Unit 3

Patterns, Relationships, and Algebraic Thinking

Focus

Connect ratios and rates to multiplication and division, use ratios to solve problems involving percent and probability, and write mathematical expressions and equations.

CHAPTER 6
Ratio, Proportion, and Functions

BIG Idea Solve problems involving ratios and rates.

BIG Idea Write mathematical expressions and equations.

CHAPTER 7
Percent and Probability

BIG Idea Solve problems involving percent and probability.

BIG Idea Solve problems involving ratios.

Problem Solving in Social Studies

Real-World Unit Project

The Nifty Fifty States Is New Jersey more crowded than Colorado? Are there more people per square mile in Ohio than in Michigan? Which states have a higher percentage of youth than older adults? Which states have a higher ratio of women compared to men? You have been selected to compile data on the population profiles of each of the fifty states. You'll be asked to write and compare ratios and find percents. Then you'll *state* your findings in a report about the nifty fifty states. Are you in the right *state* of mind? Let's begin!

IN Math Online Log on to **glencoe.com** to begin.

Ratio, Proportion, and Functions

Indiana Academic Standards

6.1.7 Interpret ratios, model ratios, and use ratios to show the relative sizes of two quantities.
• Use the notations: a/b, a to b and $a:b$.
• Write equivalent ratios.
• Express a ratio in simplest form.
• Find the ratio of two given quantities.

Key Vocabulary

proportion (p. 329)
proportional (p. 329)
ratio (p. 314)

Real-World Link

Dunes The Sleeping Bear Dunes National Lakeshore runs for 35 miles along the Michigan coastline. There are several types of park passes available including a $10 per vehicle weekly fee and a $20 per vehicle annual fee.
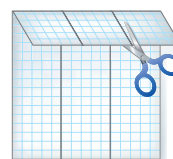
FOLDABLES Study Organizer

Ratio, Proportion, and Functions Make this Foldable to help you organize your notes. Begin with a piece of graph paper.

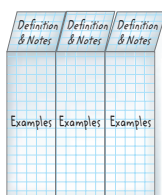**1 Fold** one sheet of graph paper in thirds lengthwise.
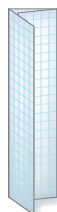
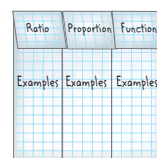**2 Unfold** lengthwise and fold one-fourth down widthwise. Cut to make three tabs as shown.

3 Unfold the tabs. Label the paper as shown.

Definition & Notes | Definition & Notes | Definition & Notes

Examples | Examples | Examples

4 Refold the tabs and label as shown.

Ratio | Proportion | Function

Examples | Examples | Examples

GET READY for Chapter 6

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICK Quiz

Write each fraction in simplest form. (Lesson 4-2)

1. $\dfrac{32}{48}$ 2. $\dfrac{7}{28}$

3. $\dfrac{15}{25}$ 4. $\dfrac{30}{35}$

5. **TRAVEL** An airplane has flown 260 miles out of a total trip of 500 miles. What fraction, in simplest form, of the trip has been completed?

Solve each equation. (Lesson 1-8)

6. $16m = 48$ 7. $5x = 40$

8. $15p = 150$ 9. $3n = 15$

10. $7y = 56$ 11. $12z = 72$

12. $8h = 96$ 13. $10e = 90$

Find the next three values in each pattern. (Lesson 1-1)

14. 4, 7, 10, 13, ...

15. 62, 66, 70, 74, ...

16. 1.8, 2.4, 3.0, 3.6, ...

17. **MUSIC** Mario played the drums for 30 minutes on Tuesday, 45 minutes on Wednesday, and 60 minutes on Thursday. At this rate, how many minutes will he play on Friday?

QUICK Review

Example 1

Write $\dfrac{40}{64}$ in simplest form.

$$\dfrac{40}{64} = \dfrac{5}{8}$$

Divide the numerator and denominator by the GCF, 8.

Since the GCF of 5 and 8 is 1, the fraction $\dfrac{5}{8}$ is in simplest form.

Example 2

Solve $14k = 84$ mentally.

$14k = 84$ **THINK** 14 times what number equals 84?

$14 \cdot 6 = 84$

$84 = 84$

The solution is 6.

Example 3

Find the next three values in the pattern 5, 16, 27, 38, ...

Look for a pattern. Each number is obtained by adding 11 to the previous number.

$$5,\ 16,\ 27,\ 38, ...$$
$$+11\ \ +11\ \ +11$$

The next three numbers are 49, 60, and 71.

6-1 Ratios and Rates

▶ MINI Lab

Consider the set of paper clips shown.

1. Compare the number of blue paper clips to the number of red paper clips using the word *more* and then using the word *times*.

2. Compare the number of red paper clips to the number of blue paper clips using the word *less* and then using a fraction.
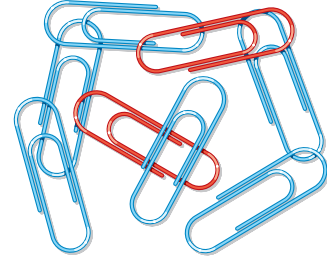
There are many different ways to compare amounts or *quantities*. A **ratio** is a comparison of two quantities by division. A ratio of 2 red paper clips to 6 blue paper clips can be written in three ways.

$$2 \text{ to } 6 \quad 2{:}6 \quad \frac{2}{6}$$

As with fractions, ratios are often expressed in simplest form.

EXAMPLE   Write a Ratio in Simplest Form

1 Write the ratio in simplest form that compares the number of red paper clips to the number of blue paper clips in the Mini Lab. Then explain its meaning.

red paper clips $\longrightarrow \dfrac{2}{6} = \dfrac{1}{3} \longleftarrow$ The GCF of 2 and 6 is 2.
blue paper clips
$\div 2$
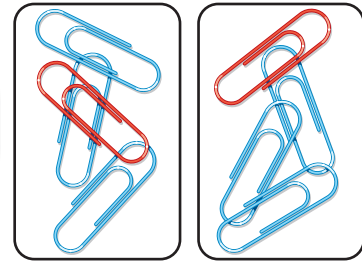
The ratio of red to blue paper clips is $\frac{1}{3}$, 1 to 3, or 1:3. This means that for every 1 red paper clip there are 3 blue paper clips.

✓ CHECK Your Progress

a. Write the ratio in simplest form that compares the number of suns to the number of moons. Then explain its meaning.
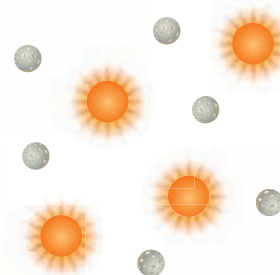
Ratios can also be used to compare a part to a whole.

EXAMPLE **Use Ratios to Compare Parts to a Whole**

2 **SURVEYS** Several students were asked to name their favorite flavor of gum. Write the ratio that compares the number of students who chose fruit to the total number of students who responded.

Favorite Flavor of Gum	
Flavor	**Number of Responses**
Peppermint	9
Cinnamon	8
Fruit	3
Spearmint	1

Three students preferred fruit out of a total of $9 + 8 + 3 + 1$ or 21 responses.

$$\text{fruit flavor responses} \longrightarrow \underset{\div 3}{\overset{\div 3}{\frac{3}{21}}} = \frac{1}{7} \quad \text{The GCF of 3 and 21 is 3.}$$

The ratio of the number of students who chose fruit to the total number of responses is $\frac{1}{7}$, 1 to 7, or 1:7. Analyzing the ratio tells us that one out of every 7 students preferred fruit-flavored gum.

CHECK Your Progress

b. **PETS** A pet store sold the animals listed in the table in one week. What was the ratio of cats to pets sold that week? Then explain its meaning.

Pet	Number Sold
Birds	10
Dogs	9
Cats	8
Gerbils	7
Lizards	2

A **rate** is a ratio comparing two quantities with different kinds of units.

Dollars and pounds are different kinds of units.   Miles and hours are different kinds of units.

$12 for 3 pounds   60 miles in 3 hours

The rate for one unit of a given quantity is called a **unit rate**.

Study Tip

Unit Rates Some common unit rates are miles per hour, miles per gallon, price per pound, and dollars per hour.

$\dfrac{\$12}{3 \text{ pounds}}$ $\dfrac{\$4}{1 \text{ pound}}$

The model shows that the dollars divided by the number of pounds is the number of dollars for 1 pound.

A unit rate of $4 for 1 pound can be read as $4 per pound.

When written as a fraction, a unit rate has a denominator of 1. Therefore, to write a rate as a unit rate, divide the numerator and denominator of the rate by the denominator. A unit rate can also be called a *rate of change*.

$$\frac{\$12}{3 \text{ pounds}} \overset{\div 3}{\underset{\div 3}{=}} \frac{\$4}{1 \text{ pound}}$$

EXAMPLE Find a Unit Rate

3 **DRAGONFLIES** Use the information at the left to find how many miles an Australian Dragonfly can fly per hour.

Write the rate that compares the number of miles to the number of hours. Then divide to find the unit rate.

$$\frac{144 \text{ miles}}{4 \text{ hours}} \overset{\div 4}{\underset{\div 4}{=}} \frac{36 \text{ miles}}{1 \text{ hour}}$$

So, an Australian Dragonfly can fly about 36 miles per hour.

The world's fastest insect is the Australian Dragonfly. It would take an Australian Dragonfly 4 hours to fly 144 miles.

CHECK Your Progress

c. **WATER PARK** For Carolina's birthday, her mom took her and 4 friends to a water park. Carolina's mom paid $40 for 5 student tickets. What was the price for one student ticket?

CHECK Your Understanding

Example 1
(p. 314)

For Exercises 1–3, write each ratio as a fraction in simplest form. Then explain its meaning.

1.

pens to pencils

2.

pennies:dimes

3. **MOVIES** A theater is showing 8 romantic comedies and 12 action thrillers. What is the ratio of action thrillers to romantic comedies?

Example 2
(p. 315)

4. **FRUIT** Last month, Amber ate 9 apples, 5 bananas, 4 peaches, and 7 oranges. Find the ratio of bananas to the total number of pieces of fruit Amber ate last month. Then explain its meaning.
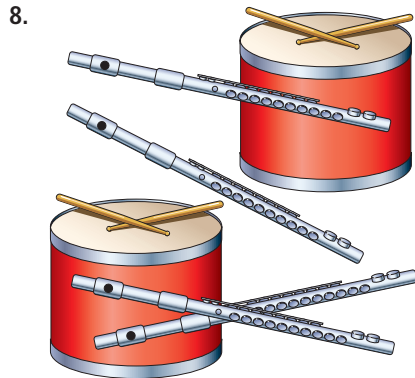
Example 3
(p. 316)

Write each rate as a unit rate.

5. $9 for 3 cases of soda

6. 25 meters in 2 seconds

7. **HEALTH** Shina's heart beats 410 times in 5 minutes. At this rate, how many times does Shina's heart beat per minute?

For Exercises 8–13, write each ratio as a fraction in simplest form. Then explain its meaning.

8.

flutes:drums

9.
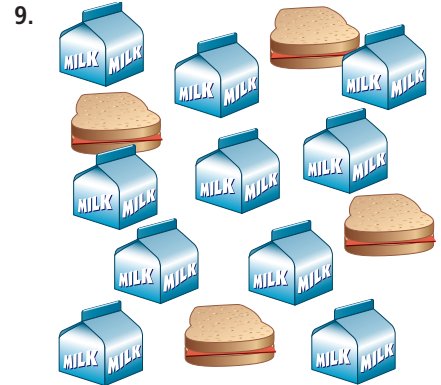
sandwiches to milk cartons

10. **SCHOOL** A class has 6 boys and 15 girls. What is the ratio of boys to girls?

11. **CARS** Audrey counted 6 motorcycles and 27 cars at the restaurant parking lot. Find the ratio of motorcycles to cars.

12. **JEWELRY** The jewelry store is having a sale on 25 emerald rings and 15 ruby rings. Find the ratio of ruby rings to emerald rings.

13. **ANIMALS** An animal shelter has 36 kittens and 12 puppies available for adoption. What is the ratio of puppies to kittens?

14. **ANALYZE TABLES** For reading class, Salvador is keeping track of the types of books he has read so far this year. Find the ratio of mystery books to the total number of books Salvador has read. Then explain its meaning.

Type	Number of Books
Mystery	10
Nonfiction	7
Science Fiction/ Fantasy	5
Western	2

15. **ANALYZE TABLES** Last week, a wireless phone company sold the cell phone covers listed in the table. Find the ratio of black cell phone covers to the total number of cell phone covers sold last week. Then explain its meaning.

