In the Concept Byte after Lesson 6-1, you solved one-variable linear equations using graphs and a graphing calculator. In the next example, you will write each side of the equation as a function and graph the functions. The x-value where the functions intersect is a solution.

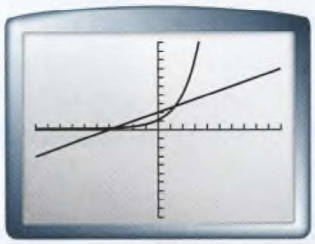
Problem 5 Solving One-Variable Equations

What is the solution or solutions of $2^x = 0.5x + 2$?

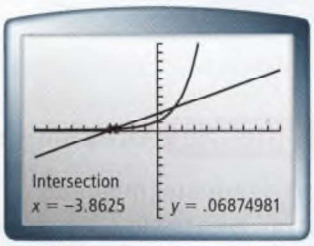
Step 1 Write each side of the equation as a function equation.

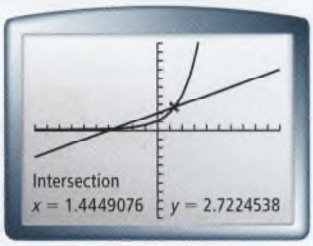
$f(x) = 2^x$ and $g(x) = 0.5x + 2$

Step 2 Graph the equations using a graphing calculator. Use y_1 for $f(x)$ and y_2 for $g(x)$.



Step 3 Use the **CALC** feature. Chose **INTERSECT** to find the points where the lines intersect.





Think

How can you check that the x-value is a solution?
Substitute for x in the original equation. Make sure you use the same x-value for each instance of x.

The solutions of $2^x = 0.5x + 2$ are about -3.86 and 1.45.

Got It? **5.** What is the solution or solutions of each equation?

a. $0.3^x = 5$ **b.** $1.25^x = -2x$ **c.** $-(2^x) = \frac{3}{4}x - 4$

Lesson Check

Do you know HOW?

Evaluate each function for the given value.

1. $f(x) = 6 \cdot 2^x$ for $x = 3$
2. $g(w) = 45 \cdot 3^w$ for $w = -2$

Graph each function.

3. $y = 3^x$
4. $f(x) = 4\left(\frac{1}{2}\right)^x$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** Describe the differences between a linear function and an exponential function.
6. **Reasoning** Is $y = (-2)^x$ an exponential function? Justify your answer.
7. **Error Analysis** A student evaluated the function $f(x) = 3 \cdot 4^x$ for $x = -1$ as shown at the right. Describe and correct the student's mistake.

$$f(-1) = 3 \cdot 4^{-1}$$
$$= 12^{-1}$$
$$= \frac{1}{12}$$

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Determine whether each table or rule represents a linear or an exponential function. Explain why or why not. See Problem 1.

8.

x	1	2	3	4
y	2	8	32	128

9.

x	0	1	2	3
y	6	9	12	15

10. $y = 4 \cdot 5^x$

11. $y = 12 \cdot x$

12. $y = -5 \cdot 0.25^x$

13. $y = 7x + 3$

Evaluate each function for the given value. See Problem 2.

14. $f(x) = 6^x$ for $x = 2$

15. $g(t) = 2 \cdot 0.4^t$ for $t = -2$

16. $y = 20 \cdot 0.5^x$ for $x = 3$

17. $h(w) = -0.5 \cdot 4^w$ for $w = 18$

18. Finance An investment of \$5000 doubles in value every decade. The function $f(x) = 5000 \cdot 2^x$, where x is the number of decades, models the growth of the value of the investment. How much is the investment worth after 30 yr?

19. Wildlife Management A population of 75 foxes in a wildlife preserve quadruples in size every 15 yr. The function $y = 75 \cdot 4^x$, where x is the number of 15-yr periods, models the population growth. How many foxes will there be after 45 yr?

Graph each exponential function. See Problem 3.

