Rational Numbers as Decimals

Write the repeating decimal as a fraction.

① $0.\overline{7}$

Let $x = 0.\overline{7}$

$10 \cdot x = 10 \cdot 0.\overline{7}$

$10x = 7.\overline{7}$

$10x - x = 7.\overline{7} - 0.\overline{7}$

$\frac{9x}{9} = \frac{7}{9}$

$\boxed{x = \frac{7}{9}}$

② $0.3\overline{8}$

Let $x = 0.3\overline{8}$

$10x = 3.\overline{8}$

$10x - x = 3.\overline{8} - .3\overline{8}$

$\frac{9x}{9} = \frac{3.5}{9}$

$3.8888\ldots$

$-.3888\ldots$

3.5

$x = \frac{3.5 \times 10}{9 \times 10} = \frac{35 \div 5}{90 \div 5} = \frac{7}{18}$

$\boxed{x = \frac{7}{18}}$

③ $8.\overline{45}$

Let $x = .\overline{45}$

$100x = 45.\overline{45}$

$100x - x = 45.\overline{45} - .\overline{45}$

$\frac{99x}{99} = \frac{45 \div 9}{99 \div 9}$

$x = \frac{5}{11}$

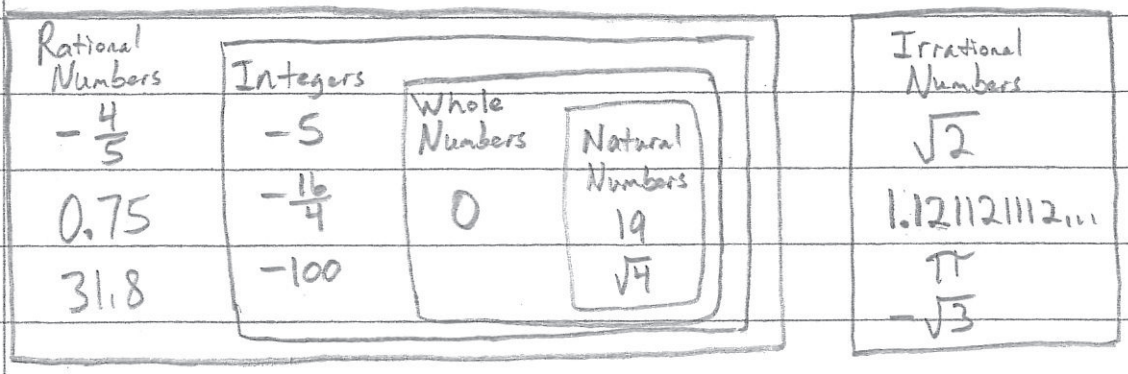
$45.454545\ldots$

$-\ .454545\ldots$

45

$\boxed{\text{Answer: } 8\frac{5}{11}}$

Irrational Numbers



Perfect Square

Ex. $49 \rightarrow \sqrt{49} = 7$

$\sqrt{1} = 1$	$\sqrt{121} = 11$
$\sqrt{4} = 2$	$\sqrt{144} = 12$
$\sqrt{9} = 3$	$\sqrt{169} = 13$
$\sqrt{16} = 4$	$\sqrt{196} = 14$
$\sqrt{25} = 5$	$\sqrt{225} = 15$
$\sqrt{36} = 6$	$\sqrt{256} = 16$
$\sqrt{49} = 7$	$\sqrt{289} = 17$
$\sqrt{64} = 8$	$\sqrt{324} = 18$
$\sqrt{81} = 9$	$\sqrt{361} = 19$
$\sqrt{100} = 10$	$\sqrt{400} = 20$

Compare and Order Real Numbers

Estimate $\sqrt{299}$.

① $\sqrt{289} < \sqrt{299} < \sqrt{324}$

$17 < \sqrt{299} < 18$

$\boxed{\sqrt{299} \approx 17}$

Approximate $\sqrt{115}$ to the nearest tenth.

② $\sqrt{100} < \sqrt{115} < \sqrt{121}$

$10 < \sqrt{115} < 11$

$\boxed{\sqrt{115} \approx 10.7}$

$10.6 \times 10.6 = 112.36$

$10.7 \times 10.7 = 114.49$

$10.8 \times 10.8 = 116.64$

Order 4π, 12.5, $12.\overline{68}$, $\sqrt{156}$ from least to greatest.

③ $4\pi = 4 \cdot (3.14159) = 12.566\ldots$

$12.5 = 12.5$

$12.\overline{68} = 12.686868\ldots$

$\sqrt{156} = 12.489996\ldots$

$\boxed{\sqrt{156},\ 12.5,\ 4\pi,\ 12.\overline{68}}$

Evaluate Square Roots and Cube Roots

Square Root $\rightarrow \sqrt{} \rightarrow$ Ex. $\sqrt{9} = 3$

Cube Root $\rightarrow \sqrt[3]{} \rightarrow$ Ex. $\sqrt[3]{125} = 5$

Evaluate.

① $\sqrt{225}$

$\sqrt{225} = \sqrt{15 \cdot 15}$

$= \sqrt{15^2}$

$= \boxed{15}$

② $\sqrt[3]{343}$

$\sqrt[3]{343} = \sqrt[3]{7 \cdot 7 \cdot 7}$

$= \sqrt[3]{7^3}$

$= \boxed{7}$

③ $\sqrt[3]{1,728}$

$\sqrt[3]{1,728} = \sqrt[3]{12 \cdot 12 \cdot 12}$

$= \sqrt[3]{12^3}$

$= \boxed{12}$

Solve Equations Using Square Roots and Cube Roots

Find the side length.