Color	Number of Cell Phone Covers
Green	5
Silver	6
Red	3
Black	4

16. **CLOTHES** For a trip, Ramona packed 6 blouses, 5 pairs of shorts, 3 pairs of jeans, and 1 skirt. Find the ratio of pairs of jeans to the total number of pieces of clothing Ramona packed. Then explain its meaning.

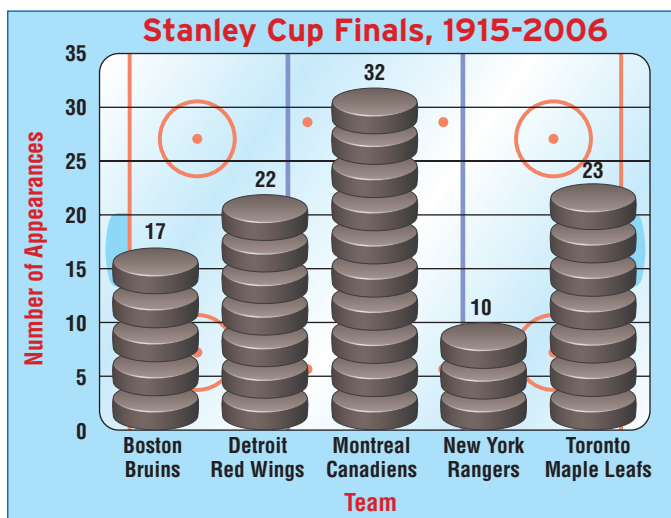**17. FOOD DRIVE** On the first day of the food drive, Mrs. Teasley's classes brought in 6 cans of fruit, 4 cans of beans, 7 boxes of noodles, and 4 cans of soup. Find the ratio of cans of fruit to the total number of food items collected. Then explain its meaning.

Write each rate as a unit rate.

18. 180 words in 3 minutes

19. $36 for 4 tickets

20. $4 for 8 bottles of water

21. $3 for a dozen eggs

22. MARATHON A marathon is approximately 26 miles. If Joshua ran the marathon in 4 hours at a constant rate, how far did he run per hour?

23. RECYCLING 340 trees are saved by recycling 20 tons of paper. How many trees are saved from 1 ton of recycled paper?

ANALYZE GRAPHS **For Exercises 24 and 25, use the graphic. Write each ratio in simplest form. Then explain its meaning.**

Stanley Cup Finals, 1915–2006

Number of Appearances

Team	Boston Bruins	Detroit Red Wings	Montreal Canadiens	New York Rangers	Toronto Maple Leafs
	17	22	32	10	23

Source: National Hockey League

24. Write the ratio that compares the appearances made by the Rangers to the appearances made by the Red Wings.

25. Write the ratio that compares the appearances made by the Maple Leafs to the appearances made by the Bruins.

26. FUNDRAISING The 24 students in Mr. Brown's homeroom sold 72 magazine subscriptions. The 28 students in Mrs. Garcia's homeroom sold 98 magazine subscriptions. Whose homeroom sold more magazine subscriptions per student? Explain your reasoning.

27. PACKAGING A 6-pack of bottled water is on sale for $3. The same bottled water is also available in a 24-pack for $10. Which is less expensive per bottle: the 6-pack or the 24-pack? Explain your reasoning.

H.O.T. Problems

28. OPEN ENDED Create three different drawings showing a number of circles and triangles in which the ratio of circles to triangles is 2:3.

29. CHALLENGE Student Council sold 8 tickets to the spring dance in 15 minutes. At this rate, how many tickets will they sell per hour?

30. FIND THE ERROR Halley and Alma are writing the rate $108 in 6 weeks as a unit rate. Who is correct? Explain your reasoning.

Halley:
$$\frac{\$108}{6 \text{ weeks}} = \frac{\$18}{1 \text{ week}}$$

Alma:
$$\frac{\$108}{6 \text{ weeks}} = \frac{\$54}{3 \text{ weeks}}$$

Halley   Alma

31. **WRITING IN MATH** What is the difference between a ratio and a rate? Give two examples of each.

ISTEP+ PRACTICE 6.1.7

32. While working out at the gym, Rodrigo spends 25 minutes on a treadmill and 35 minutes lifting weights. What is the ratio of the time Rodrigo spends on the treadmill to the time spent lifting weights?

A 2 to 3

B 5 to 7

C 4 to 5

D 1 to 7

33. The table shows the age ranges of the guests at Margo's birthday party. Which ratio accurately compares the number of guests ages 15 to 40 to the total number of guests at the party?

Age Range	Number of Guests
Under 15	11
15–40	6
41–65	3
Over 65	2

F 1:2   **H** 1:11

G 3:22   **J** 3:11

Spiral Review

Divide. Write in simplest form. (Lessons 5-9 and 5-10)

34. $\dfrac{3}{4} \div \dfrac{6}{7}$

35. $\dfrac{1}{8} \div \dfrac{1}{6}$

36. $3\dfrac{8}{9} \div 1\dfrac{2}{3}$

37. $5\dfrac{5}{8} \div 2\dfrac{1}{2}$

38. **BAKING** Viho needs $2\dfrac{1}{4}$ cups of flour for cookies, $1\dfrac{2}{3}$ cups for almond bars, and $3\dfrac{1}{2}$ cups for cinnamon rolls. How much flour does he need in all? (Lesson 5-5)

39. **DECORATING** Janie is arranging a bookshelf, a chair, and a dresser along one wall of her bedroom. Use the *make a list* strategy to find the number of ways Janie can arrange the furniture. (Lesson 4-4)

GET READY for the Next Lesson

PREREQUISITE SKILL **Write each fraction in simplest form.** (Lesson 4-2)

40. $\dfrac{6}{10}$

41. $\dfrac{15}{18}$

42. $\dfrac{3}{12}$

43. $\dfrac{25}{35}$

Math Lab
Ratios and Tangrams

A tangram is a puzzle that is made by cutting a square into seven geometric figures. The puzzle can be formed into many different figures.

In this lab, you will use a tangram to explore ratios and the relationship between ratio and area.

STEP 1 Begin with one sheet of patty paper. Fold the top left corner to the bottom right corner. Unfold and cut along the fold so that two large triangles are formed.

Fold   Cut
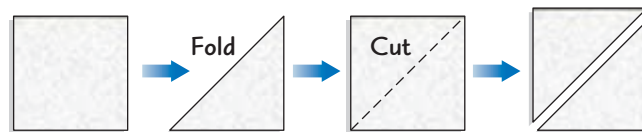
The ratio of the area of one triangle to the area of the original square is 1 to 2.

STEP 2 Use one of the cut triangles. Fold the bottom left corner to the bottom right corner. Unfold and cut along the fold. Label the triangles A and B.

Fold   Cut   A B
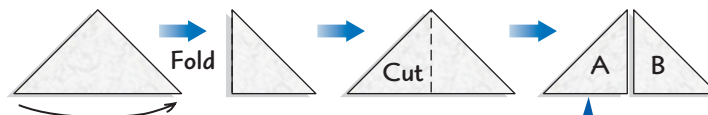
The ratio of the area of triangle A to the area of the original square is 1 to 4.

STEP 3 Use the other large triangle from step 1. Fold the bottom left corner to the bottom right corner. Make a crease and unfold. Next, fold the top down along the crease as shown. Make a crease and cut along the second crease line. Cut out the small triangle and label it C.

Fold/Unfold   Fold/Cut   C
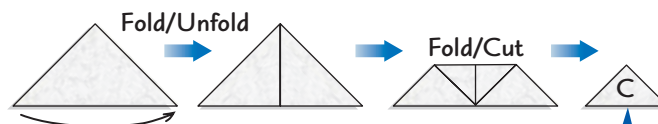
The ratio of the area of triangle C to the area of triangle A is 1 to 2.

STEP 4 Use the remaining piece. Fold it in half from left to right. Cut along the fold. Using the left figure, fold the bottom left corner to the bottom right corner. Cut along the fold and label the triangle D and the square E.

Study Tip

Ratios of Areas If the ratio of the area of triangle D to the area of square E is 1 to 2, then the area of triangle D is *half* the area of square E.

The ratio of the area of triangle D to the area of square E is 1 to 2.

The ratio of the area of triangle D to the area of triangle C is 1 to 2.

Cut

Fold   Cut   D   E
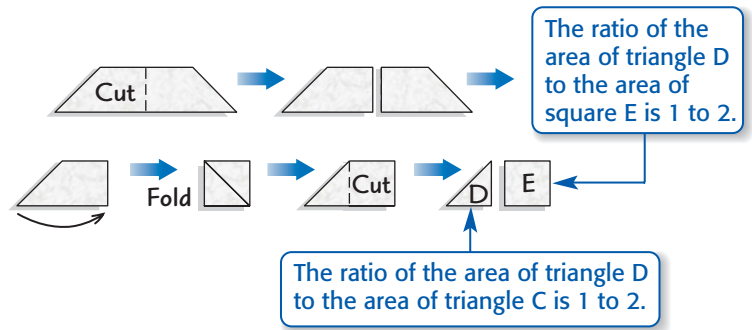
STEP 5 Use the remaining piece. Fold the bottom left corner to the top right corner. Cut along the fold. Label the triangle F and the other figure G.

Fold   Cut   F   G

The ratio of the area of triangle F to the area of the original figure is 1 to 3.
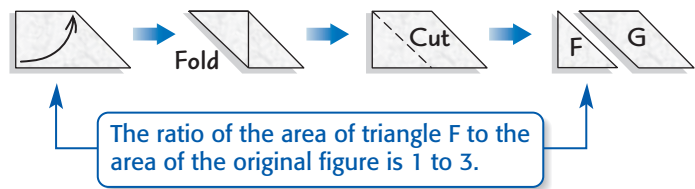
ANALYZE THE RESULTS

1. Suppose the area of triangle B is 1 square unit. Find the area of each triangle.

   a. triangle C   b. triangle F

2. Explain how the area of each of these triangles compares to the area of triangle B.

3. Explain why the ratio of the area of triangle C to the original large square is 1 to 8.

4. Tell why the area of square E is equal to the area of figure G.

5. Find the ratio of the area of triangle F to the original large square. Explain your reasoning.

6. Complete the table. Write the fraction that compares the area of each figure to the original square. What do you notice about the denominators?

Figure	A	B	C	D	E	F	G
Fractional Part of the Large Square							

6-2 Ratio Tables

GET READY for the Lesson

JUICE One can of frozen orange juice concentrate is mixed with 3 cans of water to make one batch of orange juice.

1. How many cans of juice and how many cans of water would you need to make 2 batches that have the same taste? 3 batches? Draw a picture to support your answers.

2. Find the ratio in simplest form of juice to water needed for 1, 2, and 3 batches of juice. What do you notice?

The quantities found in the activity above can be organized into a table. This table is called a **ratio table** because the columns are filled with pairs of numbers that have the same ratio.

Cans of Concentrate	1	2	3
Cans of Water	3	6	9

The ratios $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ are equivalent since each simplifies to a ratio of $\frac{1}{3}$.

Equivalent ratios express the same relationship between two quantities. You can use a ratio table to find equivalent ratios or rates.

EXAMPLE   Equivalent Ratios of Larger Quantities

1 **ICING** To make yellow icing, 6 drops of yellow food coloring are added to 1 cup of white icing. Use the ratio table to find how much yellow to add to 5 cups of white icing to get the same shade.

Cups of Icing	1				5
Drops of Yellow	6				■

METHOD 1   Find a pattern and extend it.

For 2 cups of icing, you would need a total of 6 + 6 or 12 drops.

$+1 \quad +1 \quad +1 \quad +1$

Cups of Icing	1	2	3	4	5
Drops of Yellow	6	12	18	24	30

$+6 \quad +6 \quad +6 \quad +6$

Continue this pattern until you reach 5 cups.

Multiply each quantity by the same number.

	×5 →	
Cups of Icing	1	5
Drops of Yellow	6	**30**

×5 ↗

Since $1 \times 5 = 5$, multiply each quantity by 5.

So, add 30 drops of yellow food coloring to 5 cups of icing.

Study Tip

Check for Accuracy To check your answer for Example 1, check to see if the ratio of the two new quantities is equivalent to the ratio of the original quantities.

$\frac{5}{30} = \frac{5 \div 5}{30 \div 5}$ or $\frac{1}{6}$ ✔

✓ CHOOSE Your Method

a. **NURSING** A patient receives 1 liter of IV fluids every 8 hours. At that rate, use the ratio table to find how many hours it will take to receive 4 liters of IV fluids.

IV Fluids (L)	1	4
Time (h)	8	■

You can also divide each quantity in a ratio by the same number to produce an equivalent ratio involving smaller quantities.

