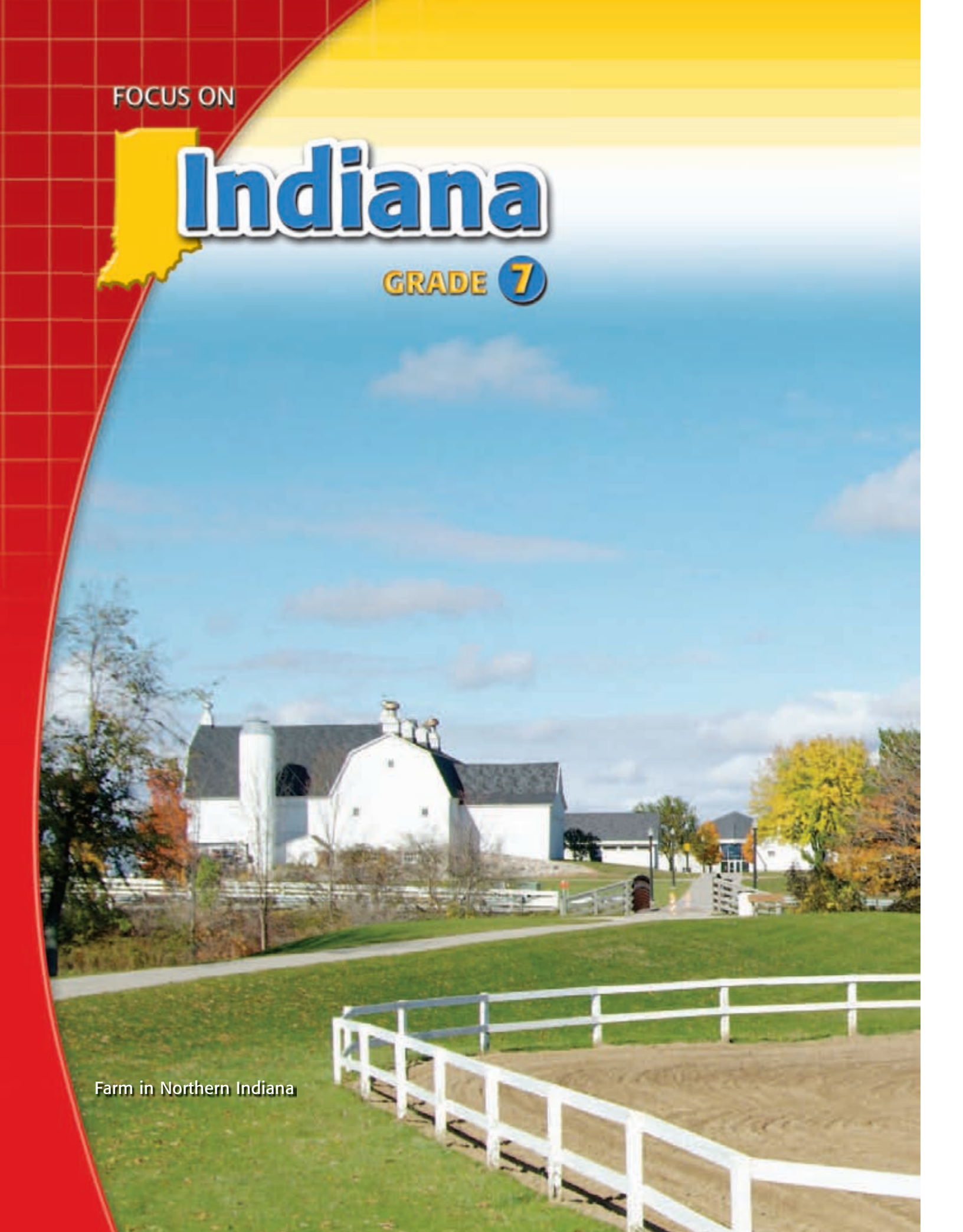


FOCUS ON

Indiana

GRADE 7

Farm in Northern Indiana



Contents

Academic Standards

7.1.1	1	Scientific NotationIN2
		<i>(Use after Lesson 1-2)</i>	
7.2.2	2	Linear InequalitiesIN6
		<i>(Use after Lesson 3-5)</i>	
7.2.4, 7.3.6	3	Literal EquationsIN10
		<i>(Use after Lesson 3-6)</i>	
7.1.7	4	Operations with DecimalsIN14
		<i>(Use after Lesson 4-9)</i>	
7.2.5	5	Slope and Similar TrianglesIN18
		<i>(Use after Lesson 6-3)</i>	
7.2.6	6	Graphs of Linear EquationsIN24
		<i>(Use after Lesson 6-3 and Indiana Lesson 5)</i>	
7.2.7	7	Direct VariationIN29
		<i>(Use after Lesson 6-3 and Indiana Lessons 5 and 6)</i>	
7.4.1	8	Choose an Appropriate DisplayIN34
		<i>(Use after Lesson 8-9)</i>	
7.4.4	9	Analyze Questions in SurveysIN39
		<i>(Use after Lesson 8-9 and Indiana Lesson 8)</i>	
7.3.1	10	Parallel Lines and TransversalsIN42
		<i>(Use after Lesson 10-1)</i>	
7.3.2	11	RotationsIN46
		<i>(Use after Lesson 10-10)</i>	
7.3.3	12	Nets of Cylinders and ConesIN50
		<i>(Use after Extend 11-6)</i>	
7.1.6, 7.2.3	13	Irrational NumbersIN54
		<i>(Use after Lesson 12-1)</i>	



Scientific Notation

MAIN IDEA

Read, write, compare, and solve problems using scientific notation.

IN Academic Standards

7.1.1 Read, write, compare and solve problems using whole numbers in scientific notation.

New Vocabulary
scientific notation

IN Math Online

glencoe.com

▶ GET READY for the Lesson

More than 425 million pounds of gold has been discovered in the world. If all this gold were in one place, it would form a cube seven stories on each side.



1. Write 425 million in standard form.
2. Complete: $4.25 \times \underline{\quad} = 425$ million.
3. Write your answer to Exercise 2 as a power of 10.

When you deal with very large numbers like 425,000,000, it can be difficult to keep track of the zeros. You can express numbers such as this in **scientific notation** by writing the number as the product of a factor and a power of 10.

Scientific Notation

Key Concept

Words A number is expressed in scientific notation when it is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

Symbols $a \times 10^n$, where $1 \leq a < 10$ and n is an integer

Example $425,000,000 = 4.25 \times 10^8$

