Problem 5 Raising a Number in Scientific Notation to a Power STEM

Aircraft The expression $\frac{1}{2}mv^2$ gives the kinetic energy, in joules, of an object with a mass of m kg traveling at a speed of v meters per second. What is the kinetic energy of an experimental unmanned jet with a mass of 1.3×10^3 kg traveling at a speed of about 3.1×10^3 m/s?

Plan

How do you raise a number in scientific notation to a power?
A number written in scientific notation is a product. Use the property for raising a product to a power.

$\frac{1}{2}mv^2 = \frac{1}{2} \cdot (1.3 \times 10^3)(3.1 \times 10^3)^2$ — Substitute the values for m and v into the expression.

$= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot (10^3)^2$ — Raise the two factors to the second power.

$= \frac{1}{2} \cdot 1.3 \cdot 10^3 \cdot 3.1^2 \cdot 10^6$ — Multiply the exponents of a power raised to a power.

$= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^3 \cdot 10^6$ — Use the Commutative Property of Multiplication.

$= \frac{1}{2} \cdot 1.3 \cdot 3.1^2 \cdot 10^{3+6}$ — Add exponents of powers with the same base.

$= 6.2465 \times 10^9$ — Simplify. Write in scientific notation.

The aircraft has a kinetic energy of about 6.2×10^9 joules.

Got It? 5. What is the kinetic energy of an aircraft with a mass of 2.5×10^5 kg traveling at a speed of 3×10^2 m/s?

Lesson Check

Do you know HOW?

Simplify each expression.

1. $(n^3)^6$
2. $(b^{-7})^3$
3. $(3a^{\frac{1}{2}})^4$
4. $(9x^{\frac{1}{2}})^2(x^2)^5$

Simplify each expression. Write each answer in scientific notation.

5. $(4 \times 10^5)^2$
6. $(2 \times 10^{-3})^5$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. **Vocabulary** Compare and contrast the property for raising a power to a power and the property for multiplying powers with the same base.
8. **Error Analysis** One student simplified $x^5 + x^5$ to x^{10}. A second student simplified $x^5 + x^5$ to $2x^5$. Which student is correct? Explain.
9. **Open-Ended** Write four different expressions that are equivalent to $(x^{\frac{2}{3}})^3$.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice Simplify each expression. See Problems 1 and 2.

10. $(n^8)^4$
11. $(n^4)^8$
12. $(c^2)^{\frac{1}{4}}$
13. $(x^{\frac{2}{5}})^{10}$
14. $(w^7)^{-1}$
15. $(x^{\frac{3}{5}})^{-\frac{1}{2}}$
16. $d(d^{-2})^{-9}$
17. $(z^8)^0 z^{\frac{1}{2}}$
18. $(a^{\frac{2}{3}})^3 c^4$
19. $(c^3)^{\frac{1}{9}}(d^3)^0$
20. $(t^2)^{-2}(t^2)^{-5}$
21. $(m^3)^{-1}(x^{\frac{1}{3}})^{\frac{1}{4}}$

Simplify each expression. See Problems 3 and 4.

22. $(3n^{-6})^{-4}$ **23.** $(7a)^{-2}$ **24.** $(5y^{\frac{1}{2}})^4$ **25.** $(36g^4)^{-\frac{1}{2}}$

26. $(2x^{\frac{1}{6}})^3x^2$ **27.** $(2y^{\frac{7}{9}})^{-3}$ **28.** $(r^{\frac{2}{5}}s)^5$ **29.** $(y^2z^{-3})^{\frac{1}{6}}(y^3)^2$

30. $(3b^{-2})^2(a^2b^4)^3$ **31.** $4j^2k^6(2j^{11})^3k^5$ **32.** $(mg^4)^{-1}(mg^4)$ **33.** $(2j^2k^4)^{-5}(k^{-1}j^7)^6$

Simplify. Write each answer in scientific notation. See Problem 5.

34. $(3 \times 10^5)^2$ **35.** $(4 \times 10^2)^5$ **36.** $(2 \times 10^{-10})^3$ **37.** $(2 \times 10^{-3})^3$

38. $(7.4 \times 10^4)^2$ **39.** $(6.25 \times 10^{-12})^{-2}$ **40.** $(3.5 \times 10^{-4})^3$ **41.** $(2.37 \times 10^8)^3$

42. Geometry The radius of a cylinder is 7.8×10^{-4} m. The height of the cylinder is 3.4×10^{-2} m. What is the volume of the cylinder? Write your answer in scientific notation. (*Hint:* $V = \pi r^2 h$)

B Apply

Complete each equation.

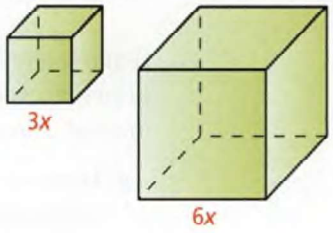
43. $(b^2)^{\blacksquare} = b^8$ **44.** $(m^{\blacksquare})^{\frac{1}{3}} = m^{-12}$ **45.** $(x^{\blacksquare})^7 = x^6$

46. $(n^9)^{\blacksquare} = n$ **47.** $(y^{-4})^{\blacksquare} = y^{\frac{1}{2}}$ **48.** $7(c^1)^{\blacksquare} = 7c^{\frac{2}{3}}$

49. $(5x^{\blacksquare})^2 = 25x^{-4}$ **50.** $(3x^3y^{\blacksquare})^3 = 27x^9$ **51.** $(m^2n^3)^{\blacksquare} = \frac{1}{m^6n^9}$

52. Think About a Plan How many times the volume of the small cube is the volume of the large cube?

- What expression can you write for the volume of the small cube? For the volume of the large cube?
- What property of exponents can you use to simplify the volume expressions?



Simplify each expression.

53. $3^2(3x)^3$ **54.** $(4.1)^5(4.1)^{-5}$ **55.** $(b^{\frac{1}{6}})^3b^{\frac{1}{6}}$

56. $(-5x)^2 + 5x^2$ **57.** $(-2a^{\frac{2}{3}}b)^3(ab^{\frac{1}{3}})^3$ **58.** $(2x^{-3})^2(0.2x)^2$

59. $4xy^20^4(-y)^{-3}$ **60.** $(10^3)^4(4.3 \times 10^{-8})$ **61.** $(3^7)^2(3^{-4})^3$

62. Reasoning Simplify $(x^2)^3$ and x^{2^3}. Are the expressions equivalent? Explain.

63. a. Error Analysis What mistake did the student make in simplifying the expression at the right?

$(2 + 3)^2 = 2^2 + 3^2$
$= 4 + 9$
$= 13$

b. What is the correct simplified form of the expression?

STEM 64. Wind Energy The power generated by a wind turbine depends on the wind speed. The expression $800v^3$ gives the power in watts for a certain wind turbine at wind speed v in meters per second. If the wind speed triples, by what factor does the power generated by the wind turbine increase?

65. Can you write the expression $49x^2y^2z^2$ using only one exponent? Show how or explain why not.