EXAMPLE   **Equivalent Ratios of Smaller Quantities**

2 **HOT DOGS** In a recent year, Takeru Kobayashi won a hot dog eating contest by eating nearly 54 hot dogs in 12 minutes. If he ate at a constant rate, use the ratio table to determine about how many hot dogs he ate every 2 minutes.

Hot Dogs	54		■
Time (min)	12		2

	÷2 → ÷3 →		
Hot Dogs	54	**27**	**9**
Time (min)	12	**6**	2

÷2 ↗ ÷3 ↗

Divide each quantity by one or more common factors until you reach a quantity of 2 minutes.

So, Kobayashi ate about 9 hot dogs every 2 minutes.

✓ CHECK Your Progress

b. **JAM** To make cranberry jam, you need 12 cups of sugar for every 16 cups of cranberries. Use the ratio table to find the amount of sugar needed for 4 cups of cranberries.

Sugar (c)	12		■
Cranberries (c)	16		4

Real-World Link . . . .

In 2007, Joey Chestnut beat Takeru Kobayashi by consuming 66 hot dogs in 12 minutes. Takeru Kobayashi ate 63 hot dogs in 12 minutes.

Source: Nathan's Famous International July Fourth Hot Dog Eating Contest

Multiplying or dividing two related quantities by the same number is called **scaling**. Sometimes you may need to *scale back* and then *scale forward* to find an equivalent ratio.

Use Scaling

3 **GROCERIES** Cans of corn are on sale at 10 for $4. Use the ratio table to find the cost of 15 cans.

Cans of Corn	10		15
Cost in Dollars	4		■

There is no whole number by which you can multiply 10 to get 15. So, scale back to 5 and then scale forward to 15.

$\div 2$ $\times 3$

Cans of Corn	10	**5**	15
Cost in Dollars	4	**2**	**6**

$\div 2$ $\times 3$

Divide each quantity by a common factor, 2.

Then, since $5 \times 3 = 15$, multiply each quantity by 3.

So, 15 cans of corn would cost $6.

CHECK Your Progress

c. **HEIGHT** A child's height measures 105 centimeters. If 25 centimeters is about 10 inches, use the ratio table to estimate the height in inches.

Height (cm)	25		105
Height (in.)	10		■

Real-World EXAMPLE **Use a Ratio Table**

4 **MONEY** On her vacation, Leya exchanged $50 American and received $90 Canadian. Use a ratio table to find how many Canadian dollars she would receive for $20 American.

Set up a ratio table.

Canadian Dollars	90		■
American Dollars	50		20

Label the rows with the two quantities being compared. Then fill in what is given.

Use scaling to find the desired quantity.

$\div 10$ $\times 4$

Canadian Dollars	90	**9**	**36**
American Dollars	50	**5**	20

$\div 10$ $\times 4$

Divide each quantity by a common factor, 10.

Then, since $5 \times 4 = 20$, multiply each quantity by 4.

Leya would receive $36 Canadian for $20 American.

Real-World Link . . . .
The U.S. dollar and the euro account for approximately half of all currency exchanged in the world.

CHECK Your Progress

d. **AUTOMOBILES** Landon owns a hybrid SUV that can travel 400 miles on a 15-gallon tank of gas. Use a ratio table to determine how many miles he can travel on 6 gallons.

For Exercises 1–3, use the ratio tables given to solve each problem.

Example 1
(pp. 322–323)

1. **MONEY** Santiago receives an allowance of $7 every week. How much total does he receive after 4 weeks?

Allowance	7			■
Number of Weeks	1			4

Example 2
(p. 323)

2. **EXERCISE** Tonya runs 8 kilometers in 60 minutes. At this rate, how long would it take her to run 2 kilometers?

Distance Run (km)	8		2
Time (min)	60		■

Example 3
(p. 324)

3. **BEVERAGES** A certain 12-ounce soft drink contains about 10 teaspoons of sugar. If you drink 18 ounces of this soft drink, how many teaspoons of sugar have you consumed?

Ounces of Soft Drink	12		18
Teaspoons of Sugar	10		■

Example 4
(p. 324)

4. **FOOD** Lamika buys 12 packs of juice boxes that are on sale and pays a total of $48. Use a ratio table to determine how much Lamika will pay to buy 8 more packs of juice boxes at the same store.

Practice and Problem Solving

For Exercises 5–10, use the ratio tables given to solve each problem.

5. **PIES** To make 5 apple pies, you need about 2 pounds of apples. How many pounds of apples do you need to make 20 apple pies?

Number of Pies	5		20
Pounds of Apples	2		■

6. **FIELD TRIP** A zoo requires that 1 adult accompany every 7 students that visit the zoo. How many adults must accompany 28 students?

Number of Adults	1			■
Number of Students	7			28

7. **MONEY** Before leaving to visit Mexico, Levant traded 270 American dollars and received 3,000 Mexican pesos. When he returned from Mexico, he had 100 pesos left. How much will he receive when he exchanges these pesos for dollars?

American Dollars	270		■
Mexican Pesos	3,000		100

8. **JEWELRY** Valentina purchased 200 beads for $48 to make necklaces. If she needs to buy 25 more beads, how much will she pay if she is charged the same rate?

Number of Beads	200		25
Cost in Dollars	48		■

9. **KNITTING** Four balls of wool will make 8 knitted caps. How many balls of wool will Malcolm need if he wants to make 6 caps?

Balls of Wool	4		■
Number of Caps	8		6

10. **BIRDS** If a hummingbird were to get all of its food from a feeder, then a 16-ounce nectar feeder could feed about 80 hummingbirds a day. How many hummingbirds would you expect to be able to feed with a 12-ounce feeder?

Ounces of Nectar	16		12
Number of Birds Fed	80		■

11. **BIKING** On a bike trip across the United States, Jason notes that he covers about 190 miles every 4 days. If he continues at this rate, use a ratio table to determine about how many miles he could bike in 6 days.

12. **PHOTOGRAPHY** When a photo is reduced or enlarged, its length to width ratio usually remains the same. Aurelia wants to enlarge a 4-inch by 6-inch photo so that it has a height of 15 inches. Use a ratio table to determine the new width of the photo.

13. **PETS** Before administering medicine, a veterinarian needs to know the animal's weight in kilograms. If 20 pounds is about 9 kilograms and a dog weighs 30 pounds, use a ratio table to find the dog's weight in kilograms. Explain your reasoning.

14. **TRAVEL** On a typical day, flights at a local airport arrive at a rate of 10 every 15 minutes. At this rate, how many flights would you expect to arrive in 1 hour?

RECIPES For Exercises 15–17, use the following information.

A punch recipe that serves 24 people calls for 4 liters of lemon-lime soda, 2 pints of sherbet, and 6 cups of ice.

15. Create a ratio table to represent this situation.

16. How much of each ingredient would you need to make an identical recipe that serves 12 people? 36 people?

17. How much of each ingredient would you need to make an identical recipe that serves 18 people? Explain your reasoning.

18. CHALLENGE Use the ratio table to determine how many people 13 subs would serve. Explain your reasoning.

Number of Subs	3	5	8	13
People Served	12	20	32	■

19. NUMBER SENSE There are 10 girls and 8 boys in Mr. Augello's class. If 5 more girls and 5 more boys join the class, will the ratio of girls to boys remain the same? Justify your answer using a ratio table.

20. WRITING IN MATH Explain two different methods that can be used to find the missing value in the ratio table.

Pages Read	60		80
Number of Days	9		■

ISTEP+ PRACTICE   6.1.7, 6.1.8

21. Paul buys 5 DVDs for $60. At this rate, how much would he pay for 3 DVDs?

A $10
B $30
C $36
D $58

22. SHORT RESPONSE Beth walks 2 blocks in 15 minutes. How many blocks would Beth walk if she walked at the same rate for an hour?

23. Jay Len is making biscuits using the recipe below.

Whole Wheat Biscuits

2 c — Whole wheat flour
4 tsp — Baking powder
$\frac{1}{2}$ tsp — Salt
2 tbsp — Shortening
1 c — Milk
1 — Small egg

Makes 20 biscuits

How many cups of flour will he need to make 30 biscuits?

F $1\frac{1}{2}$ cups   **H** 10 cups
G 3 cups   **J** 15 cups

Spiral Review

24. HORSES A horse ranch has 6 mustangs and 18 Arabians. Write the ratio of mustangs to Arabians as a fraction in simplest form. Then explain its meaning. (Lesson 6-1)

Divide. Write in simplest form. (Lesson 5-10)

25. $5\frac{3}{4} \div 3\frac{3}{4}$ **26.** $5 \div 2\frac{1}{3}$ **27.** $2\frac{1}{2} \div 1\frac{4}{5}$

28. List the next five common multiples after the LCM of 6 and 9. (Lesson 4-5)

GET READY for the Next Lesson

PREREQUISITE SKILL Write each rate as a unit rate. (Lesson 6-1)

29. $24 for 3 hats   **30.** 130 miles in 2 hours   **31.** 145 students for 5 teachers

Graphing Calculator Lab
Ratio Tables

You can use the CellSheet application on a TI-83/84 Plus graphing calculator to compare the output/input ratios of real-world functions.

ACTIVITY

1 **MOVIES** The total cost of purchasing 1, 2, 3, 4, and 5 DVDs for $19 each is found by multiplying the number of DVDs purchased by 19. Create a table to model this situation. Include a column that calculates the ratio of cost to DVDs.

STEP 1 Access CellSheet by pressing [APPS] 8 [ENTER] [ENTER].

STEP 2 Enter the heading DVDS in cell A1 by pressing 2nd [ALPHA] ["] [D] [V] [D] [S] [ENTER]. Similarly, enter the heading COST in cell B1 and RATIO in cell C1.

STEP 3 Enter the numbers 1 through 5 into cells A2 through A6, respectively. Then, insert the formula =A2*19 in cell B3 by pressing [STO▸] [ALPHA] [A] 2 [×] 19.

STEP 4 Calculate the cost for each number of DVDs by copying the formula to cells B4 through B6. Move to cell B3 and press [F3] to copy, [F1] [▼] [▼] [▼] [▼] to select the range of cells, and [F4] to paste.

STEP 5 Use a similar procedure to insert, copy, and paste the formula B2/A2 in cells C3 through C6.

ANALYZE THE RESULTS

1. Does the 2-column table of values for DVDs and Cost form a ratio table? Explain your reasoning.

2. **CLOTHING** A store offers $5 off any purchase over $10. Create a graphing calculator table that models the total cost of purchasing $11 through $14 in clothing. Include a ratio column of cost:amount.

3. Does the 2-column table of values for the amount and cost form a ratio table? Explain your reasoning.

Proportions

GET READY for the Lesson

PHOTOGRAPHY Leon spent $2 to make 10 prints from his digital camera. Later, he went back to the same store and spent $6 to make 30 prints.

Number of Prints	Cost ($)
10	2
30	6

1. Express the relationship between the total cost and number of prints he made for each situation as a rate in fraction form.

2. Compare the relationship between the numerators of each rate you wrote in Exercise 1. Compare the relationship between the denominators of these rates.

3. Are the rates you wrote in Exercise 1 equivalent? Explain.

In the situations above, there are two related quantities: the number of prints and the cost for these prints. Notice that both quantities change, but in the same way.

$$\times 3$$

Number of Prints	10	30
Cost ($)	2	6

$$\times 3$$

As the number of prints triples, the cost also triples.

By comparing these quantities as rates in simplest form, you can see that the relationship between the two quantities stays the same.

$$\frac{10 \text{ prints}}{\$2} \overset{\div 2}{\underset{\div 2}{=}} \frac{5 \text{ prints}}{\$1} \quad \text{and} \quad \frac{30 \text{ prints}}{\$6} \overset{\div 6}{\underset{\div 6}{=}} \frac{5 \text{ prints}}{\$1}$$

Two quantities are **proportional** if they have a constant ratio or rate. In the example above, the cost for making prints is proportional to the number of prints because each quantity has a constant rate of $1 for 5 prints.

A proportional relationship is often expressed by writing a proportion.

Proportion   Key Concept

Words   A **proportion** is an equation stating that two ratios or rates are equivalent.

Examples $\dfrac{2}{5} = \dfrac{6}{15}$ $\dfrac{\$2}{10 \text{ prints}} = \dfrac{\$6}{30 \text{ prints}}$

There are different ways to determine if the relationship between two quantities is proportional. One way is by examining unit rates.

EXAMPLES Use Unit Rates

Determine if the quantities in each pair of rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

1 **20 miles in 5 hours; 45 miles in 9 hours**

Write each rate as a fraction. Then find its unit rate.

$$\frac{20 \text{ miles}}{5 \text{ hours}} = \frac{4 \text{ miles}}{1 \text{ hour}} \qquad (\div 5)$$

$$\frac{45 \text{ miles}}{9 \text{ hours}} = \frac{5 \text{ miles}}{1 \text{ hour}} \qquad (\div 9)$$

Since the rates do not have the same unit rate, they are not equivalent. So, the number of miles is not proportional to the number of hours.