20. $y = 4^x$

21. $y = -4^x$

22. $y = \left(\frac{1}{3}\right)^x$

23. $y = -\left(\frac{1}{3}\right)^x$

24. $y = 10 \cdot \left(\frac{3}{2}\right)^x$

25. $y = 0.1 \cdot 2^x$

26. $y = \frac{1}{4} \cdot 2^x$

27. $y = 1.25^x$

28. Admissions A new museum had 7500 visitors this year. The museum curators expect the number of visitors to grow by 5% each year. The function $y = 7500 \cdot 1.05^x$ models the predicted number of visitors each year after x years. Graph the function. See Problem 4.

29. Environment A solid waste disposal plan proposes to reduce the amount of garbage each person throws out by 2% each year. This year, each person threw out an average of 1500 lb of garbage. The function $y = 1500 \cdot 0.98^x$ models the average amount of garbage each person will throw out each year after x years. Graph the function.

What is the solution or solutions of each equation? See Problem 5.

30. $4^x = \frac{3}{2}x + 5$

31. $x + 3 = 3^x$

B Apply

Evaluate each function over the domain $\{-2, -1, 0, 1, 2, 3\}$. As the values of the domain increase, do the values of the range *increase* or *decrease*?

32. $f(x) = 5^x$

33. $y = 2.5^x$

34. $h(x) = 0.1^x$

35. $f(x) = 5 \cdot 4^x$

36. $y = 0.5^x$

37. $y = 8^x$

38. $g(x) = 4 \cdot 10^x$

39. $y = 100 \cdot 0.3^x$

40. Compare the rule and the function table below. Which function has the greater value when $x = 12$? Explain.

Function 1

$y = 4^x$

Function 2

x	1	2	3	4
y	5	25	125	625

41. You have just read a journal article about a population of fungi that doubles every 3 weeks. The beginning population was 10. The function $y = 10 \cdot 2^{\frac{n}{3}}$ represents the population after n weeks.

a. You have a population of 15 of the same fungi. Assuming the journal articles gives the correct rate of increase, write the function that represents the population of fungi after n weeks.

b. Suppose you find another article that states that the fungi population triples every 4 weeks. If there are currently 15 fungi in your population, write the function that represents the population after n weeks.

42. Think About a Plan Hydra are small freshwater animals. They can double in number every two days in a laboratory tank. Suppose one tank has an initial population of 60 hydra. When will there be more than 5000 hydra?

- How can a table help you identify a pattern?
- What function models the situation?

Hydra

43. a. Graph $y = 2^x$, $y = 4^x$, and $y = 0.25^x$ on the same axes.

b. What point is on all three graphs?

c. Does the graph of an exponential function intersect the x-axis? Explain.

d. Reasoning How does the graph of $y = b^x$ change as the base b increases or decreases?

Which function has the greater value for the given value of x?

44. $y = 4^x$ or $y = x^4$ for $x = 2$

45. $f(x) = 10 \cdot 2^x$ or $f(x) = 200 \cdot x^2$ for $x = 7$

46. $y = 3^x$ or $y = x^3$ for $x = 5$

47. $f(x) = 2^x$ or $f(x) = 100x^2$ for $x = 10$

48. Computers A computer valued at \$1500 loses 20% of its value each year.

a. Write a function rule that models the value of the computer.

b. Find the value of the computer after 3 yr.

c. In how many years will the value of the computer be less than \$500?

49. a. Graph the functions $y = x^2$ and $y = 2^x$ on the same axes.

b. What do you notice about the graphs for the values of x between 1 and 3?

c. Reasoning How do you think the graph of $y = 8^x$ would compare to the graphs of $y = x^2$ and $y = 2^x$?

50. Writing Find the range of the function $f(x) = 500 \cdot 1^x$ using the domain $\{1, 2, 3, 4, 5\}$. Explain why the definition of *exponential function* states that $b \neq 1$.