

# Unit 2 Test Study Guide

## (Logic & Proof)

Name: \_\_\_\_\_

Date: \_\_\_\_\_ Per: \_\_\_\_\_

### Topic #1: Conjectures & Counterexamples

**Directions:** Determine if the conjecture is true or false. If false, provide a counterexample.

1. The product of any two prime numbers is always odd.

False;  $2 \cdot 5 = 10$

2. If two angles are complementary, then both angles must be acute.

True

3. The square of a number is always larger than the number.

False;  $1^2 = 1$

4. Two lines always intersect at a point.

False; 2 parallel lines never intersect

### Topic 2: Compound Statements & Truth Tables

**Conjunction:**

$p \wedge q$

Read as: p and q

True when: both statements are true.

**Disjunction:**

$p \vee q$

Read as: p or q

True when: at least one statement is true.

Use the following statements to write compound statements. Determine the truth value.

$p$ : All vegetables are green. (F)

$q$ : Vertical angles are congruent. (T)

$r$ : All integers are natural numbers. (F)

5.  $p \vee q$ : All vegetables are green or vertical angles are congruent. Truth Value: True

6.  $q \wedge r$ : Vertical angles are congruent and all integers are natural numbers. Truth Value: False

7.  $p \vee \sim q$ : All vegetables are green or vertical angles are not congruent. Truth Value: False

8.  $q \wedge \sim r$ : Vertical angles are congruent and all integers are not natural numbers. Truth Value: True

**Directions:** Complete each truth table.

9.  $\sim p \vee \sim q$

$p$	$q$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

10.  $\sim r \vee (\sim p \wedge q)$

$p$	$q$	$r$	$\sim p$	$\sim r$	$(\sim p \wedge q)$	$\sim r \vee (\sim p \wedge q)$
T	T	T	F	F	F	F
T	T	F	F	T	F	T
T	F	T	F	F	F	F
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	F	T

**Topic 3: Conditional Statements**

Conditional:	Inverse:	Converse:	Contrapositive:
$p \rightarrow q$	$\sim p \rightarrow \sim q$	$q \rightarrow p$	$\sim q \rightarrow \sim p$

Use the following statements to write conditional statements. Determine the truth value.

$p$ : two angles form a linear pair;  $q$ : they are adjacent angle

11. Conditional: If two angles form a linear pair, then they are adjacent angles. Truth Value: True

12. Inverse: If two angles do not form a linear pair, then they are not adjacent angles. Truth Value: False

13. Converse: If two angles are adjacent, then they form a linear pair. Truth Value: False

14. Contrapositive: If two angles are not adjacent, then they do not form a linear pair. Truth Value: True

Topic 4: Biconditional Statements

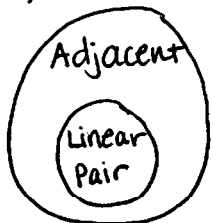

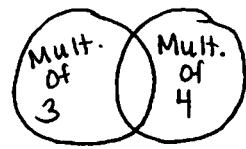
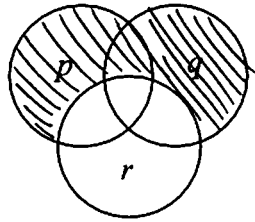
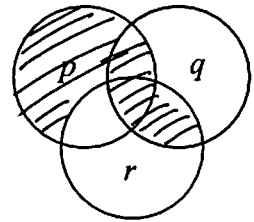
<p><b>Biconditional:</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> <math>p \leftrightarrow q</math> </div>	<p>Read as: <u><math>p</math> if and only if <math>q</math></u></p> <p>True when both conditional (<math>p \rightarrow q</math>) and converse (<math>q \rightarrow p</math>) are true!</p>
<p>Use the given statements to write the conditional, converse, and biconditional. Determine the truth value of the biconditional. Explain.</p> <p>15. <math>p</math>: a quadrilateral is a square; <math>q</math>: it has four congruent sides</p> <p>Conditional: <u>If a quadrilateral is a square, then it has 4 congruent sides.</u></p> <p>Converse: <u>If a quadrilateral has 4 congruent sides, then it is a square.</u></p> <p>Biconditional: <u>A quadrilateral is a square if and only if it has 4 congruent sides.</u></p> <p>Truth Value: <u>False; converse is false.</u></p>	

Topic 5: Deductive Reasoning

<p><b>Law of Detachment:</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> <math display="block">\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}</math> </div>	<p><b>Law of Syllogism:</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;"> <math display="block">\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}</math> </div>
<p>Use the Law of Detachment or Law of Syllogism to make a conclusion. If a conclusion cannot be made, write "no valid conclusion".</p>	
<p>16. <b>Given:</b> If it is Thursday, then Emma has drama club. (Law of Syll.)          If Emma has drama club, then she must take the late bus.</p> <p>Conclusion: <u>If it is Thursday, then Emma must take the late bus.</u></p>	
<p>17. <b>Given:</b> If two angles are vertical, then they are congruent.          Two angles are congruent.</p> <p>Conclusion: <u>No valid conclusion</u></p>	
<p>18. <b>Given:</b> If the football team raises at least \$5000, then they can buy new uniforms. (Law of Det.)          The football team raised \$5800.</p> <p>Conclusion: <u>The football team can buy new uniforms.</u></p>	
<p>19. <b>Given:</b> If you return a book to the library late, then you will pay a fee.          If you do not pay your library fees, then you may lose your library card.</p> <p>Conclusion: <u>No valid conclusion</u></p>	

