Problem 1 Finding a Break-Even Point

Think

What equations should you write?
The break-even point is when income equals expenses, so write one equation for income and one equation for expenses.

Business A fashion designer makes and sells hats. The material for each hat costs \$5.50. The hats sell for \$12.50 each. The designer spends \$1400 on advertising. How many hats must the designer sell to break even?

Step 1 Write a system of equations. Let $x =$ the number of hats sold, and let $y =$ the number of dollars of expense or income.

Expense: $y = 5.5x + 1400$ Income: $y = 12.5x$

Step 2 Choose a method. Use substitution since both equations are solved for y.

$y = 5.5x + 1400$	Start with one equation.
$12.5x = 5.5x + 1400$	Substitute $12.5x$ for y.
$7x = 1400$	Subtract $5.5x$ from each side.
$x = 200$	Divide each side by 7.

Since x is the number of hats, the designer must sell 200 hats to break even.

Got It? **1.** A puzzle expert wrote a new sudoku puzzle book. His initial costs are \$864. Binding and packaging each book costs \$.80. The price of the book is \$2. How many copies must be sold to break even?

In real-world situations, you need to consider the constraints described in the problem in order to write equations. Once you solve an equation, you need to consider the viability of the solution. For example, a solution that has a negative number of hours is not a viable solution.

Problem 2 Identifying Constraints and Viable Solutions

Think

What are the constraints of the system?
If x represents time, then $x \geq 0$. If y represents the number of gallons, then $y \geq 0$.

Zoo The local zoo is filling two water tanks for the elephant exhibit. One water tank contains 50 gal of water and is filled at a constant rate of 10 gal/h. The second water tank contains 29 gal of water and is filled at a constant rate of 3 gal/h. When will the two tanks have the same amount of water? Explain.

Step 1 Write a system of equations. Let $x =$ the number of hours the tanks are filling and let $y =$ the number of gallons in the tank.

Tank 1: $y = 10x + 50$ Tank 2: $y = 3x + 29$

Step 2 The system is easy to solve using substitution. Substitute $10x + 50$ for y in the second equation and solve for x.

$y = 3x + 29$	Write the second equation.
$10x + 50 = 3x + 29$	Substitute $10x + 50$ for y.
$7x + 50 = 29$	Subtract $3x$ from each side. Then simplify.
$7x = -21$	Subtract 50 from each side. Then simplify.
$x = -3$	Divide each side by 7.

Step 3 Substitute -3 for x in either equation and solve for y.

$y = 10(-3) + 50$	Substitute -3 for x in the first equation.
$y = 20$	Simplify.

The solution to the system is $(-3, 20)$. The solution $(-3, 20)$ is not a viable solution because it is not possible to have time be -3 hours. So, the tanks never have the same amount of water.