As an object falls, its speed continues to increase, so its height above the ground decreases at a faster and faster rate. Ignoring air resistance, you can model the object's height with the function $h = -16t^2 + c$. The height h is in feet, the time t is in seconds, and the object's initial height c is in feet.

Problem 5   Using the Falling Object Model

Nature An acorn drops from a tree branch 20 ft above the ground. The function $h = -16t^2 + 20$ gives the height h of the acorn (in feet) after t seconds. What is the graph of this quadratic function? At about what time does the acorn hit the ground?

Know
- The function for the acorn's height
- The initial height is 20 ft.

Need
The function's graph and the time the acorn hits the ground
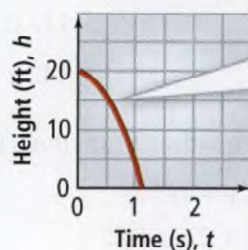
Plan
Use a table of values to graph the function. Use the graph to estimate when the acorn hits the ground.

Think

Can you choose negative values for *t*?
No. *t* represents time, so it cannot be negative.

t	$h = -16t^2 + 20$
0	20
0.5	16
1	4
1.5	−16

Graph the function using the first three ordered pairs from the table. Do not plot $(1.5, -16)$ because height cannot be negative.

The acorn hits the ground when its height above the ground is 0 ft. From the graph, you can see that the acorn hits the ground after slightly more than 1 s.

Got It? 5. a. In Problem 5 above, suppose the acorn drops from a tree branch 70 ft above the ground. The function $h = -16t^2 + 70$ gives the height h of the acorn (in feet) after t seconds. What is the graph of this function? At about what time does the acorn hit the ground?

    b. **Reasoning** What are a reasonable domain and range for the original function in Problem 5? Explain your reasoning.

Lesson Check

Do you know HOW?

Graph the parabola. Identify the vertex.

1. $y = -3x^2$

2. $y = 4x^2$

3. $y = \frac{1}{2}x^2 + 2$

4. $y = -2x^2 - 1$

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. **Vocabulary** When is the vertex of a parabola the minimum point? When is it the maximum point?

6. **Compare and Contrast** How are the graphs of $y = -\frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2 + 1$ similar? How are they different?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice Identify the vertex of each parabola. Tell whether it is a minimum or a maximum.   See Problem 1.

7.

8.
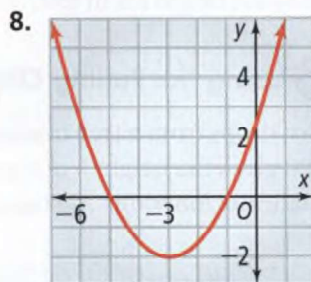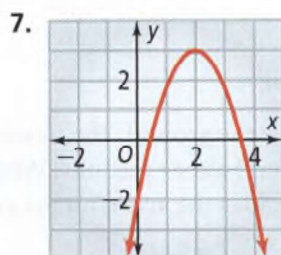
9.

x	y
0	8
1	2
2	0
3	2
4	8

Graph each function. Then identify the domain and range of the function.   See Problem 2.

10. $y = -4x^2$

11. $f(x) = 1.5x^2$

12. $f(x) = 3x^2$

13. $f(x) = \frac{2}{3}x^2$

14. $y = -\frac{1}{2}x^2$

15. $y = -\frac{1}{3}x^2$

Order each group of quadratic functions from widest to narrowest graph.   See Problem 3.

16. $y = 3x^2, y = 2x^2, y = 4x^2$

17. $f(x) = 5x^2, f(x) = -3x^2, f(x) = x^2$

18. $y = -\frac{1}{2}x^2, y = 5x^2, y = -\frac{1}{4}x^2$

19. $f(x) = -2x^2, f(x) = -\frac{2}{3}x^2, f(x) = -4x^2$

Graph each function.   See Problem 4.

20. $f(x) = x^2 + 4$

21. $y = x^2 - 7$

22. $y = \frac{1}{2}x^2 + 2$

23. $f(x) = -x^2 - 3$

24. $y = -2x^2 + 4$

25. $f(x) = 4x^2 - 5$

26. Dropped Object A person walking across a bridge accidentally drops an orange into the river below from a height of 40 ft. The function $h = -16t^2 + 40$ gives the orange's approximate height h above the water, in feet, after t seconds. Graph the function. In how many seconds will the orange hit the water?   See Problem 5.