2 **3 T-shirts for $21; 5 T-shirts for $35**

$$\frac{\$21}{3 \text{ T-shirts}} = \frac{\$7}{1 \text{ T-shirt}} \qquad (\div 3)$$

$$\frac{\$35}{5 \text{ T-shirts}} = \frac{\$7}{1 \text{ T-shirt}} \qquad (\div 5)$$

Since the rates have the same unit rate, they are equivalent. The cost is proportional to the number of T-shirts. So, $\dfrac{\$21}{3 \text{ T-shirts}} = \dfrac{\$35}{5 \text{ T-shirts}}$.

3 **READING** **Felisa read the first 60 pages of a book in 3 days. She read the last 90 pages in 6 days. Are these reading rates proportional? Explain your reasoning.**

$$\frac{60 \text{ pages}}{3 \text{ days}} = \frac{20 \text{ pages}}{1 \text{ day}} \qquad (\div 3)$$

$$\frac{90 \text{ pages}}{6 \text{ days}} = \frac{15 \text{ pages}}{1 \text{ day}} \qquad (\div 6)$$

Since the rates do not have the same unit rate, they are not equivalent. So, Felisa's reading rates were not proportional.

✓ CHECK Your Progress

a. **JEWELRY** Marcia made 10 bracelets for 5 friends. Jen made 12 bracelets for 4 friends. Are these rates proportional? Explain.

b. **FUNDRAISING** Club A raised $168 by washing 42 cars. Club B raised $152 by washing 38 cars. Are these fundraising rates proportional? Explain your reasoning.

If a unit rate is not easily found, check to see if the rates are equivalent. If they are, then the quantities are proportional.

EXAMPLES Use Equivalent Fractions

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

4 **3 free throws made out of 7 attempts;**
9 free throws made out of 14 attempts

Write each ratio as a fraction.

$$\frac{3 \text{ free throws}}{7 \text{ attempts}} \overset{?}{=} \frac{9 \text{ free throws}}{14 \text{ attempts}}$$

×3

×2

The numerator and the denominator are not multiplied by the same number. So, the fractions are not equivalent.

The number of free throws made is not proportional to the number of attempts.

5 **6 DVDs for $90; 3 DVDs for $45**

$$\frac{6 \text{ DVDs}}{\$90} \overset{?}{=} \frac{3 \text{ DVDs}}{\$45}$$

÷2

÷2

The numerator and the denominator are divided by the same number. So, the fractions are equivalent.

The number of DVDs is proportional to the cost.

CHECK Your Progress

c. 5 packs of baseball cards for $15; 10 packs of baseball cards for $30
d. 12 girls out of 16 students; 4 girls out of 8 students

CHECK Your Understanding

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

Examples 1, 2
(p. 330)

1. $24 saved after 3 weeks; $52 saved after 7 weeks

2. 270 Calories in 3 servings; 450 Calories in 5 servings

Examples 4, 5
(p. 331)

3. 3 hours worked for $12; 9 hours worked for $36

4. 16 breaths in 60 seconds; 14 breaths in 15 seconds

Example 3
(p. 330)

5. **FITNESS** Micah can do 75 push-ups in 3 minutes. Eduardo can do 130 push-ups in 5 minutes. Are these rates proportional? Explain.

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

6. $12 for 3 paperback books; $28 for 7 paperback books

7. 16 points scored in 4 games; 48 points scored in 8 games

8. 96 words typed in 3 minutes; 160 words typed in 5 minutes

9. $3 for 6 bagels; $9 for 24 bagels

10. 288 miles driven on 12 gallons of fuel; 240 miles driven on 10 gallons of fuel

11. 15 computers for 45 students; 45 computers for 135 students

12. 12 minutes to drive 30 laps; 48 minutes to drive 120 laps

13. 16 out of 28 students own pets; 240 out of 560 students own pets

14. **PHOTOGRAPHY** Jade enlarged the photograph at the right to a poster. The size of the poster is 60 inches by 100 inches. Is the size of the poster proportional to the size of the photograph? Explain your reasoning.

3 in.

5 in.

15. **SURVEY** One school survey showed that 3 out of 5 students buy their lunch. Another survey showed that 12 out of 19 students buy their lunch. Are these results proportional? Explain.

BASEBALL **For Exercises 16–18, refer to the table below. Determine if each pair of players made proportionally the same number of hits. Explain.**

16. Mark Teixeira and Eric Bruntlett

17. Hideki Matsui and Mark Teixeira

18. Brad Eldred and Ramon Santiago

2007 Spring Batting Statistics			
Player	**Team**	**At Bats**	**Hits**
Mark Teixeira	Texas Rangers	48	12
Brad Eldred	Pittsburgh Pirates	66	20
Hideki Matsui	New York Yankees	60	16
Eric Bruntlett	Houston Astros	56	14
Ramon Santiago	Detroit Tigers	33	10

Source: Major League Baseball

19. **ANALYZE TABLES** Of the players listed in the table above, did the player who made the most hits have the best record? Explain.

20. **TESTS** On a math test, it took Kiera 30 minutes to do 6 problems. Heath finished 18 problems in 40 minutes. Did the students work proportionally at the same rate? Explain your reasoning.

21. **SAVINGS** Rosalinda saved $35 in 5 weeks. Her sister saved $56 in 56 days. Did each sister save proportionally the same amount of money? Explain.

H.O.T. Problems

CHALLENGE For Exercises 22–25, use the following information to verify each proportion. Justify your response.

To verify a proportion, you can use cross products. If the product of the *means* equals the product of the *extremes*, then the two ratios form a proportion. In a proportion, the top left and bottom right numbers are the extremes. The top right and bottom left numbers are the means. In the proportion in Exercise 22, the numbers 5 and 9 are the means and the numbers 3 and 15 are the extremes.

22. $\frac{3}{5} = \frac{9}{15}$ **23.** $\frac{2}{7} = \frac{5}{21}$ **24.** $\frac{1}{8} = \frac{3}{28}$ **25.** $\frac{4}{9} = \frac{12}{27}$

26. **WRITING IN MATH** Cecil's Pizza offers two large pizzas for $15 and four large pizzas for $28. Describe and use three different ways to determine if the pair of ratios is proportional.

ISTEP+ PRACTICE   6.1.7, 6.1.8

27. The ratio of girls to boys in the junior high band is 3 to 4. Which of these shows possible numbers of the girls and boys in the band?

  A 30 girls, 44 boys

  B 27 girls, 36 boys

  C 22 girls, 28 boys

  D 36 girls, 50 boys

28. Which of the following shows an equivalent way to show the cost of the tomatoes?

```
HOME-GROWN VEGETABLES
  Cucumbers        6 for $2
  Peppers         12 for $9
  Tomatoes         6 for $4
```

  F 15 for $10   **H** 12 for $9

  G 20 for $15   **J** 8 for $6

Spiral Review

29. **SHOPPING** Walter purchased 2 CDs for $26. Use a ratio table to find how much he would pay for 6 CDs. (Lesson 6-2)

30. **MEASUREMENT** A caterpillar is 4 centimeters long, and a Monarch butterfly is 10 centimeters long. Write the ratio of the caterpillar's length to the butterfly's length as a fraction in simplest form. (Lesson 6-1)

Find the prime factorization of each number. (Lesson 1-2)

31. 15   **32.** 94   **33.** 102   **34.** 126

GET READY for the Next Lesson

PREREQUISITE SKILL Write each rate as a unit rate. (Lesson 6-1)

35. 56 wins in 8 years   **36.** $12 for 5 hot dogs   **37.** $21 for 3 hours

Algebra: Solving Proportions

▶ GET READY for the Lesson

SHOES A department store is selling flip flops for $5 a pair.

1. How many pairs of flip flops can you buy with $20? $25?

2. Write a proportion to express the relationship between the cost of 3 pairs of flip flops and the cost c of 7 pairs of flip flops.

3. How much will it cost to buy 6 pairs of flip flops?

Number of Pairs	Price ($)
1	5
2	10
3	15

When you find an unknown value in a proportion, you are *solving the proportion*. As you discovered in Lesson 6-3, there are different methods to determine if a relationship is proportional. You can use these same methods to solve a proportion.

EXAMPLES   Solve Using Equivalent Fractions

Solve each proportion.

1 $\dfrac{4}{7} = \dfrac{m}{35}$

Find a value for m so the fractions are equivalent.

$$\dfrac{4}{7} = \dfrac{m}{35} \qquad \begin{array}{c} \times 5 \\ \\ \times 5 \end{array}$$

Since $7 \times 5 = 35$, multiply the numerator and denominator by 5.

$$\dfrac{4}{7} = \dfrac{20}{35} \qquad \text{Since } 4 \times 5 = 20,\ m = 20.$$

2 $\dfrac{12}{15} = \dfrac{4}{y}$

$$\dfrac{12}{15} = \dfrac{4}{y} \qquad \begin{array}{c} \div 3 \\ \\ \div 3 \end{array}$$

Since $12 \div 3 = 4$, divide the numerator and denominator by 3.

$$\dfrac{12}{15} = \dfrac{4}{5} \qquad \text{Since } 15 \div 3 = 5,\ y = 5.$$

3 $\dfrac{x}{16} = \dfrac{7}{8}$

$\overset{\div 2}{\dfrac{x}{16}} = \dfrac{7}{8}$ Since $16 \div 2 = 8$, divide the numerator and denominator by 2.
$\underset{\div 2}{}$

$\dfrac{\mathbf{14}}{16} = \dfrac{7}{8}$ THINK What number divided by 2 is 7? The answer is 14.

So, $x = 14$.

✔ CHECK Your Progress

Solve each proportion.

a. $\dfrac{2}{3} = \dfrac{n}{9}$ b. $\dfrac{30}{54} = \dfrac{z}{9}$ c. $\dfrac{5}{8} = \dfrac{40}{x}$

Proportions can also be used to make predictions.

EXAMPLE   **Make Predictions in Proportional Situations**

4 **RESTAURANTS** In the United States, 12 out of every 15 people prefer eating at a restaurant over cooking at home. Use this ratio to predict how many people out of 500 would prefer eating at a restaurant.

Write and solve a proportion. Let p represent the number of people who can be expected to prefer eating at a restaurant.

prefer restaurants → $\dfrac{12}{15} = \dfrac{p}{500}$ ← prefer restaurants
total people →   ← total people

The denominators 15 and 500 are not easily related by multiplication. So, simplify the ratio 12 out of 15. Then solve using equivalent fractions.

$\overset{\div 3 \quad \times 100}{\dfrac{12}{15}} = \dfrac{4}{5} = \dfrac{\mathbf{400}}{500}$ Since $5 \times 100 = 500$, multiply the numerator and
$\underset{\div 3 \quad \times 100}{}$ denominator by 100.

So, about 400 out of 500 people can be expected to prefer eating at a restaurant.

✔ CHECK Your Progress

d. **ICE CREAM** There are 810 Calories in 3 scoops of vanilla ice cream. About how many Calories are there in 7 scoops of ice cream?

e. **BED TIME** If 15 out of 25 students go to bed before 10 P.M., predict how many go to bed before 10 P.M. in a school of 1,000 students.

You can also examine unit rates to solve a proportion.

EXAMPLE **Solve Using Unit Rates**

5 **DRIVING** The Millers drove 105 miles on 3 gallons of gas. At this rate, how many miles can they drive on 10 gallons of gas?

Step 1 Set up the proportion. Let a represent the number of miles that can be driven on 10 gallons of gas.

$$\frac{105 \text{ miles}}{3 \text{ gallons of gas}} = \frac{a \text{ miles}}{10 \text{ gallons of gas}}$$

Step 2 Find the unit rate.

$$\frac{105 \text{ miles}}{3 \text{ gallons of gas}} \overset{\div 3}{\underset{\div 3}{=}} \frac{35 \text{ miles}}{1 \text{ gallon of gas}}$$

Find an equivalent fraction with a denominator of 1.

Step 3 Rewrite the proportion using the unit rate and solve using equivalent fractions.

$$\frac{105 \text{ miles}}{3 \text{ gallons of gas}} \overset{\div 3}{\underset{\div 3}{=}} \frac{35 \text{ miles}}{1 \text{ gallon of gas}} \overset{\times 10}{\underset{\times 10}{=}} \frac{\textbf{350} \text{ miles}}{10 \text{ gallons of gas}}$$

So, the value of a is 350. At the given rate, the Millers can drive 350 miles on 10 gallons of gas.

CHECK Your Progress

f. **NUTRITION** Three average-size apples contain 180 Calories. How many average-size apples contain 300 Calories?

