Think

Why must like variables be grouped together?
To simplify by adding exponents, the bases must be the same.

Problem 6 Simplifying Expressions With Rational Exponents

Simplify the expression $\left(2a^{\frac{2}{3}} \cdot 3b^{\frac{1}{4}}\right)\left(a^{\frac{1}{3}} \cdot 5b^{\frac{1}{2}}\right)$.

$= (2 \cdot 3 \cdot 5)\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$ — Commutative and associative properties of multiplication

$= 30\left(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}}\right)\left(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}\right)$ — Simplify.

$= 30\left(a^{\frac{3}{3}}\right)\left(b^{\frac{3}{4}}\right)$ — Add exponents that have the same base.

$= 30ab^{\frac{3}{4}}$ — Simplify.

Got It? 6. Simplify each expression.

a. $2c^{\frac{3}{5}} \cdot 2c^{\frac{1}{5}}$

b. $n^{\frac{1}{3}} \cdot n^{\frac{4}{3}}$

c. $\left(b^{\frac{2}{3}} \cdot c^{\frac{2}{5}}\right)\left(b^{\frac{4}{9}} \cdot c^{\frac{9}{10}}\right)$

d. $\left(3j^{\frac{2}{3}} \cdot 7m^{\frac{1}{4}}\right)\left(3j^{\frac{1}{6}} \cdot 7m^{\frac{3}{2}}\right)$

Lesson Check

Do you know HOW?

1. What is $8^4 \cdot 8^8$ written using each base only once?

2. What is the simplified form of $2n^{\frac{2}{3}} \cdot 3n^{\frac{3}{4}}$?

3. What is $\left(3 \times 10^5\right)\left(8 \times 10^4\right)$ written in scientific notation?

4. **Measurement** The diameter of a penny is about 1.9×10^{-5} km. It would take about 2.1×10^9 pennies placed end to end to circle the equator once. What is the approximate length of the equator?

Do you UNDERSTAND? **MATHEMATICAL PRACTICES**

5. **Writing** Can $x^8 \cdot y^3$ be written as a single power? Explain your reasoning.

6. **Reasoning** Suppose $a \times 10^m$ and $b \times 10^n$ are two numbers in scientific notation. Is their product $ab \times 10^{m+n}$ *always*, *sometimes*, or *never* a number in scientific notation? Justify your answer.

7. **Error Analysis** Your friend says $4a^{\frac{1}{2}} \cdot 3a^{\frac{1}{5}} = 7a^{\frac{1}{7}}$. Explain your friend's error. What is the correct answer?

Practice and Problem-Solving Exercises **MATHEMATICAL PRACTICES**

A Practice Rewrite each expression using each base only once. **See Problem 1.**

8. $7^3 \cdot 7^4$

9. $(-6)^{12} \cdot (-6)^5 \cdot (-6)^2$

10. $9^6 \cdot 9^{-4} \cdot 9^{-2}$

11. $2^2 \cdot 2^7 \cdot 2^0$

12. $5^{-2} \cdot 5^{-4} \cdot 5^8$

13. $(-8)^5 \cdot (-8)^{-5}$

Simplify each expression. **See Problem 2.**

14. $m^3 m^4$

15. $5c^4 \cdot c^6$

16. $4t^{-5} \cdot 2t^{-3}$

17. $\left(x^5 y^2\right)\left(x^{-6} y\right)$

18. $\left(5x^5\right)\left(3y^6\right)\left(3x^2\right)$

19. $-m^2 \cdot 4r^3 \cdot 12r^{-4} \cdot 5m$

Write each answer in scientific notation. **See Problem 3.**

STEM 20. **Biology** A human body contains about 2.7×10^4 microliters (μL) of blood for each pound of body weight. Each microliter of blood contains about 7×10^4 white blood cells. About how many white blood cells are in the body of a 140-lb person?

STEM **21. Astronomy** The distance light travels in one second (one light-second) is about 1.86×10^5 mi. Saturn is about 475 light-seconds from the sun. About how many miles from the sun is Saturn?

Simplify each expression. ◀ **See Problem 4.**

22. $8^{\frac{1}{3}}$

23. $625^{\frac{1}{4}}$

24. $1000^{\frac{1}{3}}$

Simplify each expression. ◀ **See Problem 5.**

25. $16^{\frac{3}{4}}$

26. $9^{\frac{5}{2}}$

27. $64^{\frac{7}{3}}$

Simplify each expression. ◀ **See Problem 6.**

28. $\left(8b^{\frac{2}{3}} \cdot 9t^{\frac{1}{5}}\right)\left(8b^{\frac{5}{3}} \cdot 9t^{\frac{3}{5}}\right)$

29. $\left(7d^{\frac{3}{2}} \cdot 2g^{\frac{5}{6}}\right)\left(2g^{\frac{3}{2}} \cdot 7d^{\frac{5}{6}}\right)$

30. $\left(4r^{\frac{2}{5}} \cdot 5s^{\frac{2}{7}}\right)\left(5s^{\frac{5}{7}} \cdot 4r^{\frac{3}{5}}\right)$

B **Apply**

Complete each equation.

31. $5^2 \cdot 5^{\blacksquare} = 5^{11}$

32. $m^{\blacksquare} \cdot m^{-4} = m^{-9}$

33. $2^{\blacksquare} \cdot 2^{\frac{1}{2}} = 2^1$

34. $a^{\blacksquare} \cdot a^4 = 1$

35. $a^{\frac{2}{3}} \cdot a^{\blacksquare} = a^{\frac{5}{6}}$

36. $x^3 y^{\blacksquare} \cdot x^{\blacksquare} = y^2$

© **37. Think About a Plan** A liter of water contains about 3.35×10^{25} molecules. The Mississippi River discharges about 1.7×10^7 L of water every second. About how many molecules does the Mississippi River discharge every minute? Write your answer in scientific notation.
   • How can you use unit analysis to help you find the answer?
   • What properties can you use to make the calculation easier?

38. When you simplify an algebraic expression like $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$, you know that the bases of the expressions must be the same. You also need to rewrite the exponents so that they have a common denominator.
   a. Explain why you need to find the common denominator to simplify.
   b. Simplify the expression $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$.

Simplify each expression. Write each answer in scientific notation.

39. $\left(9 \times 10^7\right)\left(3 \times 10^{-16}\right)$

40. $\left(0.5 \times 10^{-6}\right)\left(0.3 \times 10^{-2}\right)$

41. $\left(0.2 \times 10^5\right)\left(4 \times 10^{-12}\right)$

STEM **42. Chemistry** In chemistry, a *mole* is a unit of measure equal to 6.02×10^{23} atoms of a substance. The mass of a single neon atom is about 3.35×10^{-23} g. What is the mass of 2 moles of neon atoms? Write your answer in scientific notation.

Simplify each expression.

43. $\dfrac{1}{a^4 \cdot a^{-3}}$

44. $8m^{\frac{1}{3}}\left(m^{\frac{1}{3}} + 2\right)$

45. $-4x^3\left(3x^3 - 10x\right)$

© **46. a. Open-Ended** Write y^6 as a product of two powers with the same base in four different ways. Use only positive exponents.
   b. Write y^6 as a product of two powers with the same base in four different ways, using negative or zero exponents in each product.
   c. Reasoning How many ways can you write y^6 as the product of two powers? Explain your reasoning.