Problem 4 Using Factoring to Solve a Real-World Problem

Photography You are constructing a frame for the rectangular photo shown. You want the frame to be the same width all the way around and the total area of the frame and photo to be 315 in.^{2}. What should the outer dimensions of the frame be?

Know
The size of the photo is 11 in. by 17 in. The total area is 315 in.^{2}.

Need
The outer dimensions of the frame

Plan
Write the frame's outer dimensions in terms of its width x. Use these dimensions to write an equation for the area of the frame and photo.

$(2x + 11)(2x + 17) = 315$ — Width × Length = Area

$4x^2 + 56x + 187 = 315$ — Find the product $(2x + 11)(2x + 17)$.

$4x^2 + 56x - 128 = 0$ — Subtract 315 from each side.

$4(x^2 + 14x - 32) = 0$ — Factor out 4.

$4(x + 16)(x - 2) = 0$ — Factor $x^2 + 14x - 32$.

$x + 16 = 0$ or $x - 2 = 0$ — Use the Zero-Product Property.

$x = -16$ or $x = 2$ — Solve for x.

Think
Why can you ignore the factor of 4?
By the Zero-Product Property, one of the factors, 4, $x + 16$, or $x - 2$, must equal 0. Since $4 \neq 0$, either $x + 16$ or $x - 2$ equals 0.

The only reasonable solution is 2. So the outer dimensions of the frame are $2(2) + 11$ in. by $2(2) + 17$ in., or 15 in. by 21 in.

Got It? **4.** In Problem 4, suppose the total area of the frame and photo were 391 in.^{2}. What would the outer dimensions of the frame be?

Lesson Check

Do you know HOW?

Solve each equation.

1. $(v - 4)(v - 7) = 0$

2. $t^2 + 3t - 54 = 0$

3. $3y^2 - 17y + 24 = 0$

STEM **4. Carpentry** You are making a rectangular table. The area of the table should be 10 ft^2. You want the length of the table to be 1 ft shorter than twice its width. What should the dimensions of the table be?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Give an example of how the Zero-Product Property can be used to solve a quadratic equation.

6. Compare and Contrast How is factoring the expression $x^2 - 6x + 8$ similar to solving the equation $x^2 - 6x + 8 = 0$? How is it different?

7. Reasoning Can you extend the Zero-Product Property to nonzero products of numbers? For example, if $ab = 8$, is it always true that $a = 8$ or $b = 8$? Explain.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Use the Zero-Product Property to solve each equation. See Problem 1.

8. $(x-9)(x-8)=0$ **9.** $(4k+5)(k+7)=0$ **10.** $n(n+2)=0$

11. $-3n(2n-5)=0$ **12.** $(7x+2)(5x-4)=0$ **13.** $(4a-7)(3a+8)=0$

Solve by factoring. See Problems 2 and 3.

14. $x^2+11x+10=0$ **15.** $g^2+4g-32=0$ **16.** $s^2-14s+45=0$

17. $2z^2-21z-36=0$ **18.** $3q^2+q-14=0$ **19.** $4m^2-27m-40=0$

20. $x^2+13x=-42$ **21.** $p^2-4p=21$ **22.** $c^2=5c$

23. $2w^2-11w=-12$ **24.** $3h^2+17h=-10$ **25.** $9b^2=16$

26. Geometry A box shaped like a rectangular prism has a volume of 280 in.^{3}. Its dimensions are 4 in. by $(n+2)$ in. by $(n+5)$ in. Find n. See Problem 4.

27. Knitting You are knitting a blanket. You want the area of the blanket to be 24 ft^2. You want the length of the blanket to be 2 ft longer than its width. What should the dimensions of the blanket be?

STEM **28. Construction** You are building a rectangular deck. The area of the deck should be 250 ft^2. You want the length of the deck to be 5 ft longer than twice its width. What should the dimensions of the deck be?

B Apply

Use the Zero-Product Property to solve each equation. Write your solutions as a set in roster form.

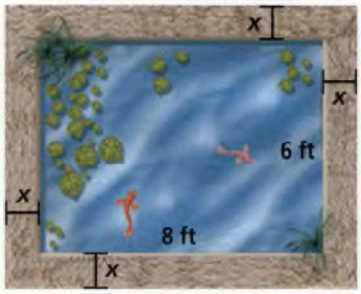
29. $x^2+6x+8=0$ **30.** $a^2+8a+12=0$ **31.** $k^2+7k+10=0$

Write each equation in standard form. Then solve.

32. $7n^2+16n+15=2n^2+3$ **33.** $4q^2+3q=3q^2-4q+18$

34. Think About a Plan You have a rectangular koi pond that measures 6 ft by 8 ft. You have enough concrete to cover 72 ft^2 for a walkway, as shown in the diagram. What should the width of the walkway be?

- How can you write the outer dimensions of the walkway?
- How can you represent the total area of the walkway and pond in two ways?



35. Reasoning Find the zeros of the function $f(x)=x^2-3x+2$ by factoring. How can you verify the zeros of the function are correct by looking at the graph?

36. Error Analysis Describe and correct the error made in solving the equation.

$2x^2+3x=20$
$x(2x+3)=20$
$x=0$ or $2x+3=0$
$x=0$ or $x=-\frac{3}{2}$

37. Reasoning How many solutions does an equation of the form $x^2-k^2=0$ have? Explain.