Recall that subtraction means to add the opposite. So when you subtract a polynomial, change each of the terms to its opposite. Then add the coefficients.

Problem 5 Subtracting Polynomials

What is a simpler form of $(x^3 - 3x^2 + 5x) - (7x^3 + 5x^2 - 12)$?

Think

Is the sum or difference of two polynomials always a polynomial?
Yes. The set of polynomials is *closed* under addition and subtraction, which means that adding or subtracting polynomials always gives you another polynomial.

Method 1 Subtract vertically.

$$\begin{array}{rl} x^3 - 3x^2 + 5x & \text{Line up like terms.} \\ -(7x^3 + 5x^2 \quad\quad - 12) & \\ \hline x^3 - 3x^2 + 5x & \text{Then add the opposite of each term in} \\ -7x^3 - 5x^2 \quad\quad + 12 & \text{the polynomial being subtracted.} \\ \hline -6x^3 - 8x^2 + 5x + 12 & \end{array}$$

Method 2 Subtract horizontally.

$$(x^3 - 3x^2 + 5x) - (7x^3 + 5x^2 - 12)$$

$= x^3 - 3x^2 + 5x - 7x^3 - 5x^2 + 12$ Write the opposite of each term in the polynomial being subtracted.

$= (x^3 - 7x^3) + (-3x^2 - 5x^2) + 5x + 12$ Group like terms.

$= -6x^3 - 8x^2 + 5x + 12$ Simplify.

Got It? **5.** What is a simpler form of $(-4m^3 - m + 9) - (4m^2 + m - 12)$?

Lesson Check

Do you know HOW?

Find the degree of each monomial.

1. $-7x^4$ **2.** $8y^2z^3$

Simplify each sum or difference.

3. $(5r^3 + 8) + (6r^3 + 3)$

4. $(x^2 - 2) - (3x + 5)$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

Vocabulary Name each polynomial based on its degree and number of terms.

5. $5x^2 + 2x + 1$ **6.** $3z - 2$

7. Compare and Contrast How are the processes of adding monomials and adding polynomials alike? How are the processes different?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

Practice Find the degree of each monomial. See Problem 1.

8. $3x$ **9.** $8a^3$ **10.** 20 **11.** $2b^8c^2$

12. $-7y^3z$ **13.** -3 **14.** $12w^4$ **15.** 0

Simplify. See Problem 2.

16. $12p^2 + 8p^2$ **17.** $2m^3n^3 + 9m^3n^3$ **18.** $8w^2x + w^2x$ **19.** $3t^4 + 11t^4$

20. $x^3 - 9x^3$ **21.** $30v^4w^3 - 12v^4w^3$ **22.** $7x^2 - 2x^2$ **23.** $5bc^4 - 13bc^4$

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms. See Problem 3.

24. $5y - 2y^2$ **25.** $-2q + 7$ **26.** $x^2 + 4 - 3x$

27. $6x^2 - 13x^2 - 4x + 4$ **28.** $c + 8c^3 - 3c^7$ **29.** $3z^4 - 5z - 2z^2$

Simplify. See Problem 4.

30.
$$\begin{array}{r} 4w - 5 \\ +\ 9w + 2 \\ \hline \end{array}$$

31.
$$\begin{array}{r} 6x^2 + 7 \\ +\ 3x^2 + 1 \\ \hline \end{array}$$

32.
$$\begin{array}{r} 2k^2 - k + 3 \\ +\ 5k^2 + 3k - 7 \\ \hline \end{array}$$

33. $(5x^2 + 3) + (15x^2 + 2)$ **34.** $(2g^4 - 3g + 9) + (-g^3 + 12g)$

35. Education The number of students at East High School and the number of students at Central High School over a 10-year period can be modeled by the following polynomials.

East High School: $-11x^2 + 133x + 1200$
Central High School: $-7x^2 + 95x + 1100$

In each polynomial, $x = 0$ corresponds to the first year in the 10-year period. What polynomial models the total number of students at both high schools?

Simplify. See Problem 5.

36.
$$\begin{array}{r} 5n - 2 \\ -(3n + 8) \\ \hline \end{array}$$

37.
$$\begin{array}{r} 6x^3 + 17 \\ -(4x^3 + 9) \\ \hline \end{array}$$

38.
$$\begin{array}{r} 2c^2 + 7c - 1 \\ -(c^2 - 10c + 4) \\ \hline \end{array}$$

39. $(14h^4 + 3h^3) - (9h^4 + 2h^3)$ **40.** $(-6w^4 + w^2) - (-2w^3 + 4w^2 - w)$

B Apply

41. Think About a Plan The perimeter of a triangular park is $16x + 3$. What is the missing length?

- What is the sum of the two given side lengths?
- What operation should you use to find the remaining side length?

42. Geometry The perimeter of a trapezoid is $39a - 7$. Three sides have the following lengths: $9a$, $5a + 1$, and $17a - 6$. What is the length of the fourth side?

43. Error Analysis Describe and correct the error in finding the difference of the polynomials.

$$(4x^2 - x + 3) - (3x^2 - 5x - 6) = 4x^2 - x + 3 - 3x^2 - 5x - 6$$
$$= 4x^2 - 3x^2 - x - 5x + 3 - 6$$
$$= x^2 - 6x - 3$$

Simplify. Write each answer in standard form.

44. $(5x^2 - 3x + 7x) + (9x^2 + 2x^2 + 7x)$

45. $(y^3 - 4y^2 - 2) - (6y^3 + 4 - 6y^2)$

46. $(-9r^3 + 2r - 1) - (-5r^2 + r + 8)$

47. $(3z^3 - 4z + 7z^2) + (8z^2 - 6z - 5)$

48. a. Is the sum of two polynomials always a polynomial? Explain.

b. Is the difference of two polynomials always a polynomial? Explain.

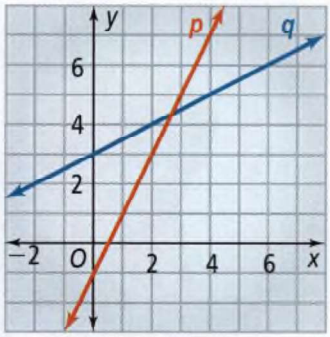
Challenge

49. a. Write the equations for line p and line q. Use slope-intercept form.

b. Use your equations from part (a) to write a function for the vertical distance $D(x)$ between points on lines p and q with the same x-value.

c. For what value of x does $D(x)$ equal zero?

d. Reasoning How does the x-value in part (c) relate to the graph?



Simplify each expression.

50. $(ab^2 + ba^3) + (4a^3b - ab^2 - 5ab)$

51. $(9pq^6 - 11p^4q) - (-5pq^6 + p^4q^4)$

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 6

Look back at Kelly's and Roberto's original plots on page 485. Choose from the following words, numbers, and expressions to complete the sentences below.

monomial	binomial	trinomial	1	2
x^2	$x + 2$	$x^2 + 2$	$x + 6$	$2x + 6$

Two polynomials that represent the length and width of Kelly's plot are **a.** ? and **b.** ? . Each of these polynomials is an example of a **c.** ? . The polynomial that represents the area of Roberto's flower bed is **d.** ? . This polynomial is an example of a **e.** ? . The degree of this polynomial is **f.** ? .