Problem 4 Simplifying an Exponential Expression

What is the simplified form of $\left(\frac{2x^6}{y^4}\right)^{-3}$?

$$\left(\frac{2x^6}{y^4}\right)^{-3} = \left(\frac{y^4}{2x^6}\right)^3 \quad \text{Rewrite using the reciprocal of } \frac{2x^6}{y^4}.$$

$$= \frac{(y^4)^3}{(2x^6)^3} \quad \text{Raise the numerator and denominator to the third power.}$$

$$= \frac{y^{12}}{8x^{18}} \quad \text{Simplify.}$$

Got It? 4. What is the simplified form of $\left(\frac{a}{5b}\right)^{-2}$?

Lesson Check

Do you know HOW?

Simplify each expression.

1. $\dfrac{y^3}{y^{10}}$

2. $\left(\dfrac{x^{\frac{1}{3}}}{3}\right)^3$

3. $\left(\dfrac{m}{n}\right)^{-3}$

4. $\left(\dfrac{3x^2}{5y^4}\right)^{-4}$

5. A large cube is made up of many small cubes. The volume of the large cube is 7.506×10^5 mm^3. The volume of each small cube is 2.78×10^4 mm^3. How many small cubes make up the large cube?

Do you UNDERSTAND?
MATHEMATICAL PRACTICES

6. Vocabulary How is the property for raising a quotient to a power similar to the property for raising a product to a power?

7. a. Reasoning Ross simplifies $\frac{a^3}{a^7}$ as shown at the right. Explain why Ross's method works.

$$\frac{a^3}{a^7} = \frac{1}{a^{7-3}} = \frac{1}{a^4}$$

b. Open-Ended Write a quotient of powers and use Ross's method to simplify it.

Practice and Problem-Solving Exercises
MATHEMATICAL PRACTICES

Ⓐ Practice Copy and complete each equation.

See Problem 1.

8. $\dfrac{5^9}{5^2} = 5^{\blacksquare}$

9. $\dfrac{2^{\frac{7}{3}}}{2^2} = 2^{\blacksquare}$

10. $\dfrac{3^2}{3^5} = 3^{\blacksquare}$

11. $\dfrac{5^2 5^3}{5^3 5^2} = 5^{\blacksquare}$

Simplify each expression.

12. $\dfrac{3^8}{3^6}$

13. $\dfrac{9^{\frac{3}{4}}}{9^{\frac{1}{4}}}$

14. $\dfrac{d^{14}}{d^{17}}$

15. $\dfrac{n^{-1}}{n^{-4}}$

16. $\dfrac{5s^{-7}}{10s^{-9}}$

17. $\dfrac{x^{11}y^3}{x^{11}y}$

18. $\dfrac{c^{\frac{2}{3}}d^{-5}}{c^{\frac{1}{6}}d^{-1}}$

19. $\dfrac{10m^6n^3}{5m^2n^7}$

20. $\dfrac{m^{\frac{2}{3}}n^2}{m^{-1}n^3}$

21. $\dfrac{3^2m^5t^6}{3^5m^7t^{-5}}$

22. $\dfrac{x^5y^{-\frac{9}{2}}z^3}{xy^{-4}z^3}$

23. $\dfrac{12a^{-1}b^6c^{-3}}{4a^5b^{-1}c^5}$

Simplify each quotient. Write each answer in scientific notation.

24. $\dfrac{5.2 \times 10^{13}}{1.3 \times 10^7}$

25. $\dfrac{3.6 \times 10^{-10}}{9 \times 10^{-6}}$

26. $\dfrac{6.5 \times 10^4}{5 \times 10^6}$

27. $\dfrac{8.4 \times 10^{-5}}{2 \times 10^{-8}}$

28. $\dfrac{4.65 \times 10^{-4}}{3.1 \times 10^2}$

29. $\dfrac{3.5 \times 10^6}{5 \times 10^8}$

30. **Computers** The average time it takes a computer to execute one instruction is measured in picoseconds. There are 3.6×10^{15} picoseconds per hour. What fraction of a second is a picosecond?

31. **Wildlife** Data from a deer count in a forested area show that an estimated 3.16×10^3 deer inhabit 7.228×10^4 acres of land. What is the density of the deer population?

STEM 32. **Astronomy** The sun's mass is 1.998×10^{30} kg. Saturn's mass is 5.69×10^{26} kg. How many times as great as the mass of Saturn is the mass of the sun?

Simplify each expression.

33. $\left(\dfrac{3}{8}\right)^2$

34. $\left(\dfrac{1}{a}\right)^3$

35. $\left(\dfrac{3x}{y}\right)^4$

36. $\left(\dfrac{2x}{3y}\right)^5$

37. $\left(\dfrac{6}{5^2}\right)^3$

38. $\left(\dfrac{2^2}{2^3}\right)^5$

39. $\left(\dfrac{8}{n^5}\right)^6$

40. $\left(\dfrac{2p}{9}\right)^3$

41. $\left(\dfrac{2}{5}\right)^{-1}$

42. $\left(\dfrac{5}{4}\right)^{-4}$

43. $\left(-\dfrac{7x^{\frac{3}{2}}}{5y^4}\right)^{-2}$

44. $\left(-\dfrac{2x^{\frac{1}{6}}}{3y^4}\right)^{-3}$

45. $\left(\dfrac{3x^{\frac{1}{2}}}{15}\right)^2$

46. $\left(\dfrac{6n^2}{3n}\right)^{-3}$

47. $\left(\dfrac{b^{\frac{4}{5}}}{b^7}\right)^{-5}$

48. $\left(\dfrac{3}{5c^2}\right)^0$

B **Apply**

Explain why each expression is *not* in simplest form.