27. Nature A bird drops a stick to the ground from a height of 80 ft. The function $h = -16t^2 + 80$ gives the stick's approximate height h above the ground, in feet, after t seconds. Graph the function. At about what time does the stick hit the ground?

B Apply

28. Error Analysis Describe and correct the error made in graphing the function $y = -2x^2 + 1$.
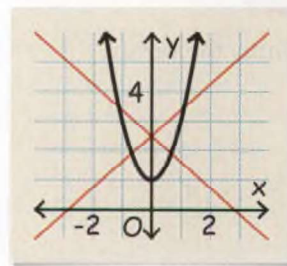

Identify the domain and range of each function.

29. $f(x) = 3x^2 + 6$

30. $y = -2x^2 - 1$

31. $y = -\frac{3}{4}x^2 - 9$

32. $y = \frac{2}{3}x^2 + 12$

33. Writing What information do the numbers a and c give you about the graph of $y = ax^2 + c$?
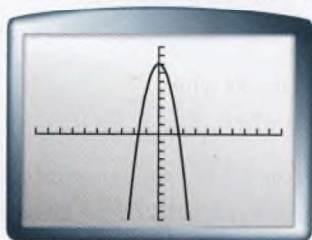
Match each function with its graph.
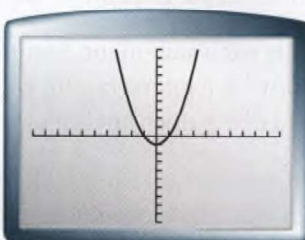
34. $f(x) = x^2 - 1$

35. $f(x) = -3x^2 + 8$

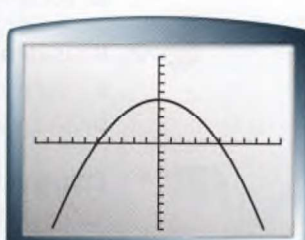**36.** $f(x) = -0.2x^2 + 5$

A.

B.

C.

37. Using a graphing calculator, graph $f(x) = x^2 + 2$.

   a. If $f(x) = x^2 + 2$ and $g(x) = 3f(x)$, write the equation for $g(x)$. Graph $g(x)$ and compare it to the graph of $f(x)$.

   b. If $f(x) = x^2 + 2$ and $h(x) = f(3x)$, write the equation for $h(x)$. Graph $h(x)$ and compare it to the graph of $f(x)$.

   c. Compare how multiplying a quadratic function by a number and multiplying the x value of a quadratic function by a number change the graphs of the quadratic functions.

© **38. Think About a Plan** Suppose a person is riding in a hot-air balloon, 154 ft above the ground. He drops an apple. The height h, in feet, of the apple above the ground is given by the formula $h = -16t^2 + 154$, where t is the time in seconds. To the nearest tenth of a second, at what time does the apple hit the ground?

   • How can you use a table to approximate the answer between two consecutive whole numbers of seconds?

   • How can you use a second table to make your approximation more accurate?

Graphing Calculator Use a graphing calculator to graph each function. Identify the vertex and axis of symmetry.

39. $y = \frac{1}{4}x^2 + 3$

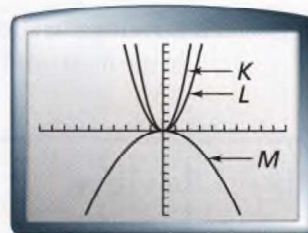**40.** $f(x) = -1.5x^2 + 5$

41. $y = -3x^2 - 6$

Three graphs are shown at the right. Identify the graph or graphs that fit each description.

42. $a > 0$

43. $a < 0$

44. $|a|$ has the greatest value.

45. $|a|$ has the least value.

STEM **46. Physics** In a physics class demonstration, a ball is dropped from the roof of a building, 72 ft above the ground. The height h, in feet, of the ball above the ground is given by the function $h = -16t^2 + 72$, where t is the time in seconds.

   a. Graph the function.

   b. How far has the ball fallen from time $t = 0$ to $t = 1$?

© **c. Reasoning** Does the ball fall the same distance from time $t = 1$ to $t = 2$ as it does from $t = 0$ to $t = 1$? Explain.