7)(x-7) = x^2 - 49$$



# Factoring $(x^2 + bx + c)$

$$\textcircled{1} x^2 + 11x + 28 = (x + \square)(x + \square) \\ = \boxed{(x + 4)(x + 7)}$$

Factors of 28	Sum
1, 28	29 x
2, 14	16 x
4, 7	11 ✓

$$\textcircled{2} x^2 - 10x + 24 = (x - \square)(x - \square) \\ = \boxed{(x - 4)(x - 6)}$$

Factors of 24	Sum
1, 24	25 x
2, 12	14 x
3, 8	11 x
4, 6	10 ✓

$$\textcircled{3} x^2 + 2x - 15 = (x + \square)(x - \square) \\ = \boxed{(x + 5)(x - 3)}$$

Factors of -15	Sum
1, -15	-14
-1, 15	14
3, -5	-2
-3, 5	2

$$x^2 - 2x - 15 = (x + \square)(x - \square) \\ = \boxed{(x + 3)(x - 5)}$$

## Factoring $(ax^2+bx+c)$

$$\begin{aligned} \textcircled{1} \quad 5x^2+11x+2 &= (5x+\square)(x+\square) \\ &= (5x+2)(x+1) \text{ or } \boxed{(5x+1)(x+2)} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 3x^2+4x-15 &= (3x+\square)(x-\square) \text{ or } (3x-\square)(x+\square) \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad \boxed{(3x-5)(x+3)} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 18x^2-33x+12 &= 3(6x^2-11x+4) \\ &= 3(6x-\square)(x-\square) \text{ or } 3(3x-\square)(2x-\square) \\ &\qquad\qquad\qquad \downarrow \\ &\qquad\qquad\qquad \boxed{3(3x-4)(2x-1)} \end{aligned}$$