

Name: Key

Date:

Topic:

Class:

Main Ideas/Questions

Notes/Examples

**Systems with
3 VARIABLES**

- ① **CHOOSE TWO EQUATIONS** in which you can easily eliminate a variable. Let's call this new equation after the elimination **Equation A**.
- ② **ELIMINATE THE SAME VARIABLE** from a combination of the equation you didn't use and one of the ones you did. Let's call this equation after the elimination **Equation B**.
- ③ **PUT EQUATION A AND EQUATION B TOGETHER** and **SOLVE THAT SYSTEM**. You now have two of your variables!
- ④ **SUBSTITUTE** your answers from step 3 into either original equation to find the remaining variable. Write your solution as (x, y, z).

:: EXAMPLES ::

$$\begin{cases} 2x - 6y + 9z = -8 \\ 5x + y + 2z = 10 \\ 3x + y - 8z = -28 \end{cases} \begin{matrix} A \\ B \end{matrix}$$

$$\begin{aligned} 5(-1) + y + 2(4) &= 10 \\ -5 + y + 8 &= 10 \\ y + 3 &= 10 \\ y &= 7 \end{aligned}$$

$(-1, 7, 4)$

$$\begin{aligned} A: 2x - 6y + 9z &= -8 \\ 6(5x + y + 2z &= 10) \\ \hline 2x - 6y + 9z &= -8 \\ + 30x + 6y + 12z &= 60 \\ \hline 32x + 21z &= 52 \end{aligned}$$

$$\begin{aligned} 5(32x + 21z &= 52) \\ -8(20x - 39z &= -176) \\ \hline 160x + 105z &= 260 \\ + -160x + 312z &= 1408 \\ \hline 417z &= 1668 \\ z &= 4 \end{aligned}$$

$$\begin{aligned} B: 2x - 6y + 9z &= -8 \\ 6(3x + y - 8z &= -28) \\ \hline 2x - 6y + 9z &= -8 \\ + 18x + 6y - 48z &= -168 \\ \hline 20x - 39z &= -176 \end{aligned}$$

$$\begin{aligned} 32x + 21(4) &= 52 \\ 32x + 84 &= 52 \\ 32x &= -32 \\ x &= -1 \end{aligned}$$

$$\begin{cases} 4x + y - 3z = 45 \\ 6x - 4y + 3z = 41 \\ 2x - 7y - 11z = 85 \end{cases} \begin{matrix} A \\ B \end{matrix}$$

$$\begin{aligned} 4(8) + y - 3(-5) &= 45 \\ 30 + y - 15 &= 45 \\ y + 15 &= 45 \\ y &= 30 \end{aligned}$$

$(8, -2, -5)$

$$\begin{aligned} A: 4x + y - 3z &= 45 \\ + 6x - 4y + 3z &= 41 \\ \hline 10x - 3y &= 86 \end{aligned}$$

$$\begin{aligned} 36(10x - 3y &= 86) \\ -5(72x - 65y &= 706) \\ \hline 360x - 108y &= 3096 \\ + -360x + 325y &= -3530 \\ \hline 217y &= -434 \\ y &= -2 \end{aligned}$$

$$\begin{aligned} B: (6x - 4y + 3z &= 41) \parallel \\ (2x - 7y - 11z &= 85) \parallel 3 \\ \hline 66x - 44y + 33z &= 451 \\ + 6x - 21y - 33z &= 255 \\ \hline 72x - 65y &= 706 \end{aligned}$$

$$\begin{aligned} 10x - 3(-2) &= 86 \\ 10x + 6 &= 86 \\ 10x &= 80 \\ x &= 8 \end{aligned}$$

$$\begin{cases} 3. \ 5x-3y+8z=-59 \\ \quad 9x-4y-z=-60 \\ \quad 5x+2y+4z=-47 \end{cases} \begin{matrix} A \\ B \end{matrix}$$

$$\begin{aligned} 9(-7) - 4(0) - z &= -60 \\ -63 - z &= -60 \\ -z &= 3 \\ z &= -3 \end{aligned}$$

$$\boxed{(-7, 0, -3)}$$

$$\begin{aligned} A: \ 5x-3y+8z &= -59 \\ 8(9x-4y-z) &= -60 \end{aligned}$$

$$\begin{aligned} 5x-3y+8z &= -59 \\ + 72x-32y-8z &= -480 \\ \hline 77x-35y &= -539 \end{aligned}$$

$$\begin{aligned} -2(77x-35y) &= -539 \\ 5(41x-14y) &= -287 \end{aligned}$$

$$\begin{aligned} -154x+70y &= 1078 \\ + 205x-70y &= -1435 \\ \hline 51x &= -357 \\ x &= -7 \end{aligned}$$

