

Sometimes solving an inequality gives a statement that is *always* true, such as $4 > 1$. In that case, the solutions are all real numbers. If the statement is *never* true, as is $9 \leq -5$, then the inequality has no solution.

Think

Is there another way to solve this inequality?

Yes. Instead of using the Distributive Property, you can first divide each side by 2.

Think

Without solving, how can you tell that this inequality has no solution?

The variable terms on each side of the inequality are equal, but -5 is not greater than 7.

Problem 5 Inequalities With Special Solutions

A What are the solutions of $10 - 8a \geq 2(5 - 4a)$?

$$10 - 8a \geq 2(5 - 4a)$$

$$10 - 8a \geq 10 - 8a \quad \text{Distributive Property}$$

$$10 - 8a + 8a \geq 10 - 8a + 8a \quad \text{Add } 8a \text{ to each side.}$$

$$10 \geq 10 \quad \text{Simplify.}$$

Since the inequality $10 \geq 10$ is always true, the solutions of $10 - 8a \geq 2(5 - 4a)$ are all real numbers.

B What are the solutions of $6m - 5 > 7m + 7 - m$?


$$6m - 5 > 7m + 7 - m$$

$$6m - 5 > 6m + 7 \quad \text{Simplify.}$$

$$6m - 5 - 6m > 6m + 7 - 6m \quad \text{Subtract } 6m \text{ from each side.}$$

$$-5 > 7 \quad \text{Simplify.}$$

Since the inequality $-5 > 7$ is never true, the inequality $6m - 5 > 7m + 7 - m$ has no solution.

 **Got It?** 5. What are the solutions of each inequality?

a. $9 + 5n \leq 5n - 1$

b. $8 + 6x \geq 7x + 2 - x$

Lesson Check

Do you know HOW?

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

1. $7 + 6a > 19$

2. $2(t + 2) - 3t \geq -1$


3. $6z - 15 < 4z + 11$


4. $18x - 5 \leq 3(6x - 2)$


5. The perimeter of a rectangle is at most 24 cm. Two opposite sides are both 4 cm long. What are the possible lengths of the other two sides?

Do you UNDERSTAND?



 6. **Reasoning** How can you tell that the inequality $3t + 1 > 3t + 2$ has no solution just by looking at the terms in the inequality?

 7. **Reasoning** Can you solve the inequality $2(x - 3) \leq 10$ *without* using the Distributive Property? Explain.

 8. **Error Analysis** Your friend says that the solutions of the inequality $-2(3 - x) > 2x - 6$ are all real numbers. Do you agree with your friend? Explain. What if the inequality symbol were \geq ?



Practice and Problem-Solving Exercises



A Practice

Solve each inequality. Check your solutions.

← See Problem 1.

9. $5f + 7 \leq 22$

10. $6n - 3 > -18$

11. $-5y - 2 < 8$

12. $6 - 3p \geq -9$

13. $9 \leq -12 + 6r$

14. $6 \leq 12 + 4j$

Write and solve an inequality.

← See Problem 2.

15. **Family Trip** On a trip from Buffalo, New York, to St. Augustine, Florida, a family wants to travel at least 250 mi in the first 5 h of driving. What should their average speed be in order to meet this goal?

16. **Geometry** An isosceles triangle has at least two congruent sides. The perimeter of a certain isosceles triangle is at most 12 in. The length of each of the two congruent sides is 5 in. What are the possible lengths of the remaining side?

Solve each inequality.

← See Problems 3 and 4.

17. $3(k - 5) + 9k \geq -3$

18. $-(7c - 18) - 2c > 0$

19. $-3(j + 3) + 9j < -15$

20. $-4 \leq 4(6y - 12) - 2y$

21. $30 > -(5z + 15) + 10z$

22. $-4(d + 5) - 3d > 8$

23. $4x + 3 < 3x + 6$

24. $4v + 8 \geq 6v + 10$

25. $5f + 8 \geq 2 + 6f$

26. $6 - 3p \leq 4 - p$

27. $3m - 4 \leq 6m + 11$

28. $4t + 17 > 7 + 5t$

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

← See Problem 5.

29. $-3(w - 3) \geq 9 - 3w$

30. $-5r + 6 \leq -5(r + 2)$

31. $-2(6 + s) \geq -15 - 2s$

32. $9 + 2x < 7 + 2(x - 3)$

33. $2(n - 8) < 16 + 2n$

34. $6w - 4 \leq 2(3w + 6)$

B Apply

Solve each inequality, if possible. If the inequality has no solution, write *no solution*. If the solutions are all real numbers, write *all real numbers*.

35. $-3(x - 3) \geq 5 - 4x$

36. $3s + 6 \leq -5(s + 2)$

37. $3(2 + t) \geq 15 - 2t$

38. $\frac{4}{3}s - 3 < s + \frac{2}{3} - \frac{1}{3}s$

39. $4 - 2n \leq 5 - n + 1$

40. $-2(0.5 - 4t) \geq -3(4 - 3.5t)$

41. $4(a - 2) - 6a \leq -9$

42. $4(3n - 1) \geq 2(n + 3)$

43. $17 - (4k - 2) \geq 2(k + 3)$

44. **Think About a Plan** Your cell phone plan costs \$39.99 per month plus \$.15 for each text message you send or receive. You have at most \$45 to spend on your cell phone bill. What is the maximum number of text messages that you can send or receive next month?

- What information do you know? What information do you need?
- What inequality can you use to find the maximum number of text messages that you can send or receive?
- What are the solutions of the inequality? Are they reasonable?