A function can be thought of as a rule that you apply to the input in order to get the output. You can describe a nonlinear function with words or with an equation, just as you did with linear functions.

Problem 3 Writing a Rule to Describe a Nonlinear Function

The ordered pairs $(1, 2)$, $(2, 4)$, $(3, 8)$, $(4, 16)$, and $(5, 32)$ represent a function. What is a rule that represents this function?

Make a table to organize the x- and y-values. For each row, identify rules that produce the given y-value when you substitute the x-value. Look for a pattern in the y-values.

x	y
1	2
2	4
3	8
4	16
5	32

What rule produces 2, given an x-value of 1? The rules $y = 2x$, $y = x + 1$, and $y = 2^x$ work for $(1, 2)$.

$y = x + 1$ does not work for $(2, 4)$. $y = 2x$ works for $(2, 4)$, but not for $(3, 8)$. $y = 2^x$ works for all three pairs.

$8 = 2 \cdot 2 \cdot 2$ and $16 = 2 \cdot 2 \cdot 2 \cdot 2$. The pattern of the y-values matches 2^1, 2^2, 2^3, 2^4, 2^5, or $y = 2^x$.

The function can be represented by the rule $y = 2^x$.

Got It? 3. What is a rule for the function represented by the ordered pairs $(1, 1)$, $(2, 4)$, $(3, 9)$, $(4, 16)$, and $(5, 25)$?

Lesson Check

Do you know HOW?

1. Graph the function represented by the table below. Is the function *linear* or *nonlinear*?

x	0	1	2	3	4
y	12	13	14	15	16

2. The ordered pairs $(0, -2)$, $(1, 1)$, $(2, 4)$, $(3, 7)$, and $(4, 10)$ represent a function. What is a rule that represents this function?
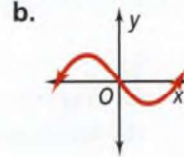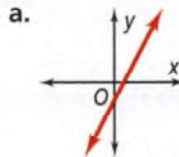
3. Which rule could represent the function shown by the table below?

x	0	1	2	3	4
y	0	-1	-4	-9	-16

A. $y = x^2$ **B.** $y = -x^3$ **C.** $y = -x^2$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary Does the graph represent a *linear function* or a *nonlinear function*? Explain.

a.

b.

5. Error Analysis A classmate says that the function shown by the table at the right can be represented by the rule $y = x + 1$. Describe and correct your classmate's error.

x	y
0	1
1	2
2	5
3	10
4	17

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

The cost C, in dollars, for pencils is a function of the number n of pencils purchased. The length L of a pencil, in inches, is a function of the time t, in seconds, it has been sharpened. Graph the function shown by each table below. Tell whether the function is *linear* or *nonlinear*.

See Problem 1.

6.

Pencil Cost					
Number of Pencils, n	6	12	18	24	30
Cost, C	\$1	\$2	\$3	\$4	\$5

7.

Pencil Sharpening					
Time (s), t	0	3	6	9	12	15
Length (in.), L	7.5	7.5	7.5	7.5	7.4	7.3

Graph the function shown by each table. Tell whether the function is *linear* or *nonlinear*.

8.

x	y
0	5
1	5
2	5
3	5

9.

x	y
0	−4
1	−3
2	0
3	5

10.

x	y
0	0
1	1
2	−5
3	8

11.

x	y
0	0
1	3
2	6
3	9

12. For the diagram below, the table gives the total number of small triangles y in figure number x. What pattern can you use to complete the table? Represent the relationship using words, an equation, and a graph.
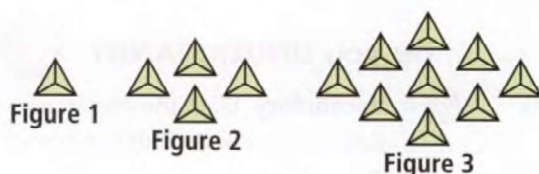
See Problem 2.

Figure 1   Figure 2   Figure 3

Figure Number, x	Total Small Triangles, y	Ordered Pair (x, y)
1	3	(1, 3)
2	12	(2, 12)
3	27	(3, 27)
4	■	■
5	■	■

Each set of ordered pairs represents a function. Write a rule that represents the function.

See Problem 3.

13. $(0, 0), (1, 4), (2, 16), (3, 36), (4, 64)$

14. $\left(1, \frac{2}{3}\right), \left(2, \frac{4}{9}\right), \left(3, \frac{8}{27}\right), \left(4, \frac{16}{81}\right), \left(5, \frac{32}{243}\right)$

15. $(1, 2), (2, 16), (3, 54), (4, 128), (5, 250)$

16. $(0, 0), (1, 0.5), (2, 2), (3, 4.5), (4, 8)$

B Apply

17. **Writing** The rule $V = \frac{4}{3}\pi r^3$ gives the volume V of a sphere as a function of its radius r. Identify the independent and dependent variables in this relationship. Explain your reasoning.

18. **Open-Ended** Write a rule for a nonlinear function such that y is negative when $x = 1$, positive when $x = 2$, negative when $x = 3$, positive when $x = 4$, and so on.