

Rational Exponents and Radicals

$$a^{\frac{1}{n}} = \sqrt[n]{a} \text{ and } a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

What is the simplified form of each expression?

$$\begin{aligned} \textcircled{1} \sqrt[3]{64} &= \sqrt[3]{4 \cdot 4 \cdot 4} \\ &= (4 \cdot 4 \cdot 4)^{\frac{1}{3}} \\ &= \boxed{4} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sqrt[5]{243} &= \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} \\ &= (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)^{\frac{1}{5}} \\ &= \boxed{3} \end{aligned}$$

Write each expression in radical form.

$$\textcircled{3} 5x^{\frac{1}{3}} = \boxed{5\sqrt[3]{x}}$$

$$\begin{aligned} \textcircled{4} (54y)^{\frac{2}{3}} &= (27^{\frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= (3^{3 \cdot \frac{2}{3}})(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= (3^2)(2^{\frac{2}{3}})(y^{\frac{2}{3}}) \\ &= 9\sqrt[3]{2^2 \cdot y^2} = \boxed{9\sqrt[3]{4y^2}} \end{aligned}$$

Write each expression in exponential form.

$$\textcircled{5} \sqrt[3]{d^2} = \boxed{d^{\frac{2}{3}}}$$

$$\begin{aligned} \textcircled{6} \sqrt{(4y)^5} &= (4y)^{\frac{5}{2}} \\ &= 2^2 \cdot 2^{\frac{5}{2}} y^{\frac{5}{2}} \\ &= 2^5 y^{\frac{5}{2}} \\ &= \boxed{32y^{\frac{5}{2}}} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \sqrt[4]{(32m)^3} &= (32^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= (16^{\frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= (2^{4 \cdot \frac{3}{4}})(2^{\frac{3}{4}})(m^{\frac{3}{4}}) \\ &= 2^3 \cdot 2^{\frac{3}{4}} \cdot m^{\frac{3}{4}} \\ &= \boxed{8(2m)^{\frac{3}{4}}} \end{aligned}$$