CHECK Your Understanding

Examples 1–3
(pp. 334–335)

Solve each proportion.

1. $\dfrac{3}{4} = \dfrac{x}{20}$ 2. $\dfrac{5}{4} = \dfrac{a}{36}$ 3. $\dfrac{18}{20} = \dfrac{9}{n}$

Example 4
(p. 335)

4. **PETS** Out of 30 students surveyed, 17 have a dog. Based on these results, predict how many of the 300 students in the school have a dog.

5. **LOCKERS** If one out of 12 students at a school share a locker, predict how many share a locker in a school of 456 students.

Example 5
(p. 336)

6. **PARTIES** If 84 cookies will serve 28 students, how many cookies are needed for 30 students?

7. **SUNGLASSES** Chet spent $24 on two pairs of sunglasses. At this rate, how much would 6 pairs of sunglasses cost?

Practice and Problem Solving

Solve each proportion.

8. $\dfrac{2}{5} = \dfrac{w}{15}$

9. $\dfrac{3}{4} = \dfrac{z}{28}$

10. $\dfrac{7}{d} = \dfrac{35}{10}$

11. $\dfrac{4}{x} = \dfrac{16}{28}$

12. $\dfrac{p}{3} = \dfrac{25}{15}$

13. $\dfrac{h}{8} = \dfrac{6}{16}$

14. $\dfrac{6}{7} = \dfrac{18}{c}$

15. $\dfrac{21}{35} = \dfrac{3}{r}$

16. NEWSPAPER A recent survey reported that out of 50 teenagers, 9 said they get most of their news from a newspaper. At this rate, how many out of 300 teenagers would you expect to get their news from a newspaper?

17. HORSES A Clydesdale drinks about 120 gallons of water every 4 days. At this rate, about how many gallons of water does a Clydesdale drink in 28 days?

18. DVDs Nata spent $28 on 2 DVDs. At this rate, how much would 5 DVDs cost?

19. LUNCH Four students spent $12 on school lunch. At this rate, find the amount 10 students would spend on the same school lunch.

20. BASEBALL If 15 baseballs weigh 75 ounces, how many baseballs weigh 15 ounces?

21. HEALTH In 10 minutes, a heart can beat 700 times. At this rate, in how many minutes will a heart beat 140 times?

Solve each proportion.

22. $\dfrac{11}{13} = \dfrac{x}{91}$

23. $\dfrac{96}{128} = \dfrac{12}{c}$

24. $\dfrac{5}{12} = \dfrac{x}{6}$

Real-World Link ····
A Clydesdale consumes about 20 quarts of feed and 40 to 50 pounds of hay each day.

25. SCHOOL Suppose 8 out of every 20 students are absent from school less than five days a year. Predict how many students would be absent from school less than five days a year in a school system of 40,000 students.

26. ANALYZE TABLES The table shows which school subjects are favored by a group of students. Write and solve a proportion that could be used to predict the number of students out of 400 that would pick science as their favorite subject.

Favorite School Subject	
Subject	**Number of Responses**
Math	6
Science	3
English	4
History	7

27. YOGA Liliana takes 4 breaths per 10 seconds during yoga. At this rate, about how many breaths would Liliana take in 2 minutes of yoga?

28. CONTESTS For a store contest, 4 out of every 65 people who visit a store will receive a free DVD. If 455 people have visited the store, how many DVDs have been given away?

29. ANALYZE TABLES There were 340,000 cattle placed on feed in Texas in February 2005. Write a proportion that could be used to find how many of these cattle were between 700 and 799 pounds. How many of the 340,000 cattle placed on feed were between 700 and 799 pounds?

30. FIND THE DATA Refer to the Data File on pages 16–19. Choose some data and write a real-world problem in which you would write and solve a proportion.

Cattle Placed on Feed in Texas, February 2005	
Weight Group	**Fraction of Total Cattle**
Less than 600 pounds	$\frac{1}{5}$
600–699 pounds	$\frac{11}{50}$
700–799 pounds	$\frac{2}{5}$
800 pounds	$\frac{9}{50}$

Source: National Agriculture Statistics Service

H.O.T. Problems

31. OPEN ENDED One rate of a proportion is $\frac{9}{n}$. Select two other rates to form the proportion, one that can be solved using equivalent fractions and the other that can be solved with unit rates. Then solve the proportion using each method.

32. FIND THE ERROR James and Akiko are setting up a proportion to solve the following problem. Who set up their proportion correctly? Explain your reasoning.

Angelina's mom teaches at a preschool. There is 1 teacher for every 12 students at the preschool. There are 276 students at the preschool. How many teachers are there at the preschool?

$$\frac{12}{1} = \frac{x}{276}$$

James

$$\frac{1}{12} = \frac{x}{276}$$

Akiko

33. REASONING Tell whether the following statement is *sometimes*, *always*, or *never* true for numbers greater than zero. Explain.

In a proportion, if the numerator of the first ratio is greater than the denominator of the first ratio, then the numerator of the second ratio is greater than the denominator of the second ratio.

34. CHALLENGE Suppose 25 out of 175 people said they like to play disc golf and 5 out of every 12 of the players have a personalized flying disc. At the same rate, in a group of 252 people, predict how many you would expect to have a personalized flying disc.

35. WRITING IN MATH Jonah can run 3 laps in 24 minutes. At the same rate, about how many laps can Jonah run in 50 minutes? Explain your reasoning.

36. A spinner is divided into equal sections. There are 6 green sections and 4 yellow sections. Damon spins the spinner 30 times. Which proportion can be used to find y, the number of times that the spinner can be expected to land on a yellow section?

   A $\dfrac{y}{30} = \dfrac{4}{6}$

   B $\dfrac{y}{30} = \dfrac{6}{10}$

   C $\dfrac{y}{30} = \dfrac{4}{10}$

   D $\dfrac{y}{30} = \dfrac{6}{4}$

37. **SHORT RESPONSE** Student Council sells bottled water at the cheerleading competition. They sold 3 cases in 20 minutes. If they continue selling bottled water at this rate, how many cases of bottled water would they sell in 3 hours?

38. The ratio of green pepper plants to red pepper plants in Adeline's garden is about 3 to 5. If there are 20 red pepper plants, about how many green pepper plants are there?

   F 35   H 12

   G 16   J   6

Spiral Review

Determine if the quantities in each pair of rates are proportional. Explain your reasoning and express each proportional relationship as a proportion. (Lesson 6-3)

39. $36 for 4 baseball hats; $56 for 7 baseball hats

40. 12 posters for 36 students; 21 posters for 63 students

41. **TUTORS** A tutor charges $30 for 2 hours. Use the ratio table to determine how much she charges for 5 hours. (Lesson 6-2)

Cost	30		■
Number of Hours	2		5

42. Order $\dfrac{1}{4}, \dfrac{7}{40}, \dfrac{1}{5}, \dfrac{3}{20}$ from least to greatest. (Lesson 4-8)

WEATHER For Exercises 43 and 44, refer to the table at the right. (Lesson 2-2)

43. Make a line graph of the data.

44. Describe the change in the daily high temperature from Tuesday to Friday.

5-Day Forecast	
Day	High Temperature (°F)
Monday	76
Tuesday	81
Wednesday	78
Thursday	62
Friday	53

GET READY for the Next Lesson

45. **PREREQUISITE SKILL** The Willis Tower in Chicago has 30 stories more than the Aon Center in Chicago. Together, both buildings have 190 stories. Find the number of stories in each building. Use the *guess and check* strategy. (Lesson 1-7)

1. **CLASSES** Tyson's math class has 12 boys and 8 girls. What is the ratio of boys to girls? (Lesson 6-1)

2. **SALES** At a bake sale, 15 cookies and 40 brownies were sold. What is the ratio of cookies sold to brownies sold? (Lesson 6-1)

Write each rate as a unit rate. (Lesson 6-1)

3. 171 miles in 3 hours

4. $15 for 3 pounds

5. **MULTIPLE CHOICE** A hockey team made four of their 10 attempted goals. Which ratio compares the number of goals made to the number of goals attempted? (Lesson 6-1)

   A $\frac{4}{5}$

   B $\frac{3}{5}$

   C $\frac{5}{2}$

   D $\frac{2}{5}$

For Exercises 6 and 7, use the ratio tables given to solve each problem. (Lesson 6-2)

6. **MONEY** Peyton spends $15 on lunch every week. At this rate, how much money will he spend in 5 weeks?

Number of Weeks	1				5
Money Spent ($)	15				■

7. **DISHES** Charlee washes 10 dishes in 8 minutes. At this rate, how long will it take her to wash 25 dishes?

Number of Dishes	10		25
Number of Minutes	8		■

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion. (Lesson 6-3)

8. $4 for 12 doughnuts; $9 for 36 doughnuts

9. 24 pages read in 8 minutes; 72 pages read in 24 minutes

10. 48 out of 64 students own cell phones; 192 out of 258 students own cell phones

11. **MULTIPLE CHOICE** The ratio of brown tiles to tan tiles in a mosaic is 2 to 3. Which of these shows the possible numbers of brown tiles and tan tiles in the mosaic? (Lesson 6-3)

    F 16 brown tiles, 24 tan tiles

    G 14 brown tiles, 20 tan tiles

    H 12 brown tiles, 19 tan tiles

    J 8 brown tiles, 9 tan tiles

Solve each proportion. (Lesson 6-4)

12. $\frac{x}{6} = \frac{12}{18}$

13. $\frac{8}{20} = \frac{30}{x}$

14. $\frac{3}{d} = \frac{9}{15}$

15. $\frac{24}{72} = \frac{x}{6}$

16. **MULTIPLE CHOICE** Christina made 4 bracelets in 36 minutes. At this rate, how many bracelets would she make in 108 minutes? (Lesson 6-4)

    A 8

    B 9

    C 12

    D 16

17. **ACTIVITIES** Suppose 8 out of 24 students in a classroom participate in after-school activities. At the same rate, predict how many students in a school of 960 can be expected to participate in after-school activities. (Lesson 6-4)

Problem-Solving Investigation

MAIN IDEA: Solve problems by looking for a pattern.

Academic Standards **P.1.3** Apply and adapt a variety of appropriate strategies to solve problems. **P.1.4** Monitor and reflect on the process of mathematical problem solving. *Also addresses P.1.1.*
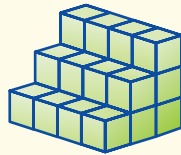
P.S.I. TEAM +

e-Mail: LOOK FOR A PATTERN

LISETE: I'm building a model of a set of stairs using cubes. I used 4 cubes to make the first step, 8 cubes for the second step, and 12 cubes for the third step.

YOUR MISSION: Use the look for a pattern strategy to find how many cubes will be used to make the eighth step.

Understand	You know how many cubes were used to make the first three steps. You need to find how many cubes are needed to make the eighth step.
Plan	Look for a pattern to find the total number of cubes needed.
Solve	Use a table to find the pattern.

Step Number	Number of Cubes
1	4
2	8
3	12
⋮	⋮
8	■

The number of cubes is 4 times the step number. So, the number of cubes needed for the eighth step is equal to 8×4, or 32 cubes.

Check	Draw a sketch of all eight steps. Count the total number of cubes. Since there are 32 cubes altogether, the answer is correct. ✔

Analyze The Strategy

1. Explain when you would use the *look for a pattern* strategy to solve a problem.

2. **WRITING IN MATH** Write a problem that can be solved by looking for a pattern. Then write the steps you would take to find the solution.

Mixed Problem Solving

Use the *look for a pattern* strategy to solve Exercises 3–5.

3. **MONEY** Every year, Miguel receives $20 for his birthday, plus $1 for each year of his age. Lauren receives $10 for her birthday and $2 for each year of her age. In 2009, Miguel is 10, and Lauren is 8. In what year will they both receive the same amount of money?

4. **GEOMETRY** Draw the next two figures in the pattern below.

15   10   6
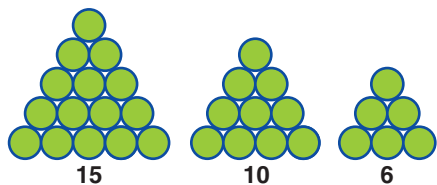
5. **NUMBER SENSE** Describe the pattern below. Then find the next three numbers.

3, 6, 10, 15, 21, ■, ■, ■

Use any strategy to solve Exercises 6–14. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES
• Guess and check.
• Look for a pattern.
• Act it out.

6. **FOOD** Which is more, $\frac{3}{8}$ of a pizza or $\frac{1}{3}$ of a pizza?

7. **MONEY** The admission for a fair is $6 for adults, $4 for children, and $3 for senior citizens. Twelve people paid a total of $50 for admission. If 8 children attended, how many adults and senior citizens attended?