$$\begin{aligned} B: \ 9x-4y-z &= -60 \\ 5x+2y+4z &= -47 \end{aligned}$$

$$\begin{aligned} 36x-16y-4z &= -240 \\ + 5x+2y+4z &= -47 \\ \hline 41x-14y &= -287 \end{aligned}$$

$$\begin{aligned} 41(-7)-14y &= -287 \\ -287-14y &= -287 \\ -14y &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{cases} 4. \ x-5y+3z=39 \\ \quad 3x+8y-z=-19 \\ \quad 2x-6y-5z=-29 \end{cases} \begin{matrix} A \\ B \end{matrix}$$

$$\begin{aligned} x-5(-2)+3(9) &= 39 \\ x+10+27 &= 39 \\ x &= 2 \end{aligned}$$

$$\boxed{(2, -2, 9)}$$

$$\begin{aligned} A: \ x-5y+3z &= 39 \\ 3x+8y-z &= -19 \end{aligned}$$

$$\begin{aligned} -3x+15y-9z &= -117 \\ + 3x+8y-z &= -19 \\ \hline 23y-10z &= -136 \end{aligned}$$

$$\begin{aligned} -4(23y-10z) &= -136 \\ 23(4y-11z) &= -107 \end{aligned}$$

$$\begin{aligned} -92y+40z &= 544 \\ + 92y-253z &= -2461 \\ \hline -213z &= -1917 \\ z &= 9 \end{aligned}$$

$$\begin{aligned} B: \ x-5y+3z &= 39 \\ 2x-6y-5z &= -29 \end{aligned}$$

$$\begin{aligned} -2x+10y-6z &= -78 \\ + 2x-6y-5z &= -29 \\ \hline 4y-11z &= -107 \end{aligned}$$

$$\begin{aligned} 4y-11(9) &= -107 \\ 4y-99 &= -107 \\ 4y &= -8 \\ y &= -2 \end{aligned}$$

$$\begin{cases} 5. \ x+y+3z=12 \\ \quad -3x+7y-2z=-35 \\ \quad 7x-6y-5z=32 \end{cases} \begin{matrix} A \\ B \end{matrix}$$

$$\begin{aligned} x-2+3(3) &= 12 \\ x+7 &= 12 \\ x &= 5 \end{aligned}$$

$$\boxed{(5, -2, 3)}$$

$$\begin{aligned} A: \ x+y+3z &= 12 \\ -3x+7y-2z &= -35 \end{aligned}$$

$$\begin{aligned} 3x+3y+9z &= 36 \\ + -3x+7y-2z &= -35 \\ \hline 10y+7z &= 1 \end{aligned}$$

$$\begin{aligned} 13(10y+7z) &= 13 \\ 10(-13y-26z) &= -52 \end{aligned}$$

$$\begin{aligned} 130y+91z &= 13 \\ + -130y-260z &= -520 \\ \hline -169z &= -507 \\ z &= 3 \end{aligned}$$

$$\begin{aligned} B: \ x+y+3z &= 12 \\ 7x-6y-5z &= 32 \end{aligned}$$

$$\begin{aligned} -7x-7y-21z &= -84 \\ + 7x-6y-5z &= 32 \\ \hline -13y-26z &= -52 \end{aligned}$$

$$\begin{aligned} 10y+7(3) &= 1 \\ 10y &= -20 \\ y &= -2 \end{aligned}$$

:: APPLICATIONS ::

6. The theater sells three types of tickets: general admission, senior citizen, and child. Mark bought 9 general admission, 2 senior citizen, and 3 child tickets and paid \$170. Sarah bought 5 general admission, 4 senior citizen, and 7 child tickets and paid \$173. Kyle bought 4 general admission, 1 senior citizen, and 6 child tickets and paid \$116.50. Find the cost of each type of ticket.

x = gen. admission

y = sr. citizen

z = child

$$\begin{cases} A \begin{cases} 9x + 2y + 3z = 170 \\ 5x + 4y + 7z = 173 \end{cases} \\ B \begin{cases} 4x + y + 6z = 116.50 \end{cases} \end{cases}$$

Gen admission cost \$13.50 per ticket, senior citizen cost \$11.50 per ticket, and children cost \$8.50 per ticket.

A: $(9x + 2y + 3z = 170) - 2$

$$5x + 4y + 7z = 173$$

$$\underline{-18x - 4y - 6z = -340}$$

$$+ \quad 5x + 4y + 7z = 173$$

$$\underline{-13x + z = -167}$$

$$17(-13x + z = -167)$$

$$\underline{-11x - 17z = -293}$$

$$-221x + 17z = -2839$$

$$+ \quad -11x - 17z = -293$$

$$\underline{-232x = -3132}$$

$$x = 13.5$$

$$9(13.5) + 2y + 3(8.5) = 170$$

$$2y + 147 = 170$$

$$2y = 23$$

$$y = 11.5$$

B: $5x + 4y + 7z = 173$

$$\underline{(4x + y + 6z = 116.50) \cdot 4}$$

$$5x + 4y + 7z = 173$$

$$+ \quad -16x - 4y - 24z = -466$$

$$\underline{-11x - 17z = -293}$$

$$-13(13.5) + z = -167$$

$$-175.5 + z = -167$$

$$z = 8.5$$

7. Jamie has a collection of 78 nickels, dimes, and quarters worth \$12.40. If the number of quarters is doubled, the value becomes \$22.15. How many of each type of coin does Jamie have?