EXAMPLE Express Large Numbers in Standard Form

1 Express 2.16×10^5 in standard form.

$$2.16 \times 10^5 = 2.16 \times 100,000 \quad 10^5 = 100,000$$

$$= \underline{216,000}$$

Move the decimal point 5 places to the right.

In standard form, 2.16×10^5 is 216,000.

Scientific notation is also used to express very small numbers. Study the pattern of products at the right. Notice that multiplying by a negative power of 10 moves the decimal point to the left the same number of places as the absolute value of the exponent. For example, the number 1.25×10^{-2} has an exponent with an absolute value of 2. The decimal point in 1.25 will be moved to the left two places to become 0.0125.

$$1.25 \times 10^3 = 1,250$$

$$1.25 \times 10^2 = 125$$

$$1.25 \times 10^1 = 12.5$$

$$1.25 \times 10^0 = 1.25$$

$$1.25 \times 10^{-1} = 0.125$$

$$1.25 \times 10^{-2} = 0.0125$$

$$1.25 \times 10^{-3} = 0.00125$$

Study Tip

Move the Decimal Point
The exponent tells you how many places to move the decimal point.

- If the exponent is positive, move the decimal point to the right.
- If the exponent is negative, move the decimal point to the left.

EXAMPLE

Express Small Numbers in Standard Form

2 Express 5.8×10^{-3} in standard form.

$$\begin{aligned} 5.8 \times 10^{-3} &= 5.8 \times 0.001 & 10^{-3} &= 0.001 \\ &= 0.0058 & & \end{aligned}$$

Move the decimal point 3 places to the left.

In standard form, 5.8×10^{-3} is 0.0058.

To write a number in scientific notation, place the decimal point after the first nonzero digit. Then find the power of 10.

EXAMPLES

Express Numbers in Scientific Notation

Express each number in scientific notation.

3 1,457,000

$$\begin{aligned} 1,457,000 &= 1.457 \times 1,000,000 & \text{The decimal point moves 6 places to the left.} \\ &= 1.457 \times 10^6 & \text{The exponent is positive.} \end{aligned}$$

4 0.00063

$$\begin{aligned} 0.00063 &= 6.3 \times 0.0001 & \text{The decimal point moves 4 places to the right.} \\ &= 6.3 \times 10^{-4} & \text{The exponent is negative.} \end{aligned}$$

To compare numbers in scientific notation, first compare the exponents. With positive numbers, any number with a greater exponent is greater. If the exponents are the same, compare the factors.