8. **FOOD** About how much more money is spent on strawberry and grape jelly than the other types of jelly each year?

Yearly Jelly Sales (thousands)	
strawberry and grape	$366.2
all others	$291.5

Source: Nielson Marketing Research

9. **NUMBER THEORY** The triangle below is known as *Pascal's Triangle*. If the pattern continues, what will the numbers in the next row be from left to right?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
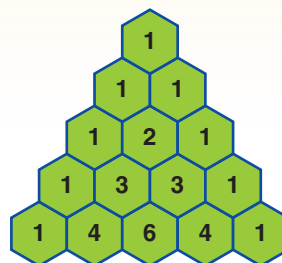
10. **LANGUAGE ARTS** On Monday, 86 science fiction books were sold at a book sale. This is 8 more than twice the amount sold on Thursday. How many science fiction books were sold on Thursday?

11. **PATTERNS** Find the number of toothpicks needed to create figure 8 in the pattern below.

Figure 1   Figure 2   Figure 3

12. **TICKETS** The total price, including a service fee, of a concert ticket is $66.45. If the service fee was $4.95, what was the original cost of the concert ticket?

13. **TRAVEL** Mr. Ishikawa left Kansas City, Missouri, at 3:00 P.M. and arrived in Tulsa, Oklahoma, at 8:00 P.M., driving a distance of approximately 240 miles. During his trip, he took a one-hour dinner break. What was Mr. Ishikawa's average speed in miles per hour?

14. **FIELD TRIPS** Mrs. Samuelson had $350 to spend on a field trip for herself and 18 students. The rate of admission was $13 per person and lunch cost about $5 per person. How much money was left after the trip?

Sequences and Expressions

GET READY for the Lesson

PIZZA The table shows the number of slices of pizza for different numbers of large pizzas.

The Pizza Palace	
Number of Pizzas	**Number of Slices**
1	8
2	16
3	24
4	32

1. Find the rate of slices to the number of pizzas for each row in the table.

2. Is the number of pizzas proportional to the number of slices? Explain your reasoning.

3. Make an ordered list of the number of slices and describe the pattern between consecutive numbers in this list.

4. What relationship appears to exist between the pattern found in Exercise 3 and the rates found in Exercise 1?

The number of slices in the table above is an example of a sequence. A **sequence** is a list of numbers in a specific order. Each number in the list at the right is called a **term** of the sequence.

sequence

8, **16**, 24, 32, …
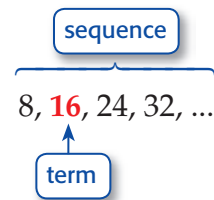
term

This sequence is an **arithmetic sequence** because each term is found by adding the same number to the previous term.

8,   16,   24,   32, …

+8   +8   +8
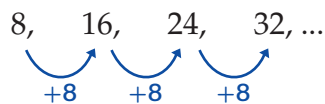
There are several ways of showing a sequence. In addition to being shown as a list, a sequence can also be shown in a table. The table gives both the position of each term in the list and the value of the term.

List

8, 16, 24, 32, …

Table

Position	1	2	3	4
Value of Term	8	16	24	32

You can also write an algebraic expression to describe a sequence. The value of each term can be described as a function of its position in the sequence.

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the tenth term in the sequence.

1
Position	1	2	3	4	n
Value of Term	3	6	9	12	■

Notice that the value of each term is 3 times its position number. So, the value of the term in position n is $3n$.

Position	Multiply by 3	Value of Term
1	1×3	3
2	2×3	6
3	3×3	9
4	4×3	12
n	$n \times 3$	$3n$

Now find the value of the tenth term.

$3n = 3 \cdot 10$ Replace n with 10.
$\quad = 30$ Multiply.

The value of the tenth term in the sequence is 30.

2
Position	6	7	8	9	n
Value of Term	2	3	4	5	■

The value of each term is 4 less than its position number. So, the value of the term in position n is $n - 4$.

Position	Subtract 4	Value of Term
6	$6 - 4$	2
7	$7 - 4$	3
8	$8 - 4$	4
9	$9 - 4$	5
n	$n - 4$	$n - 4$

Study Tip

Arithmetic Sequences Some arithmetic sequences are proportional. In Example 2, $\frac{6}{2} \neq \frac{7}{3} \neq \frac{8}{4} \neq \frac{9}{5}$. However, all arithmetic sequences can be described by an algebraic expression.

Now find the value of the tenth term.

$n - 4 = 10 - 4$ Replace n with 10.
$\qquad = 6$ Subtract.

The value of the tenth term in the sequence is 6.

✓**CHECK Your Progress**

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the eighth term in the sequence.

a.
Position	10	11	12	13	n
Value of Term	7	8	9	10	■

b.
Position	2	3	4	5	n
Value of Term	12	18	24	30	■

3 **MEASUREMENT** There are 12 inches in 1 foot. Make a table and write an algebraic expression relating the number of feet to the number of inches. Then find Becca's height in feet if she is 60 inches tall.

Inches	Feet
12	1
24	2
36	3
48	4
n	$n \div 12$

Notice that the number of inches divided by 12 gives the number of feet. So, to find Becca's height, use the expression $n \div 12$.

$n \div 12 = 60 \div 12$ Replace n with 60.

$ = 5$ Divide.

So, Becca is 5 feet tall.

✓ **CHECK Your Progress**

c. **RUNNING** There are 3 feet in 1 yard. Make a table and write an algebraic expression relating the number of feet to the number of yards. Then find how many feet Summer ran if she ran 400 yards.

ISTEP+ EXAMPLE   **6.2.5**

4 Which expression was used to create the table at the right?

Position	Value of Term
1	5
2	7
3	9
n	■

A $n + 3$ **C** $2n + 3$

B $2n$ **D** $n - 3$

Read the Item To find the expression, determine the function.

Solve the Item Notice that the values 5, 7, 9, ... increase by 2, so the rule contains $2n$. Therefore, choices A and D can be eliminated.

If the rule were simply $2n$, then the value for position 1 would be 2×1 or 2. But this value is 5. So, choice B can be eliminated.

This leaves choice C. Test a few values.

Row 1: $2n + 3 = 2(1) + 3 = 2 + 3$ or 5
Row 3: $2n + 3 = 2(3) + 3 = 6 + 3$ or 9

So, the answer is C.

✓ **CHECK Your Progress**

d. Which expression was used to create the table at the right?

Position	Value of Term
1	4
2	9
3	14
n	■

F $5n - 1$ **H** $n + 1$

G $5n$ **J** $n + 5$

✓ CHECK Your Understanding

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the fifteenth term in the sequence.

1.

Position	1	2	3	4	n
Value of Term	2	4	6	8	■

2.

Position	3	4	5	6	n
Value of Term	10	11	12	13	■

3. MEASUREMENT There are 16 ounces in 1 pound. Make a table and write an algebraic expression relating the number of ounces to the number of pounds. Then find the number of ounces of potatoes Mr. Padilla bought if he bought a ten-pound bag of potatoes.

4. MULTIPLE CHOICE The table at the right shows the fee for overdue books at a library, based on the number of weeks the book is overdue. Which expression was used to find the fee for a book that is n weeks overdue?

A $n + 1$ **C** $n - 1$

B $2n$ **D** $2n + 1$

Weeks Overdue	Fee ($)
1	3
2	5
3	7
4	9
n	■

▶ Practice and Problem Solving

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the twelfth term in the sequence.

5.

Position	3	4	5	6	n
Value of Term	12	13	14	15	■

6.

Position	6	7	8	9	n
Value of Term	2	3	4	5	■

7.

Position	1	2	3	4	n
Value of Term	5	10	15	20	■

8.

Position	2	3	4	5	n
Value of Term	24	36	48	60	■

9. MEASUREMENT There are 60 minutes in 1 hour. Make a table and write an algebraic expression relating the number of hours to the number of minutes. Then find the duration of the movies in hours if Hannah and her friends watched two movies that together were 240 minutes long.

10. **MEASUREMENT** There are 12 months in 1 year. Make a table and write an algebraic expression relating the number of months to the number of years. Then find Andre's age in months if he is 12 years old.

:: **ANALYZE TABLES** Use the table at the right and the following information for Exercises 11 and 12.

The table shows the amount it costs to rock climb at an indoor rock climbing facility, based on the number of hours.

Time (h)	Amount ($)
1	13
2	21
3	29
4	37
n	■

11. How does the cost change with each additional hour?

12. What is the rule to find the amount charged to rock climb for n hours?

Determine how the next term in each sequence can be found. Then find the next two terms in the sequence.

13. 1, 4, 7, 10, ...

14. 4, 16, 28, 40, ...

15. 2.3, 3.2, 4.1, 5.0, ...

16. 1.5, 3.9, 6.3, 8.7, ...

17. $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, ...

18. $2\frac{1}{4}$, $2\frac{3}{4}$, $3\frac{1}{4}$, $3\frac{3}{4}$, ...

Find the missing number in each sequence.

19. 7, ■, 16, $20\frac{1}{2}$, ...

20. 14.6, ■, 24, 28.7

21. 30, ■, 19, $13\frac{1}{2}$, ...

22. 43.8, 36.7, ■, 22.5, ...

23. **MEASUREMENT** There are 24 hours in 1 day. Make a table and write an algebraic expression relating the number of hours to the number of days. Then find the number of hours in 1 week.

24. **GEOMETRY** Assume the pattern below continues. Write an algebraic expression to find the number of squares in Figure 8. Then find the number of squares in Figure 8.

Figure 1   Figure 2   Figure 3
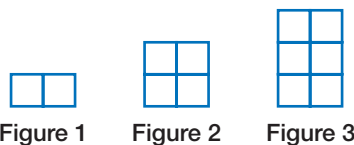
H.O.T. Problems

25. **OPEN ENDED** Create a sequence in which $1\frac{1}{4}$ is added to each number.

26. **SELECT A TECHNIQUE** Gene charges a base fee of $5 for each lawn he mows plus $2 for the number of hours it takes to complete the job. Which of the following techniques might Gene use to determine an expression he can use to represent the total charge for mowing a lawn based on the number of hours? Justify your selection(s). Then use the technique(s) to solve the problem.

mental math   number sense   estimation

27. ANALYZE TABLES Use words and symbols to generalize the relationship of each term as a function of its position. Then determine the value of the term when $n = 100$.

Position	1	2	3	4	5	n
Value of Term	1	4	9	16	25	■

28. **WRITING IN** **MATH** Write a problem about a real-world situation in which you would use a sequence to describe a pattern.

ISTEP+ PRACTICE 6.2.5

29. What is the rule to find the value of the missing term in the sequence below?

Position, x	Value of Term
1	1
2	5
3	9
4	13
5	17
x	■

A $x + 4$

B $4x - 3$

C $4x$

D $x - 3$

30. The table shows Samantha's age and Ling's age over four consecutive years.

Samantha's Age, x (years)	Ling's Age, y (years)
9	6
10	7
11	8
12	9

Which expression represents Ling's age in terms of Samantha's age?

F $y - 3$

G $3x$

H $3y$

J $x - 3$

Spiral Review

31. ART Mr. Torres is hanging his students' drawings in rows on one wall of his classroom. He places 1 drawing on the top row, 3 on the second row, and 5 on the third row. If this pattern continues, how many drawings will be on the seventh row? (Lesson 6-5)

Solve each proportion. (Lesson 6-4)

32. $\dfrac{3}{4} = \dfrac{a}{28}$

33. $\dfrac{5}{x} = \dfrac{45}{63}$

34. $\dfrac{24}{38} = \dfrac{12}{m}$

35. $\dfrac{x}{75} = \dfrac{5}{25}$

GET READY for the Next Lesson

PREREQUISITE SKILL Determine what number should be added to the first number to get the second number. (Lesson 5-5)

36. $4\dfrac{1}{4}, 4\dfrac{1}{2}$

37. $8\dfrac{1}{2}, 10$

38. $9, 12\dfrac{1}{2}$

39. $1\dfrac{2}{3}, 2\dfrac{1}{3}$

Proportions and Equations

GET READY for the Lesson

BABYSITTING The table at the right shows the amount of money Carli earns based on the number of hours she babysits.

Hours Babysitting	Earnings ($)
1	5
2	10
3	15
4	20

1. Write a sentence that describes the relationship between the number of hours she babysits and her earnings.

2. Is the relationship proportional? Explain.

3. What is the rule to find the amount Carli earns for babysitting h hours?

4. If e represents the amount Carli earns, what equation can you use to represent this situation?

In Lesson 6-6, you used algebraic expressions to determine how to find the value of a term in a sequence. Similarly, you can use an equation to represent a function.