49. $5^3 m^3$

50. $x^5 y^{-2}$

51. $(2c)^4$

52. $x^0 y$

53. $\dfrac{d^7}{d}$

54. **Think About a Plan** During one year, about 163 million adults over 18 years old in the United States spent a total of about 93 billion hours online at home. On average, how many hours per day did each adult spend online at home?
   • How do you write each number in scientific notation?
   • How do you convert the units to hours per day?

55. **Television** During one year, people in the United States older than 18 years old watched a total of 342 billion hours of television. The population of the United States older than 18 years old was about 209 million people.
   a. On average, how many hours of television did each person older than 18 years old watch that year? Round to the nearest hour.
   b. On average, how many hours per week did each person older than 18 years old watch that year? Round to the nearest hour.

Which property or properties of exponents would you use to simplify each expression?

56. 2^{-3}

57. $\dfrac{2^2}{2^5}$

58. $\dfrac{1}{2^{-4}2^7}$

59. $\dfrac{(2^{\frac{2}{3}})^3}{2^{15}}$

Simplify each expression.

60. $\dfrac{3n^2(5^0)}{2n^3}$

61. $\left(\dfrac{2m^4}{m^2}\right)^{-4}$

62. $\dfrac{3x^3}{(3x)^3}$

63. $\dfrac{(2a^6)(4a)}{8a^3}$

64. $\left(\dfrac{9t^{\frac{2}{3}}}{36t}\right)^3$

65. $\left(\dfrac{a^4a}{a^2}\right)^{-3}$

66. $\left(\dfrac{2x^2}{5x^3}\right)^{-2}$

67. $\dfrac{4x^{-2}y^4}{8x^3(y^{-2})^3}$

ⓒ 68. a. Open-Ended Write three numbers greater than 1000 in scientific notation.
  b. Divide each number by 2.
  c. Reasoning Is the exponent of the power of 10 divided by 2 when you divide a number in scientific notation by 2? Explain.

69. Simplify the expression $\left(\dfrac{3}{x^2}\right)^{-3}$ in three different ways. Justify each step.
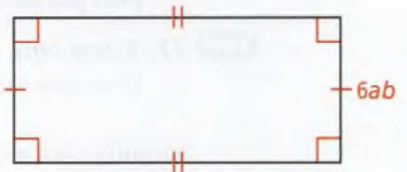
70. Geometry The area of the rectangle is $72a^3b^4$. What is the length of the rectangle?

  Ⓐ $\dfrac{a^3b^4}{12}$

  Ⓒ $\dfrac{12}{a^3b^4}$

  Ⓑ $12a^2b^3$

  Ⓓ $12a^3b^4$

6ab

Simplify each expression.

71. $\left(\dfrac{3xy^5}{x^4y}\right)^{-2}$

72. $\dfrac{m^4n^3p^{-3}}{m^{-2}n^7p^{-8}}$

73. $\dfrac{\left(\frac{1}{4}\right)^{-2}}{\left(\frac{1}{6}\right)^{-3}}$

74. $\dfrac{0.2^3 \cdot 0.2^4}{0.2^7}$

75. $\left(\dfrac{a^{-1}b^3c}{a^2b^4}\right)^6$

76. $\left(\dfrac{(-4)^2}{(-3)^{-3}}\right)^2$

77. $\left(\dfrac{(4x)^2y}{xy^4}\right)^{-2}$

78. $\dfrac{(6a^3)(8b^4)}{(2a^4)(36b^{-1})}$

STEM 79. Physics The wavelength of a radio wave is defined as speed divided by frequency. An FM radio station has a frequency of 9×10^7 waves per second. The speed of the waves is about 3×10^8 meters per second. What is the wavelength of the station?

ⓒ 80. a. Error Analysis What mistake did the student make in simplifying the expression at the right?
  b. What is the correct simplified form of the expression?

$$5^4 \div 5 = \dfrac{5^4}{5}$$
$$= 1^4$$
$$= 1$$

ⓒ 81. Writing Suppose $\dfrac{a^x}{a^y} = a^3$ and $\dfrac{a^x}{a^{3y}} = a^{-5}$. Find the values of x and y. Explain how you found your answer.

82. a. Finance In 2000, the United States government owed about $5.63 trillion to its creditors. The population of the United States was 282.4 million people. How much did the government owe per person in 2000? Round to the nearest dollar.
  b. In 2005, the debt had grown to $7.91 trillion, with a population of 296.9 million. How much did the government owe per person? Round to the nearest dollar.
  c. What was the percent increase in the average amount owed per person from 2000 to 2005?

Write each expression with only one exponent. You may need to use parentheses.

83. $\dfrac{m^7}{n^7}$

84. $\dfrac{10^7 \cdot 10^0}{10^{-3}}$

85. $\dfrac{27x^3}{8y^3}$

86. $\dfrac{4m^2}{169m^4}$