Real-World Link

At the deepest point in the ocean, the pressure is greater than 8 tons per square inch and the temperature is only a few degrees above freezing.

Source: Ocean Planet Smithsonian



Real-World EXAMPLES

Compare Numbers in Scientific Notation

Refer to the table at the right which gives the approximate area in square kilometers of Earth's oceans.

5 **OCEANS** Which ocean has the greater area, the Arctic or the Southern?

Since both exponents are the same, compare the factors.

$$1.41 < 2.03 \rightarrow 1.41 \times 10^7 < 2.03 \times 10^7$$

So, the Southern Ocean has the greater area.

6 Which ocean has the greater area, the Arctic or the Pacific?

Compare the exponents.

$$8 > 7 \rightarrow 1.56 \times 10^8 > 1.41 \times 10^7$$

So, the Pacific Ocean has the greater area.

Earth's Oceans	
Ocean	Approximate Area (km ²)
Arctic	1.41×10^7
Atlantic	7.68×10^7
Indian	6.86×10^7
Pacific	1.56×10^8
Southern	2.03×10^7

CHECK Your Understanding

Examples 1, 2
(pp. IN2–IN3)

Express each number in standard form.

1. 3.754×10^5
2. 8.34×10^6
3. 1.5×10^{-4}
4. 2.68×10^{-3}

Examples 3, 4
(p. IN3)

Express each number in scientific notation.

5. 4,510,000
6. 0.00673
7. 0.000092
8. 11,620,000

9. **PHYSICAL SCIENCE** Light travels 300,000 kilometers per second. Write this number in scientific notation.

Example 5
(p. IN3)

10. **TECHNOLOGY** The distance between tracks on a CD and DVD are shown in the table. Which disc has the greater distance between tracks?

Disc	Distance (mm)
CD	1.6×10^{-3}
DVD	7.4×10^{-4}

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

11. $2.3 \times 10^5 \bullet 1.7 \times 10^5$
12. $0.012 \bullet 1.4 \times 10^{-1}$

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
13–22	1, 2
23–32	3, 4
33–39	5

Express each number in standard form.

13. 6.1×10^4
14. 5.72×10^6
15. 3.3×10^{-1}
16. 5.68×10^{-3}
17. 9.014×10^{-2}
18. 1.399×10^5
19. 2.505×10^3
20. 7.4×10^{-5}

21. **SPIDERS** The diameter of a spider's thread is 1.0×10^{-3} inch. Write this number in standard form.



22. **DINOSAURS** The *Gigantosaurus* dinosaur weighed about 1.4×10^4 pounds. Write this number in standard form.

Express each number in scientific notation.

23. 499,000
24. 2,000,000
25. 0.006
26. 0.0125
27. 50,000,000
28. 39,560
29. 0.000078
30. 0.000425

31. **CHESS** The number of possible ways that a player can play the first four moves in a chess game is 3 billion. Write this number in scientific notation.

32. **SCIENCE** A particular parasite is approximately 0.025 inch long. Write this number in scientific notation.

Find each of the following. Write in standard form.

33. $(8 \times 10^0) + (4 \times 10^{-3}) + (3 \times 10^{-5})$
34. $(4 \times 10^4) + (8 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (6 \times 10^0)$

SPORTS For Exercises 35 and 36, use the table. Determine which category in each pair had a greater amount of sales.

Category	Sales (\$)
Camping	1.547×10^9
Golf	3.243×10^9
Tennis	3.73×10^8

Source: National Sporting Goods Assoc.

35. golf or tennis 36. camping or golf

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

37. $1.8 \times 10^3 \bullet 1.9 \times 10^{-1}$ 38. $5.2 \times 10^2 \bullet 5000$
 39. $0.00701 \bullet 7.1 \times 10^{-3}$ 40. $6.49 \times 10^4 \bullet 649 \times 10^2$

41. **MEASUREMENT** The table at the right shows the values of different prefixes that are used in the metric system. Write the units attometer, gigameter, kilometer, nanometer, petameter, and picometer in order from greatest to least measure.