① Square

$A = 144\ cm^2$

② Cube

$V = 216\ ft^3$

$A = s^2$

$\sqrt{144} = \sqrt{s^2}$

$\pm 12 = s$

$s = 12\ cm$

$V = s^3$

$\sqrt[3]{216} = \sqrt[3]{s^3}$

$6 = s$

$s = 6\ ft$

Solve for x.

③ $x^2 = 113$

$\sqrt{x^2} = \sqrt{113}$

$x = \pm\sqrt{113}$

④ $x^3 = 19$

$\sqrt[3]{x^3} = \sqrt[3]{19}$

$x = \sqrt[3]{19}$

Using Properties of Integer Exponents

Write an equivalent expression.

① $5^2 \times 5^4 = (5\times5)\times(5\times5\times5\times5)$

$= 5^{2+4}$

$= \boxed{5^6}$

② $(8^3)^4 = (8^3)(8^3)(8^3)(8^3)$

$= 8^{3+3+3+3}$ or $8^{3\times4}$

$= \boxed{8^{12}}$

③ $7^8 \div 7^5 = \frac{7^8}{7^5}$

$= \frac{7\times7\times7\times7\times7\times7\times7\times7}{7\times7\times7\times7\times7}$

$= 7^{8-5}$

$= \boxed{7^3}$

④ $9^3 \times 9^{12} = \boxed{9^{15}}$

⑤ $9^3 \div 9^{12} = \frac{9^3}{9^{12}} = \boxed{\frac{1}{9^9}}$

More Properties of Integer Exponents

Evaluate.

① $4^0 = \boxed{1}$ ② $(-12)^0 = \boxed{1}$ ③ $7^5 \div 7^5 = 7^{5-5}$

$= 7^0 = \boxed{1}$

Write each expression using positive exponents.

④ $12^{-3} = \boxed{\frac{1}{12^3}}$ ⑤ $8^{-7} = \boxed{\frac{1}{8^7}}$

Simplify each expression for $y = 5$.

⑥ $10y^{-3} = \frac{10}{y^3}$

$= \frac{10}{5^3}$

$= \frac{10 \div 5}{125 \div 5} = \boxed{\frac{2}{25}}$

⑦ $5y^0(y^{-2}) = \frac{5 \cdot y^0}{y^2}$

$= \frac{5 \cdot 1}{5^2}$

$= \frac{5 \div 5}{25 \div 5} = \boxed{\frac{1}{5}}$

Use Powers of 10 to Estimate Quantities

① Estimate Large Quantities

Compare the populations of China and Japan.

China = 1,402,941,487 Japan = 126,818,019

Round → 1,000,000,000 100,000,000

Count Zeros → 9 zeros → 1×10^9 ← 8 zeros → 1×10^8 ←

$1 \times 10^9 > 1 \times 10^8$, China has a greater population.

② Estimate Small Quantities

Sheet of Paper = 0.0013 m Human Hair = 0.000177 m

Round → 0.001 0.0002

1×10^{-3} 2×10^{-4}

$1 \times 10^{-3} > 2 \times 10^{-4}$, a sheet of paper is thicker than human hair

③ How many times greater is the greater number?

a) 7×10^8, 7×10^6 b) 2×10^{-5}, 2×10^{-8}

100 times greater 1,000 times greater

Scientific Notation

① Large Numbers in Scientific Notation

Distance from Earth to the Sun

$92,960,000$ miles

(7 spots)

9.296×10^{7} miles

↑ Greater than or equal to 1 and less than 10

② Small Numbers in Scientific Notation

Width of a red blood cell

0.00000703 m

(6 spots)

7.03×10^{-6} m

③ Scientific Notation → Standard Form

a) $3.5 \times 10^{15} = 3,500,000,000,000,000$ (10^{15})

b) $8 \times 10^{-9} = 0.000000008$ (10^{-9})

Operations with Numbers in Scientific Notation

1) $2.5 \times 10^7 + 2.25 \times 10^8$

Method #1

$$
\begin{array}{r}
225{,}000{,}000 \\
+\ 25{,}000{,}000 \\
\hline
250{,}000{,}000
\end{array}
$$

$\boxed{2.5 \times 10^8}$

Method #2

$$
\begin{array}{r}
22.5 \times 10^7 \\
+\ 2.5 \times 10^7 \\
\hline
25.0 \times 10^7
\end{array}
$$

$\boxed{2.5 \times 10^8}$

2) $(8.2 \times 10^2) \times 43$

$$
\begin{array}{r}
8.2 \\
\times 43 \\
\hline
246 \\
+3280 \\
\hline
352.6
\end{array}
$$

352.6×10^2

$\boxed{3.526 \times 10^4}$

3) $(1.83 \times 10^6) \div 30$

$$
\begin{array}{r}
.061 \\
30\,\overline{)\,1.830} \\
-\ 0 \\
\hline
183 \\
-180 \\
\hline
30 \\
-30 \\
\hline
0
\end{array}
$$

$.061 \times 10^6$

$\boxed{6.1 \times 10^4}$