Topic 6: Venn Diagrams

**Draw a Venn diagram to represent each relationship.**

<p>20. All linear pairs are adjacent.</p> 	<p>21. No rectangles are trapezoids.</p> 	<p>22. Some multiples of 3 are also multiples of 4.</p> 
<p><b>Shade the indicated region on the Venn diagram given by the compound statement.</b></p>		
<p>23. <math>\sim r \wedge (p \vee q)</math></p> 	<p>24. <math>(p \wedge \sim r) \vee (q \wedge r)</math></p> 	

Topic 7: Algebraic Proof

**KNOW THE FOLLOWING PROPERTIES OF EQUALITY:** Addition, Subtraction, Multiplication, Division, Distributive, Substitution, Reflexive, Symmetric, and Transitive

25. Given:  $8(x - 1) = 5x - 35$ ; Prove:  $x = -9$

Statements	Reasons
1. $8(x - 1) = 5x - 35$	1. Given
2. $8x - 8 = 5x - 35$	2. Distributive Property
3. $3x - 8 = -35$	3. Subtraction Property
4. $3x = -27$	4. Addition Property
5. $x = -9$	5. Division Property

26. Given:  $4c - d = f$ ,  $d = c - e$ ,  $f = d + e$ ; Prove:  $3c = d$

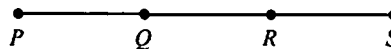
Statements	Reasons
1. $4c - d = f$	1. Given
2. $d = c - e$	2. Given
3. $4c - (c - e) = f$	3. Substitution Property
4. $f = d + e$	4. Given
5. $4c - (c - e) = d + e$	5. Substitution Property
6. $4c - c + e = d + e$	6. Distributive Property
7. $3c + e = d + e$	7. Simplify
8. $3c = d$	8. Subtraction Property

**Topic 8: Segment Proofs**

**KNOW THE FOLLOWING:**

Segment Addition Postulate, Definition of Midpoint, Definition of Congruence, all Properties of Equality, and Properties of Congruence (Reflexive, Symmetric, and Transitive)

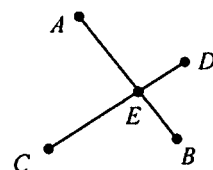
27. **Given:**  $Q$  is the midpoint of  $\overline{PR}$   
 $R$  is the midpoint of  $\overline{QS}$



**Prove:**  $PQ = \frac{1}{2}QS$

Statements	Reasons
1. $Q$ is the midpoint of $\overline{PR}$	1. Given
2. $R$ is the midpoint of $\overline{QS}$	2. Given
3. $PQ = QR$ ; $QR = RS$	3. Def. of Midpoint
4. $PQ = RS$	4. Transitive Property
5. $PQ + PQ = PQ + RS$	5. Addition Property
6. $2PQ = PQ + RS$	6. Simplify
7. $2PQ = QR + RS$	7. Substitution Property
8. $QR + RS = QS$	8. Segment Addition Postulate
9. $2PQ = QS$	9. Transitive Property
10. $PQ = \frac{1}{2}QS$	10. Division Property

28. **Given:**  $\overline{AB} \cong \overline{CD}$ ;  $\overline{CE} \cong \overline{AE}$   
**Prove:**  $\overline{ED} \cong \overline{EB}$



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$ ; $\overline{CE} \cong \overline{AE}$	1. Given
2. $AB = CD$ ; $CE = AE$	2. Def. of Congruence
3. $AE + EB = AB$ ; $CE + ED = CD$	3. Segment Addition Postulate
4. $CE + EB = CD$	4. Substitution Property
5. $CE + ED = CE + EB$	5. Transitive Property
6. $ED = EB$	6. Subtraction Property
7. $\overline{ED} \cong \overline{EB}$	7. Def. of Congruence

**Topic 9: Angle Proofs**

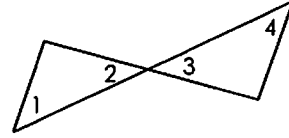
**KNOW THE FOLLOWING:**

Definition of Congruence  
 Definition of Angle Bisector  
 Definition of Complementary Angles  
 Definition of Supplementary Angles

Definition of Perpendicular  
 Definition of Right Angle  
 Angle Addition Postulate  
 Vertical Angles Theorem

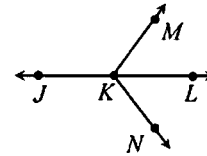
Complement Theorem  
 Linear Pair (Supplement) Theorem  
 Congruent Complements Theorem  
 Congruent Supplements Theorem

- 29. Given:**  $\angle 1$  and  $\angle 2$  are complementary;  $\angle 1 \cong \angle 4$   
**Prove:**  $\angle 3$  and  $\angle 4$  are complementary



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary	1. Given
2. $m\angle 1 + m\angle 2 = 90^\circ$	2. Def. of Complementary Angles
3. $\angle 1 \cong \angle 4$	3. Given
4. $m\angle 1 = m\angle 4$	4. Def. of congruence
5. $\angle 2 \cong \angle 3$	5. Vertical Angles Theorem
6. $m\angle 2 = m\angle 3$	6. Def. of congruence
7. $m\angle 4 + m\angle 3 = 90^\circ$	7. Substitution Property
8. $\angle 3$ and $\angle 4$ are complementary	8. Def. of Complementary Angles

- 30. Given:**  $\angle JKM$  and  $\angle MKL$  form a linear pair;  
 $\angle JKM$  and  $\angle LKN$  are supplementary  
**Prove:**  $\angle MKL \cong \angle LKN$



Statements	Reasons
1. $\angle JKM$ and $\angle MKL$ form a linear pair	1. Given
2. $\angle JKM$ and $\angle MKL$ are supplementary	2. Linear Pair (Supp.) Theorem
3. $\angle JKM$ and $\angle LKN$ are supplementary	3. Given
4. $\angle MKL \cong \angle LKN$	4. Congruent Supplements Thm.