Metric Measures	
Prefix	Meaning
atto	10^{-18}
giga	10^9
kilo	10^3
nano	10^{-9}
peta	10^{15}
pico	10^{-12}

42. **NUMBER SENSE** Write the product of 0.00004 and 0.0008 in scientific notation.

43. **NUMBER SENSE** Order 6.1×10^4 , 6100, 6.1×10^{-5} , 0.0061, and 6.1×10^{-2} from least to greatest.

H.O.T. Problems

44. **REASONING** Which is a better estimate for the number of times per year that a person blinks, 6.25×10^{-2} times or 6.2×10^6 ? Explain your reasoning.

45. **CHALLENGE** Convert the numbers in each expression to scientific notation. Then evaluate the expression. Express in scientific notation and in decimal notation.

a. $\frac{(420,000)(0.015)}{0.025}$

b. $\frac{(0.078)(8.5)}{0.16(250,000)}$

46. **OPEN ENDED** Describe a real-life value or measure using numbers in scientific notation and in standard form.

47. **WRITING IN MATH** Explain the relationship between a number in standard form and the sign of the exponent when the number is written in scientific notation.

ISTEP+ PRACTICE 7.1.1

48. Which of the following expressions is the GREATEST: 3.2×10^4 , 9.8×10^{-1} , 5.6×10^2 , or 1.7×10^5 ?
- A 1.7×10^5
 B 3.2×10^4
 C 5.6×10^2
 D 9.8×10^{-1}

49. Which of the following expressions is equivalent to 4.01×10^3 ?
- F 0.00401
 G 40.1
 H 4,010
 J 40,100



Linear Inequalities

MAIN IDEA

Write and solve two-step linear inequalities.

IN Academic Standards

7.2.2 Write and solve two-step linear equations and inequalities in one variable.

IN Math Online

glencoe.com

▶ GET READY for the Lesson

MEASUREMENT The perimeter of a rectangle is less than 34 centimeters. The inequality $2\ell + 2w < 34$ represents this situation.

$$P < 34 \text{ cm}$$

1. The length of the rectangle is 12 centimeters. Write the inequality by replacing ℓ with 12.
2. What operations are used in the inequality you wrote in Exercise 1?
3. Describe how you would solve the equation $24 + 2w = 34$.
4. List three different possible widths w that would result in a perimeter less than 34 centimeters if the length is 12 centimeters.

You learned how to solve a two-step linear equation in Lesson 3-5. Recall that to solve a two-step linear equation, undo the addition or subtraction first. Then undo the multiplication or division.

You can use a similar method to solve a two-step linear inequality. You already learned how to solve a one-step linear inequality in *Indiana Math Connects*, Course 1.

EXAMPLE Solve Two-Step Inequalities

- 1 MEASUREMENT** The perimeter of a rectangle is less than 34 centimeters. The length of the rectangle is 12 centimeters. Solve the two-step linear inequality $24 + 2w < 34$ to find an expression that gives the width of the rectangle. Check your solution.

$$\begin{array}{r} 24 + 2w < 34 \\ - 24 \quad \quad - 24 \\ \hline \frac{2w}{2} < \frac{10}{2} \\ w < 5 \end{array}$$

Write the inequality.
Subtract 24 from each side.
Divide each side by 2.
Simplify.

Check $24 + 2w < 34$
 $24 + 2(4) < 34$

Write the inequality.
Replace w with a number less than 5, such as 4.
Simplify. This statement is true.

$$32 < 34 \quad \checkmark$$

The solution is $w < 5$.

So, the width of the rectangle must be less than 5 centimeters.

Recall from *Indiana Math Connects*, Course 1, that the direction of the inequality symbol is reversed when multiplying or dividing each side by a negative number. This is demonstrated below using the inequality $-3x \geq 12$.

$$-3x \geq 12$$

Write the inequality.

$$\frac{-3x}{3} \leq \frac{12}{-3}$$

Divide each side by -3 . Reverse the direction of the inequality symbol.

$$x \leq -4$$

Simplify.

Study Tip

Solving Inequalities
Remember to reverse the direction of the inequality only when you are multiplying or dividing by a negative number.

In the inequality $3x < -9$, you would not reverse the direction of the inequality even though there is a negative number, -9 . You are dividing both sides by a positive number, 3 .

EXAMPLES

Solve Two-Step Inequalities

Solve each two-step linear inequality. Check your solution.