EXAMPLE Write an Equation for a Function

1 Write an equation to represent the function displayed in the table.

Input, x	1	2	3	4	5
Output, y	9	18	27	36	45

Examine how the value of each input and output changes. Each output y is equal to 9 times the input x. So, the equation that represents the function is $y = 9x$.

Input, x	Multiply by 9	Output, y
1	1×9	9
2	2×9	18
3	3×9	27
4	4×9	36
5	5×9	45

CHECK Your Progress

a. Write an equation to represent the function displayed in the table.

Input, x	1	2	3	4	5
Output, y	16	32	48	64	80

You can often use an equation to represent a real-world situation.

Real-World EXAMPLES

FUNDRAISING The cheerleading squad is holding a car wash to raise money. They are charging $7 for each car they wash.

2 Make a table to show the relationship between the number of cars washed c and the total amount earned t.

The total earned (output) is equal to $7 times the number of cars washed (input).

Cars Washed, c	Multiply by 7	Total Earned ($), t
1	1×7	7
2	2×7	14
3	3×7	21
4	4×7	28

3 Write an equation to find the total amount earned t for washing c cars.

Study the table from Example 2. The total earned equals $7 times the number of cars washed.

Words	Total earned equals $7 times the number of cars washed.
Variable	Let t represent the total earned and c represent the number of cars washed.
Equation	$t \quad = \quad 7 \quad \cdot \quad c$

So, the equation is $t = 7c$.

4 How much will the cheerleading squad earn if they wash 25 cars?

$t = 7c$ Write the equation.

$t = 7 \cdot 25$ Replace c with 25.

$t = 175$ Multiply.

The cheerleading squad will earn $175 for washing 25 cars.

CHECK Your Progress

BALD EAGLE While in normal flight, a bald eagle flies at an average speed of 30 miles per hour.

b. Make a table to show the relationship between the total distance d that a bald eagle can travel in h hours.

c. Write an equation to find the total distance d that a bald eagle can travel in h hours while in normal flight.

d. How many miles can a bald eagle travel in 2 hours?

Real-World Link......
Bald eagles can dive at speeds up to 200 miles per hour.
Source: San Diego Zoo

5) JET SKIING The cost of renting a jet ski at a local marina is shown in the table. Write a sentence and an equation to describe the data. Then find the total cost of renting a jet ski for 6 hours.

Number of Hours, h	Total Cost ($), t
1	10
2	20
3	30
4	40

The cost of renting a jet ski is $10 for each hour. The total cost t is $10 times the number of hours h. Therefore, $t = 10h$. Use this equation to find the total cost t of renting a jet ski for 6 hours.

$t = 10h$ Write the equation.

$t = 10 \cdot 6$ or 60   Replace h with 6. Multiply.

The total cost of renting a jet ski for 6 hours is $60.

✓ CHECK Your Progress

e. **FITNESS** A fitness center charges the amount shown in the table for using the facility. Write a sentence and an equation to describe the data. Then find the total cost of using the fitness center for 12 months.

Number of Months, m	Total Fee ($), t
1	25
2	50
3	75
4	100

✓ CHECK Your Understanding

Example 1 (p. 349)

Write an equation to represent the function displayed in each table.

1.
Input, x	0	1	2	3	4
Output, y	0	4	8	12	16

2.
Input, x	1	2	3	4	5
Output, y	8	16	24	32	40

Examples 2–4 (p. 350)

LUNCH Use the following information for Exercises 3–5.

The school cafeteria sells lunch passes that allow a student to purchase any number of lunches in advance for $3 a lunch.

3. Make a table to show the relationship between the number of lunches n and the total cost t.

4. Write an equation to find t, the total cost in dollars for a lunch card with n lunches.

5. If Lolita buys a lunch pass for 20 lunches, how much will it cost?

Example 5 (p. 351)

6. **CARNIVAL** The general admission to a local carnival is shown in the table. Write a sentence and an equation to describe the data. Then find the total cost of admission for 7 people.

Number of People, n	Total Admission ($), t
1	4
2	8
3	12
4	16

Practice and Problem Solving

Write an equation to represent the function displayed in each table.

7.

Input, x	1	2	3	4	5
Output, y	6	12	18	24	30

8.

Input, x	0	1	2	3	4
Output, y	0	11	22	33	44

9.

Input, x	0	1	2	3	4
Output, y	0	15	30	45	60

10.

Input, x	1	2	3	4	5
Output, y	10	20	30	40	50

VIDEO GAMES Use the following information for Exercises 11–13.

In a video game, each player earns 15 points for each coin they collect.

11. Make a table to show the relationship between the number of coins collected c and the total points p.

12. Write an equation to find p, the total points for collecting c coins.

13. How many points will a player earn if she collects 21 coins?

ELEPHANTS Use the following information for Exercises 14–16.

An African elephant eats at a rate of 400 pounds of vegetation each day.

14. Make a table to show the relationship between the number of pounds v an African elephant eats in days d.

15. Write an equation to find v, the number of pounds of vegetation an African elephant eats in d days.

16. How many pounds of vegetation does an African elephant eat in 5 days?

17. ENTERTAINMENT The disc jockey hired for the spring dance charges the amount shown in the table. Write a sentence and an equation to describe the data. At this rate, how much will it cost to hire the disc jockey for 5 hours?

Number of Hours, h	Total Charge ($), t
1	35
2	70
3	105

18. FOOD A catering service provides the table shown as a guide to help its customers decide how many pans of lasagna to order for events. Write a sentence and an equation to describe the data. At this rate, how many people would 8 pans of lasagna serve?

Number of Pans, p	Total Number Served, n
1	24
2	48
3	72

19. RESEARCH Use the Internet or another source to find the average amount of food that another animal eats per day. Then write an equation to find f, the amount of food the animal eats in d days.

Write an equation to represent the function displayed in each table.

20.

Input, x	2	4	6	8	10
Output, y	1	2	3	4	5

21.

Input, x	3	6	9	12	15
Output, y	1	2	3	4	5

22. FIND THE DATA Refer to the Data File on pages 16–19. Choose some data and write a real-world problem in which you would write an equation using two variables to represent a problem situation.

23. WEATHER Write an equation to find the total precipitation t in inches for Burbank in m months. How much precipitation does Burbank receive in 4 months? Compare this to the total precipitation in 4 months for Coronado.

City	Average Annual Precipitation (in.)
Burbank	12
Coronado	9
Pasadena	20

Source: Weatherbase

H.O.T. Problems

24. OPEN ENDED Write about a real-world situation that can be represented by the equation $y = 5x$. Be sure to explain what the variables represent in the situation.

25. CHALLENGE Write an equation to represent the function in the table.

Input, x	6	8	10	12	14	16
Output, y	0	1	2	3	4	5

26. WRITING IN MATH Choose an exercise from this lesson, and explain why the relationship is proportional.

ISTEP+ PRACTICE 6.2.5

27. The table shows admission prices at a local zoo based on the number of guests.

Number of Guests, x	Total Admission ($), y
1	7
2	14
3	21
4	28

Which equation can be used to find y, the total admission for x guests?

A $x = 7y$ **C** $y = 7x$

B $y = 7 + x$ **D** $x = 7 + y$

28. If the cost of snorkeling is $5 for the equipment plus an additional $7 for each hour that you snorkel, which equation represents c, the cost in dollars for snorkeling for h hours?

F $c = 7h + 5$

G $c = 5h + 7$

H $c = 7(h + 5)$

J $c = 5(h + 7)$

Spiral Review

29. Find the next two terms of the sequence 3, 11, 19, 27, ... (Lesson 6-6)

30. MONEY Maxine withdraws the same amount of money from her savings account each month. In March, her account balance was $100. Her next monthly balances were $85, $70, and $55. At this rate, what will her account balance be in September? (Lesson 6-5)

Graphing Calculator Lab
Graphing Proportional Relationships

You can use a TI-83/84 Plus graphing calculator to graph an equation
that represents a proportional relationship.

ACTIVITY

CORN DOGS Suppose Booth A at a fair sells corn dogs for $2 each
while Booth B charges $3 for each corn dog. Write and graph an
equation representing the total cost y for selling x corn dogs.

STEP 1 Write an equation to show the relationship between the
number of corn dogs sold x and total cost y at each booth.

Booth A	Booth B
total cost $2 per corn dog	total cost $3 per corn dog
$y = 2x$	$y = 3x$

STEP 2 Press $\boxed{Y=}$ on your calculator and enter the expression $2x$
into Y_1 and $3x$ into Y_2.

STEP 3 Next, change the view of your graph by pressing $\boxed{\text{WINDOW}}$
and entering the values shown below.

STEP 4 Finally, graph the equations by pressing $\boxed{\text{Graph}}$.


```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=10
 Yscl=1
 Xres=1
```
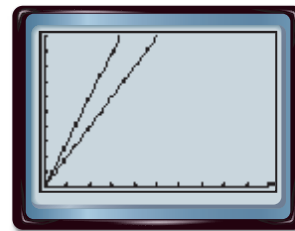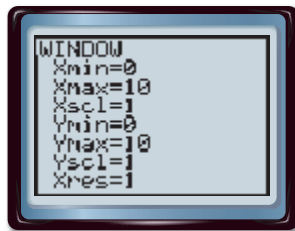
ANALYZE THE RESULTS

1. **MAKE A CONJECTURE** Which equation is represented by the steeper
   line? Explain. To check your conjecture, press $\boxed{\text{Trace}}$ and then use
   $\boxed{\uparrow}$ and $\boxed{\downarrow}$ keys to switch between the two lines.

2. Press $\boxed{\text{2nd}}$ [CALC] 1 and enter a value of 3. Switch between the
   two lines. What do the x- and y-values displayed represent?

3. **MAKE A CONJECTURE** Suppose Booth C sells corn dogs for
   $2.50 each. How should the graph of this equation appear in
   relationship to the graphs of the other two equations? Explain.
   Check your answer by entering the appropriate equation into Y_3,
   pressing $\boxed{\text{Graph}}$, and switching between the equations.

FOLDABLES
Study Organizer

▶ GET READY to Study

Be sure the following Big Ideas are noted in your Foldable.

Ratio	Proportion	Function
Examples	Examples	Examples

BIG Ideas

Ratios (Lessons 6-1 and 6-2)
- Ratios are a comparison of two quantities by division.

- Equivalent ratios express the same relationship between two quantities.

- To find an equivalent ratio, multiply or divide each quantity in the ratio by the same number.

Rates (Lesson 6-1)
- Rates are a ratio comparing two quantities with different kinds of units.

- A unit rate is a rate for one unit of a given quantity.

Proportions (Lessons 6-3 and 6-4)
- Proportions are equations stating that two ratios are equivalent.

- To solve a proportion, use either an equivalent fraction or a unit rate.

Sequences (Lesson 6-6)
- Sequences are a list of numbers in a specific order.

- Each number listed in a sequence is called a term.

- Each term of an arithmetic sequence is found by adding the same number to the previous term.

- Algebraic expressions can be used to describe a sequence.

Equations (Lesson 6-7)
- An equation can be used to represent a function.

Key Vocabulary

arithmetic sequence (p. 343)	ratio (p. 314)
equivalent ratio (p. 322)	ratio table (p. 322)
proportion (p. 329)	scaling (p. 323)
proportional (p. 329)	sequence (p. 343)
rate (p. 315)	term (p. 343)
rate of change (p. 343)	unit rate (p. 315)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. A ratio is a comparison of two numbers by <u>multiplication</u>.

2. A <u>rate</u> is the ratio of two measurements that have different units.

3. Two quantities are said to be <u>proportional</u> if they have a constant ratio or rate.

4. A proportion is an equation stating that two ratios or rates are <u>not equal</u>.

5. <u>Adding or subtracting</u> two related quantities by the same number is called scaling.

6. A <u>sequence</u> is a list of numbers in a specific order.

7. In an arithmetic sequence, each term is found by <u>multiplying</u> the same number.

8. In a ratio table, the columns are filled with pairs of numbers that have the same <u>term</u>.

Lesson-by-Lesson Review

6-1 | Ratios and Rates (pp. 314–319)

6.1.7

Write each ratio or rate as a fraction in simplest form.

9. 12 blue marbles out of 20 marbles

10. 9 goldfish out of 36 fish

11. 18 boys out of 21 students

Write each rate as a unit rate.

12. 3 inches of rain in 6 hours

13. 189 pounds of garbage in 12 weeks

14. 78 erasers in 3 packages

15. **DVDs** Rick has 12 action, 15 comedy, and 9 drama DVDs. Find the ratio of action DVDs to the total number of DVDs. Then explain its meaning.