x = nickel

y = dime

z = quarter

$$A \begin{cases} x + y + z = 78 \end{cases}$$

$$B \begin{cases} .05x + .10y + .25z = 12.40 \\ .05x + .10y + .25(2z) = 22.15 \end{cases}$$

A: $(x + y + z = 78) \cdot .05$

$$.05x + .10y + .25z = 12.40$$

$$\underline{-.05x - .05y - .05z = -3.90}$$

$$+ \quad .05x + .10y + .25z = 12.40$$

$$\underline{.05y + .20z = 8.50}$$

$$.05y + .20(39) = 8.5$$

$$.05y + 7.80 = 8.5$$

$$.05y = .70$$

$$y = 14$$

B: $.05x + .10y + .25z = 12.40$

$$\underline{-(.05x + .10y + .50z = 22.15)}$$

$$\underline{-.25z = -9.75}$$

$$z = 39$$

$$x + 14 + 39 = 78$$

$$x + 53 = 78$$

$$x = 25$$

25 nickels, 14 dimes, 39 quarters

Examples in which SUBSTITUTION may be the BETTER METHOD

8. Mason has a collection 49¢ stamps, 20¢ stamps, and 3¢ stamps worth \$23.55. He has 56 total 49¢ and 20¢ stamps and the number of 3¢ stamps is nine more than the number of 20¢ stamps. If how many of each does he have?

$x = 49¢ \text{ stamps}$
 $y = 20¢ \text{ stamps}$
 $z = 3¢ \text{ stamps}$

$$\begin{aligned} .49x + .20y + .03z &= 23.55 \\ x + y &= 56 \rightarrow x = 56 - y \\ z &= y + 9 \end{aligned}$$

40 - 49¢ stamps,
 16 - 20¢ stamps,
 25 - 3¢ stamps

$$\begin{aligned} .49(56 - y) + .20y + .03(y + 9) &= 23.55 \\ 27.44 - .49y + .20y + .03y + .27 &= 23.55 \\ -0.26y + 27.71 &= 23.55 \\ -0.26y &= -4.16 \\ y &= 16 \\ z &= 16 + 9 \\ z &= 25 \\ x &= 56 - 16 \\ x &= 40 \end{aligned}$$

9. The three medals earned in the Olympics are the gold, silver, and bronze medals. In the 2014 Winter Olympics, the United States had 28 total medal winners. There were two more gold medals than silver medals and the number of bronze medals was four less than the sum of the number of gold and silver medals. How many winners were there of each type of medal?

$x = \text{gold}$
 $y = \text{silver}$
 $z = \text{bronze}$

$$\begin{aligned} x + y + z &= 28 \\ x &= y + 2 \\ z &= x + y - 4 \rightarrow z = (y + 2) + y - 4 \\ z &= 2y - 2 \end{aligned}$$

9 gold medals,
 7 silver medals,
 and 12 bronze medals.

$$\begin{aligned} (y + 2) + y + (2y - 2) &= 28 \\ 4y &= 28 \\ y &= 7 \\ x &= 7 + 2 \\ x &= 9 \\ z &= 2(7) - 2 \\ z &= 12 \end{aligned}$$

10. Enrique has a pay-per-use plan on his cell phone. It costs \$0.10 per minute talk, \$0.20 per text message, and \$0.25 per picture message. His total bill last month was \$30.95. The total number of text and picture messages sent was 87. The cost for the use of minutes was \$1.60 less than the cost of the text messages. Find the number of talk minutes, text messages, and picture messages he used.

$x = \text{talk}$
 $y = \text{text message}$
 $z = \text{picture message}$

$$\begin{aligned} .10x + .20y + .25z &= 30.95 \\ y + z &= 87 \rightarrow z = 87 - y \\ .10x &= .20y - 1.60 \rightarrow x = 2y - 16 \end{aligned}$$

Enrique used 128 talk minutes, 72 text messages, and 15 picture messages.

$$\begin{aligned} .10(2y - 16) + .20y + .25(87 - y) &= 30.95 \\ .20y - 1.60 + .20y + 21.75 - .25y &= 30.95 \\ .15y + 20.15 &= 30.95 \\ .15y &= 10.80 \\ y &= 72 \\ z &= 87 - 72 \\ z &= 15 \\ x &= 2(72) - 16 \\ x &= 128 \end{aligned}$$