2 $\frac{y}{-5} + 3 > -7$

$$\frac{y}{-5} + 3 > -7$$

$$\underline{-3 \quad -3}$$

$$\frac{y}{-5} > -10$$

$$(-5)\frac{y}{-5} < (-5)(-10)$$

$$y < 50$$

Check $\frac{y}{-5} + 3 > -7$

$$\frac{40}{-5} + 3 \stackrel{?}{>} -7$$

$$-5 > -7 \quad \checkmark$$

The solution is $y < 50$.

Write the inequality.

Subtract 3 from each side.

Simplify.

Multiply both sides by -5 . Reverse the direction of the inequality symbol.

Simplify.

Write the inequality.

Replace y with a number less than 50, such as 40.

Simplify. This statement is true.

3 $8 - 4a \leq 24$

$$8 - 4a \leq 24$$

$$\underline{-8 \quad -8}$$

$$-4a \leq 16$$

$$\frac{-4a}{-4} \geq \frac{16}{-4}$$

$$a \geq -4$$

Check $8 - 4a \leq 24$

$$8 - 4(0) \stackrel{?}{\leq} 24$$

$$8 \leq 24 \quad \checkmark$$

The solution is $a \geq -4$.

Write the inequality.

Subtract 8 from each side.

Simplify.

Divide each side by -4 . Reverse the direction of the inequality symbol.

Simplify.

Write the inequality.

Replace a with a number less than 24, such as 0.

Simplify. This statement is true.



Real-World EXAMPLE

- 4 SNOWBOARDING** Three friends went snowboarding. The admission price was \$5 each. Two of the friends rented boards. If they spent a total of at least \$19, write and solve an inequality to find the dollar amount that each board rental could cost.

Words The cost of two rentals and three admissions is at least \$19.

Variable Let s = cost for snowboard rental.

Inequality $\underbrace{\text{Two rentals at } \$s \text{ each}}_{2s}$ $\underbrace{\text{plus admission}}_{+ 3(5)}$ $\underbrace{\text{is at least } \$19}_{\geq 19}$

$$2s + 15 \geq 19$$

Write the inequality.

$$\underline{-15} \quad \underline{-15}$$

Subtract 15 from each side.

$$2s \geq 4$$

Simplify.

$$\frac{2s}{2} \geq \frac{4}{2}$$

Divide each side by 2.

$$s \geq 2$$

Simplify.

So, one snowboard rental is at least \$2.

Real-World Link

In a recent year, snowboarding was the fastest growing sport in the United States, with over 7.2 million participants.

CHECK Your Understanding

Examples 1–3
(pp. IN6–IN7)

Solve each two-step linear inequality. Check your solution.

- $2a + 5 < 13$
- $7 + 3p \leq -14$
- $-4m - 1 \geq 11$
- $\frac{k}{8} + 1 > 5$
- $-4 + \frac{y}{-3} < 6$
- $\frac{h}{-9} + 6 \leq 5$

Example 4
(p. IN8)

7. **ZOOS** Jenna visited the zoo with both of her parents. They paid a total of at least \$24 and Jenna's admission cost \$6. Each parent paid the same adult admission amount. Write and solve an inequality to find the dollar amount that each adult admission could cost.

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
8–19	1–3
20–23	4

Solve each two-step linear inequality. Check your solution.

- $6d + 1 < 19$
- $-n - 5 \geq 7$
- $3 + 2q \leq -9$
- $-10y - 4 \geq 56$
- $11c + 2 < 35$
- $1 + 8x \leq -39$
- $\frac{8}{7} + 3 > 6$
- $-1 + \frac{p}{-4} < 2$
- $\frac{t}{5} + 3 > 12$
- $\frac{z}{9} - 2 \geq 9$
- $\frac{w}{4} + 6 \leq 4$
- $8 + \frac{j}{-6} > 11$

20. **GROCERIES** Carter bought 6 pounds of fruit and four potatoes that each weighed the same amount. If the total weight of the items he bought was no more than 7 pounds, write and solve an inequality to find the number of pounds that each potato could weigh.
21. **TENNIS** On Saturday, Danielle played tennis at the local community center. Racket rental was \$7 and court time cost \$27 per hour. If the total cost was less than \$88, write and solve an inequality to find the number of hours Danielle could have spent playing tennis.
22. **PIZZA** Three friends shared the cost of an extra-large pizza. In addition, each friend spent \$2 on a beverage. If each friend paid no more than \$7, write and solve an inequality to find the dollar amount that the pizza could cost.
23. **ALGEBRA** The mean of five numbers less four is greater than 20. Write and solve an inequality to find how large the sum of these numbers could be.

