

### Problem 3 Modeling Exponential Decay STEM

**Physics** The kilopascal is a unit of measure for atmospheric pressure. The atmospheric pressure at sea level is about 101 kilopascals. For every 1000-m increase in altitude, the pressure decreases about 11.5%. What is the approximate pressure at an altitude of 3000 m?

**Relate**  $y = a \cdot b^x$  Use an exponential function.



**Define** Let  $x$  = the altitude (in thousands of meters).  
Let  $y$  = the atmospheric pressure (in kilopascals).  
Let  $a$  = the initial pressure (in kilopascals), 101.  
Let  $b$  = the decay factor, which is  $1 - 0.115 = 0.885$ .

**Write**  $y = 101 \cdot 0.885^x$

Use the equation to estimate the pressure at an altitude of 3000 m.

$$\begin{aligned} y &= 101 \cdot 0.885^x \\ &= 101 \cdot 0.885^3 \quad \text{Substitute 3 for } x. \\ &\approx 70 \quad \text{Round to the nearest kilopascal.} \end{aligned}$$

The pressure at an altitude of 3000 m is about 70 kilopascals.

-   **Got It?** 3. a. What is the atmospheric pressure at an altitude of 5000 m?  
b. **Reasoning** Why do you subtract the percent decrease from 1 to find the decay factor?

### Think

**Will the pressure ever be negative?**

No. The range of an exponential decay function is all positive real numbers. The graph of an exponential decay function approaches but does not cross the  $x$ -axis.

### Lesson Check

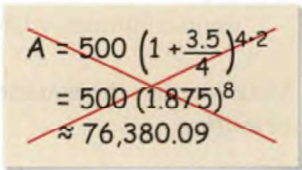
#### Do you know HOW?

1. What is the growth factor in the equation  $y = 34 \cdot 4^x$ ?
2. What is the initial amount in the function  $y = 15 \cdot 3^x$ ?
3. What is the decay factor in the function  $y = 17 \cdot 0.2^x$ ?
4. A population of fish in a lake decreases 6% annually. What is the decay factor?
5. Suppose your friend's parents invest \$20,000 in an account paying 5% interest compounded annually. What will the balance be after 10 yr?

#### Do you UNDERSTAND?



6. **Vocabulary** How can you tell if an exponential function models growth or decay?
7. **Reasoning** How can you simplify the compound interest formula when the interest is compounded annually? Explain.
8. **Error Analysis** A student deposits \$500 into an account that earns 3.5% interest compounded quarterly. Describe and correct the student's error in calculating the account balance after 2 yr.


$$\begin{aligned} A &= 500 \left(1 + \frac{3.5}{4}\right)^{4 \cdot 2} \\ &= 500 (1.875)^8 \\ &\approx 76,380.09 \end{aligned}$$



## Practice and Problem-Solving Exercises



### Practice

Identify the initial amount  $a$  and the growth factor  $b$  in each exponential function.

← See Problem 1.

9.  $g(x) = 14 \cdot 2^x$

10.  $y = 150 \cdot 1.0894^x$

11.  $y = 25,600 \cdot 1.01^x$

12.  $f(t) = 1.4^t$

13. **College Enrollment** The number of students enrolled at a college is 15,000 and grows 4% each year.

a. The initial amount  $a$  is  $\blacksquare$ .

b. The percent rate of change is 4%, so the growth factor  $b$  is  $1 + \blacksquare = \blacksquare$ .

c. To find the number of students enrolled after one year, you calculate  $15,000 \cdot \blacksquare$ .

d. Complete the equation  $y = \blacksquare \cdot \blacksquare$  to find the number of students enrolled after  $x$  years.

e. Use your equation to predict the number of students enrolled after 25 yr.

14. **Population** A population of 100 frogs increases at an annual rate of 22%. How many frogs will there be in 5 years? Write an expression to represent the equivalent monthly population increase rate.

Find the balance in each account after the given period.

← See Problem 2.

15. \$4000 principal earning 6% compounded annually, after 5 yr

16. \$12,000 principal earning 4.8% compounded annually, after 7 yr

17. \$500 principal earning 4% compounded quarterly, after 6 yr

18. \$20,000 deposit earning 3.5% compounded monthly, after 10 yr

19. \$5000 deposit earning 1.5% compounded quarterly, after 3 yr

20. \$13,500 deposit earning 3.3% compounded monthly, after 1 yr

21. \$775 deposit earning 4.25% compounded annually, after 12 yr

22. \$3500 deposit earning 6.75% compounded monthly, after 6 months

Identify the initial amount  $a$  and the decay factor  $b$  in each exponential function.

← See Problem 3.

23.  $y = 5 \cdot 0.5^x$

24.  $f(x) = 10 \cdot 0.1^x$

25.  $g(x) = 100\left(\frac{2}{3}\right)^x$

26.  $y = 0.1 \cdot 0.9^x$

27. **Population** The population of a city is 45,000 and decreases 2% each year. If the trend continues, what will the population be after 15 yr?



### Apply

State whether the equation represents *exponential growth*, *exponential decay*, or *neither*.

28.  $y = 0.93 \cdot 2^x$

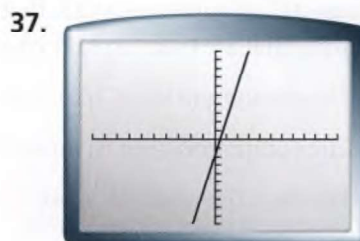
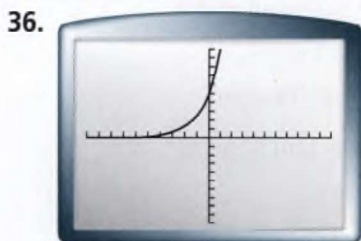
29.  $y = 2 \cdot 0.68^x$

30.  $y = 68 \cdot x^2$

31.  $y = 68 \cdot 0.2^x$

32. **Sports** In a single-elimination tournament starting with 128 teams, half of the remaining teams are eliminated in each round.
- Make a table, a scatter plot, and a function rule to represent the situation.
  - Is it possible for 24 teams to remain after a round? Which representation in part (a) made it the easiest to answer the question?
  - What is the domain of the function? What does the domain represent?
  - How many teams will be left after 5 rounds?
33. **Car Value** A family buys a car for \$20,000. The value of the car decreases about 20% each year. After 6 yr, the family decides to sell the car. Should they sell it for \$4000? Explain.
- © 34. **Think About a Plan** You invest \$100 and expect your money to grow 8% each year. About how many years will it take for your investment to double?
- What function models the growth of your investment?
  - How can you use a table to find the approximate amount of time it takes for your investment to double?
  - How can you use a graph to find the approximate amount of time it takes for your investment to double?
- © 35. **Reasoning** Give an example of an exponential function in the form  $y = a \cdot b^x$  that is neither an exponential growth function nor an exponential decay function. Explain your reasoning.

State whether each graph shows an *exponential growth function*, an *exponential decay function*, or *neither*.



38. Use a table and a scatter plot to answer each question.
- You play a game of musical chairs in which 32 players start and you remove 2 chairs in each round. How many rounds will you play before two players are left?
  - In another game of musical chairs, you take away half of the chairs each time. If the game begins with 32 players, how many rounds will it take to get down to two players?
  - Will a game where you remove half of the chairs always end more quickly than one in which you take the same number of chairs each time? Give an example.
39. **Business** Suppose you start a lawn-mowing business and make a profit of \$400 in the first year. Each year, your profit increases 5%.
- Write a function that models your annual profit.
  - If you continue your business for 10 yr, what will your *total* profit be?