Example 1 Write the ratio 30 sixth graders out of 45 students as a fraction in simplest form.

$$\frac{30}{45} = \frac{2}{3} \quad \text{The GCF of 30 and 45 is 15.}$$

($\div 15$)

Example 2 Write the rate 150 miles in 4 hours as a unit rate.

$$\frac{150 \text{ miles}}{4 \text{ hours}} = \frac{37.5 \text{ miles}}{1 \text{ hour}}$$

($\div 4$)

Divide the numerator and the denominator by 4 to get the denominator of 1.

6-2 | Ratio Tables (pp. 322–327)

6.1.7,
6.1.8

For Exercises 16 and 17, use the ratio tables given to solve each problem.

16. **MONEY** Arthur bought 5 notebooks for $3. How much will he spend on 10 notebooks?

Number of Notebooks	5	10
Money Spent ($)	3	■

17. **TRUCKS** In a parking lot, 3 out of 8 vehicles were trucks. If there were 128 vehicles, how many were trucks?

Number of Trucks	3		■
Number of Vehicles	8		128

18. **TICKETS** Roman spent $306 for 17 baseball tickets. Use a ratio table to determine how much each ticket costs.

Example 3 Boston received 6 inches of rain in 24 hours. If it rained at a constant rate, use a ratio table to determine how much rain Boston would receive in 48 hours.

Set up a ratio table. Label the rows with the two quantities being compared. Then fill in what is given.

Inches of Rain	6	■
Number of Hours	24	48

Use scaling to find the desired quantity.

×2

Inches of Rain	6	12
Number of Hours	24	48

×2

So, Boston would receive 12 inches of rain in 48 hours.

6-3 **Proportions** (pp. 329–333)

6.1.7,
6.1.8

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

19. 220 ft to 25 in.; 88 ft to 10 in.

20. 14 girls to 21 boys; 21 girls to 34 boys

21. **JEWELRY** Stacey made 8 necklaces in 48 minutes. Nick made 4 necklaces in 24 minutes. Are these rates proportional? Explain your reasoning.

Example 4 Are the ratios 18 weeks to 3 years and 54 weeks to 10 years proportional? Explain your reasoning.

Find the unit ratio for each ratio.

$$\overset{\div 3}{\frac{18 \text{ weeks}}{3 \text{ years}}} = \frac{6 \text{ weeks}}{1 \text{ year}} \qquad \overset{\div 10}{\frac{54 \text{ weeks}}{10 \text{ years}}} = \frac{5.4 \text{ weeks}}{1 \text{ year}}$$

Since the ratios do not share the same unit ratio, they are not equivalent. Therefore, they are not proportional.

6-4 **Algebra: Solving Proportions** (pp. 334–339)

6.1.8,
6.2.1

Solve each proportion.

22. $\frac{7}{11} = \frac{m}{33}$ 23. $\frac{3}{20} = \frac{15}{k}$

24. $\frac{g}{20} = \frac{9}{60}$ 25. $\frac{25}{h} = \frac{5}{7}$

26. **DOGS** In the sixth grade, 12 out of 27 students have a dog. If there are 162 students, predict how many have a dog.

Example 5 Solve the proportion $\frac{9}{12} = \frac{g}{4}$.

$$\overset{\div 3}{\frac{9}{12}} = \frac{3}{4}$$ Since $12 \div 3 = 4$, divide the numerator and denominator by 3.

So, $g = 3$.

6-5 **PSI: Look for a Pattern** (pp. 341–342)

P.1.3,
P.1.4

Solve by looking for a pattern.

27. **LAPS** Cheyenne ran 1 lap on day 1. She ran 2 laps on day 2, 4 laps on day 3, and 8 laps on day 4. If this pattern continues, how many laps will she run on day 7?

28. **BASEBALL** The table shows the scores at a baseball game. If the scoring pattern continues, how many runs will the Reds score in the 7th inning?

Inning	1	2	3	4	5	6	7
Cubs	0	1	2	4	0	1	3
Reds	4	3	2	0	4	3	■

Example 6 A display of cans is stacked in the shape of a pyramid. There are 2 cans in the top row, 4 in the second row, 6 in the third row, and so on. The display contains 7 rows. How many cans are in the display?

Make a table to see the pattern.

Row	1	2	3	4	5	6	7
Number of Cans	2	4	6	8	10	12	14

+2 +2 +2 +2 +2 +2

There are $2 + 4 + 6 + 8 + 10 + 12 + 14$ or 56 cans in the display.

Mixed Problem Solving
For mixed problem-solving practice,
see page 711.

Chapter 6 Study Guide and Review **357**

6-6 Sequences and Expressions (pp. 343–348)

6.2.5

Use words and symbols to describe the value of each term as a function of its position. Then find the value of the sixteenth term in the sequence.

29.

Position	22	23	24	25	n
Value of Term	15	16	17	18	■

30.

Position	10	11	12	13	n
Value of Term	28	31	34	37	■

31. **BUSINESS** Jennifer earns $12 for every dog she walks. Write an algebraic expression to find the amount of money she would earn for walking n number of dogs. Then find the amount of money she would earn for walking 45.

Example 7 Use words and symbols to describe the value of each term as a function of its position. Then find the value of the ninth term in the sequence.

Position	5	6	7	8	n
Value of Term	20	24	28	32	■

Study the relationship between each position and the value of its term.

Position	Multiply by 4	Value of Term
5	5×4	20
6	6×4	24
7	7×4	28
8	8×4	32
n	$n \times 4$	**4n**

Notice that the value of each term is 4 times its position number.

$4n = 4 \cdot 9$ Replace n with 9.
$= 36$ Multiply.

The value of ninth term in the sequence is 36.

6-7 Proportions and Equations (pp. 349–353)

6.2.5

Write an equation to represent the function displayed in each table.

32.

Input, x	1	2	3	4	5
Output, y	6	12	18	24	30

33.

Input, x	1	2	3	4	5
Output, y	7	14	21	28	35

34. **BOWLING** A bowling alley charges $4 per game. Write an equation to find the total cost in dollars t for the number of games bowled n. Then find the total cost for 8 games bowled.

Example 8 Write an equation to represent the function displayed in the table.

Examine how the value of each input and output changes.

+1 +1 +1 +1

Input, x	1	2	3	4	5
Output, y	9	18	27	36	45

+9 +9 +9 +9

Each output is equal to 9 times the input. So, the equation that represents the function is $y = 9x$.

Write each ratio as a fraction in simplest form.

1. 12 red blocks out of 20 blocks

2. 24 potato chips out of 144 chips

3. 65 rotten apples out of 520 apples

4. **WORD PROCESSING** The world record for the fastest typing speed is 212 words per minute. How many words per second is this? Round to the nearest tenth.

Write each rate as a unit rate.

5. $2 for 36 erasers

6. 180 pages in 90 minutes

7. **MULTIPLE CHOICE** Candace buys 12 cans of orange juice for $6. At this rate, how much would she pay for 48 cans of orange juice?

   A $20

   B $22

   C $24

   D $30

Determine if the quantities in each pair of ratios or rates are proportional. Explain your reasoning and express each proportional relationship as a proportion.

8. 32 pencils for $8; 16 pencils for $4

9. 72 out of 90 students have siblings; 362 out of 450 students have siblings

10. 524 Calories for 4 servings; 786 Calories for 6 servings

Solve each proportion.

11. $\dfrac{4}{6} = \dfrac{x}{12}$

12. $\dfrac{10}{p} = \dfrac{2}{8}$

13. $\dfrac{n}{13} = \dfrac{8}{52}$

14. $\dfrac{7}{13} = \dfrac{a}{52}$

15. **SEASONS** If 7 of the 28 students in a class prefer the winter months, predict how many would prefer the winter months in a school of 400 students.

16. **DISCOUNT** Ellie is using the following table to help her calculate the discount on baseball caps. Mr. Gomez would like to order 8 baseball caps. How much of a discount should Ellie give him?

Baseball caps	1	2	3
Discount ($)	2	3	4

17. **MULTIPLE CHOICE** Which expression was used to create the table?

Position, x	Value of Term
3	11
4	14
5	17
6	20
7	23
x	■

   F $x + 8$

   G $2x + 3$

   H $x - 8$

   J $3x + 2$

READING For Exercises 18–20, use the following information.

Darnell reads an average of 2 hours each day.

18. Make a table to show the relationship between the number of h hours Darnell spends reading in d days.

19. Write an equation to find h, the number of hours Darnell spends reading in d days.

20. On average, how many hours will Darnell spend reading in 12 days?

PART 1 Multiple Choice

Read each question. Then fill in the correct answer on the answer sheet provided by your teacher or on a sheet of paper.

1. Dante made a scale model of a tree. The actual tree is 32 feet tall, and the height of the model is 2 feet. How many actual feet does one foot on the model represent?

   A 2 feet

   B 8 feet

   C 16 feet

   D 32 feet

2. The ratio of cats to dogs seen by a vet in a day is about 2 to 5. If a vet saw 40 animals in one day, about how many were dogs?

   F 5

   G 12

   H 29

   J 40

3. Which statement about the mixed number $1\frac{5}{8}$ is true?

   A $1\frac{5}{8} > \frac{1}{2}$

   B $1\frac{5}{8} > 2$

   C $1 > 1\frac{5}{8}$

   D $1\frac{5}{8} < \frac{1}{4}$

4. Mr. Lee's car odometer read 55,085.4 miles. After his trip, it read 56,002.8 miles. Choose the best estimate for the number of miles the car was driven on his trip.

   F 100 miles

   G 150 miles

   H 1,000 miles

   J 2,000 miles

5. David's time in a race was 9 minutes. Armando's time was two minutes less than Rachel's time, and Rachel's time was 4 minutes more than David's time. Which table could be used to find Armando's time?

   A

Name	Time (min)
Armando	9 − 4 + 2
Rachel	4 − 2
David	9

   B

Name	Time (min)
Armando	4 − 2
Rachel	9 + 4
David	9

   C

Name	Time (min)
Armando	9 + 4 − 2
Rachel	9 + 4
David	9

   D

Name	Time (min)
Armando	4 + 2
Rachel	9 + 4
David	9

6. On Monday, Ted ran $3\frac{1}{2}$ miles. Then on Wednesday he ran $1\frac{2}{3}$ miles and on Friday he ran $2\frac{1}{4}$ miles. Find the total number of miles Ted ran.

   F $6\frac{1}{2}$ miles   H $7\frac{1}{2}$ miles

   G $7\frac{5}{12}$ miles   J $8\frac{1}{12}$ miles

7. At a concert, backstage passes were awarded to the person sitting in the seat numbered with the least common multiple of 9, 12, and 18. Find the number of the prizewinning seat.

   A 3   C 36

   B 12   D 45

8. Juanita was making a cake. She added $\frac{1}{4}$ cup of brown sugar, $1\frac{1}{3}$ cups of flour, and $\frac{3}{4}$ cup of white sugar. Which procedure can you use to find the total number of cups Juanita used?

   F Divide the sum of the whole numbers by the sum of the fractions, using a common denominator when necessary.

   G Find the difference between the sum of the whole numbers and the sum of the fractions, using a common denominator when necessary.

   H Multiply the sum of the whole numbers by the sum of the fractions, using a common denominator when necessary.

   J Add the sum of the whole numbers and the sum of the fractions, using a common denominator when necessary.

9. At a sports camp, there is 1 counselor for every 12 campers. If there are 156 campers attending the camp, which proportion can be used to find x, the number of counselors?

   A $\dfrac{x}{12} = \dfrac{1}{156}$

   B $\dfrac{12}{1} = \dfrac{x}{156}$

   C $\dfrac{1}{12} = \dfrac{x}{156}$

   D $\dfrac{x}{1} = \dfrac{12}{156}$

TEST-TAKING TIP

Question 9 When setting up a proportion, make sure the numerators and denominators in each ratio have the same units, respectively.

PART 2   Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Which is the prime factorization of 252 using exponents?

11. When Shawnel held her cat and stepped onto a scale, the scale's display read 107.2 pounds. If Shawnel weighs 97.9 pounds, how many pounds does her cat weigh?

12. Find the greatest common factor of 15, 20, and 25.

PART 3   Extended Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper. Show your work.

13. Samantha's class sorted books in the library. The class sorted 45 books in 90 minutes.

    a. Write a proportion to find how long it would take to sort 120 books.

    b. How many hours did it take the class to sort 120 books?

    c. Suppose their rate slowed to 30 books in 90 minutes. How long would it take the class to sort the 120 books?

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson...	6-1	6-4	4-6	3-4	5-5	5-5	4-5	1-4	6-4	1-3	3-5	4-1	6-4
IN Academic Standards	6.1.7	6.1.8	6.1.1	6.1.6	6.1.6	6.1.6	6.1.6	6.2.3	6.1.8	7.1.3	6.1.6	6.1.6	6.1.8