H.O.T. Problems

24. **CHALLENGE** Use what you know about the Distributive Property and solving two-step linear inequalities to solve the inequality $2(n - 9) > -4$.
25. **OPEN ENDED** Write a two-step inequality involving division and addition. Solve your inequality.
26. **WRITING IN MATH** Explain the similarities and differences in the methods used to solve the equation $19 = -5x + 4$ and the inequality $19 \leq -5x + 4$.

ISTEP+ PRACTICE 7.2.2

27. Felisa solved the linear inequality $\frac{b}{-2} - 1 \leq 3$ by first adding 1 to each side and then by multiplying both sides by -2 . Her result was $b \leq -8$. Which of the following BEST describes her error?
- A She added 1 to each side when she should have multiplied each side by -1 .
- B She multiplied both sides by -2 when she should have divided both sides by -2 .
- C She added 1 to each side when she should have subtracted 1 from each side.
- D She did not reverse the direction of the inequality symbol.
28. Which of the following is the correct FIRST step in solving the linear inequality $3x - 9 > 12$?
- F Divide both sides by 3.
- G Multiply both sides by 3.
- H Add 9 to each side.
- J Subtract 9 from each side.
29. Hakim bought 3 DVDs. Each DVD was the same price. He also spent \$15 on a CD. He spent less than \$45 altogether. Which of the following inequalities represents this situation?
- A $3x - 15 < 45$
- B $3x + 15 < 45$
- C $3x + 15 \leq 45$
- D $3x + 15 > 45$



Literal Equations

MAIN IDEA

Solve equations and formulas with two variables for a particular variable.

IN Academic Standards

7.2.4 Solve an equation or formula with two variables for a particular variable.
Also addresses 7.3.6.

New Vocabulary

literal equation

IN Math Online

glencoe.com

▶ GET READY for the Lesson

TORNADOES Most tornadoes travel at an average speed of 30 miles per hour. The formula $d = rt$ gives the distance d traveled given a rate r and a time t .

1. An F5 tornado is traveling at a speed of 30 miles per hour. Write the equation that gives the distance d traveled by this tornado for a time of t hours.
2. If the tornado in Exercise 1 covers a distance of 5.4 miles, write the equation that gives the distance traveled by this tornado for a time of t hours.
3. Solve the equation you wrote in Exercise 2 for t . What operation(s) did you perform to solve the equation?



Common Literal Equations	
Area of a Rectangle	$A = \ell w$
Area of a Triangle	$A = \frac{1}{2}bh$
Distance, Rate, and Time	$d = rt$
Perimeter of a Rectangle	$P = 2\ell + 2w$

The formula $d = rt$ represents a literal equation. A **literal equation** is an equation or formula that contains more than one variable. Some common literal equations are shown in the table.

Often, you may need to solve a literal equation for a particular variable when the other variables still remain unknown. This process is called *solving a literal equation*. You can solve a literal equation using the same methods you have been using to solve one- and two-step equations.

EXAMPLE Solve Literal Equations

- 1 TORNADOES** Solve the literal equation $d = rt$ for t . Then use the equation to find the time t that it takes an F2 tornado traveling at 25 miles per hour to cover a distance of 2.7 miles.

Solve the literal equation $d = rt$ for t .

$$d = rt \quad \text{Write the equation.}$$

$$\frac{d}{r} = \frac{rt}{r} \quad \text{To solve for } t, \text{ divide both sides of the equation by } r.$$

$$\frac{d}{r} = t \quad \text{Since the variables remain unknown, you cannot simplify } \frac{d}{r}.$$

The literal equation $d = rt$ solved for t is $\frac{d}{r} = t$ or $t = \frac{d}{r}$.

Use this equation to find the time t that it takes an F2-tornado traveling at 25 miles per hour to cover a distance of 2.7 miles.

$$t = \frac{d}{r} \quad \text{Write the equation.}$$

$$t = \frac{2.7}{25} \quad \text{Replace } d \text{ with 2.7 and } r \text{ with 25.}$$

$$t = 0.108 \quad \text{Divide.}$$

It takes the tornado 0.108 hour, or 6.48 minutes, to travel a distance of 2.7 miles.

EXAMPLES Solve Multi-Step Literal Equations

Solve each literal equation for the indicated variable.

2 $P = 2\ell + 2w$, for w

$$P = 2\ell + 2w \quad \text{Write the equation.}$$

$$\begin{array}{r} -2\ell \quad -2\ell \\ \hline P - 2\ell = 2w \end{array} \quad \begin{array}{l} \text{Subtract } 2\ell \text{ from each side.} \\ \text{You cannot simplify } P - 2\ell. \end{array}$$

$$\frac{P - 2\ell}{2} = \frac{2w}{2} \quad \text{Divide both sides by 2.}$$

$$\frac{P - 2\ell}{2} = w \quad \text{You cannot simplify } \frac{P - 2\ell}{2}.$$

The literal equation solved for w is $\frac{P - 2\ell}{2} = w$, or $w = \frac{P - 2\ell}{2}$.

3 $A = \frac{1}{2}bh$, for b

$$A = \frac{1}{2}bh \quad \text{Write the equation.}$$

$$(2)A = (2)\frac{1}{2}bh \quad \text{Dividing both sides by } \frac{1}{2} \text{ is the same as multiplying both sides by 2.}$$

$$2A = bh \quad \text{Simplify.}$$

$$\frac{2A}{h} = \frac{b\cancel{h}}{\cancel{h}} \quad \text{Divide both sides by } h.$$

$$\frac{2A}{h} = b \quad \text{You cannot simplify } \frac{2A}{h}.$$

The literal equation solved for b is $\frac{2A}{h} = b$, or $b = \frac{2A}{h}$.

Check You can check the answer by substituting values for b , h , and A .

For example, if the base of a triangle is 6 units and the height is 4 units, then the area is $\frac{1}{2}(6)(4)$, or 12 square units.

$$b = \frac{2A}{h} \quad \text{Write the answer.}$$

$$6 \stackrel{?}{=} \frac{2(12)}{4} \quad \text{Replace } b \text{ with 6, } h \text{ with 4, and } A \text{ with 12.}$$

$$6 \stackrel{?}{=} \frac{24}{4} \quad \text{Multiply.}$$

$$6 = 6 \quad \checkmark \quad \text{Divide. This statement is true.}$$



Real-World EXAMPLES

SCIENCE The *momentum* of a moving object can be described as the level of energy carried by the object. It is the product of an object's mass and velocity. The formula for the momentum M of a moving object with a mass m and a velocity v is given by $M = mv$.



- 4** Solve the equation $M = mv$ for v .

$$M = mv \quad \text{Write the equation.}$$

$$\frac{M}{m} = \frac{mv}{m} \quad \text{Divide both sides by } m.$$

$$\frac{M}{m} = v \quad \text{You cannot simplify } \frac{M}{m} \text{ because they are different variables.}$$

The literal equation solved for v is $\frac{M}{m} = v$, or $v = \frac{M}{m}$.

- 5** Use the equation to find the velocity of a black bear whose mass is 185 kilograms and whose momentum is 1,517 kilograms-kilometers per hour.

$$v = \frac{M}{m} \quad \text{Write the equation.}$$

$$v = \frac{1,517}{185} \quad \text{Replace } M \text{ with } 1,517 \text{ and } m \text{ with } 185.$$

$$v = 8.2 \quad \text{Divide.}$$

The velocity of the black bear is 8.2 kilometers per hour.

Check

$$M = mv \quad \text{Write the original equation.}$$

$$1,517 \stackrel{?}{=} 185(8.2) \quad \text{Replace } M \text{ with } 1,517, m \text{ with } 185, \text{ and } v \text{ with } 8.2.$$

$$1,517 \stackrel{?}{=} 1,517 \quad \checkmark \quad \text{Multiply. This statement is true.}$$



CHECK Your Understanding

Examples 1–4
(pp. IN10–IN12)

Solve each literal equation for the indicated variable.

1. $F = ma$, for a

2. $P = \frac{F}{A}$, for F

3. $x = \frac{1}{4}w + z$, for w

4. $4x + 3y = 12$, for y

Examples 1, 4, 5
(pp. IN10, IN12)

SCIENCE The amount of work W done on an object is given by the formula $W = Fd$, where F is the force applied to an object and d is the distance the object moved. The amount of work W is measured in joules. The amount of force applied on the object is measured in newtons.

5. Solve the equation $W = Fd$ for F .

6. Find the amount of force applied on an object if the distance the object moved was 2.1 meters and the work done on the object was 19.53 joules.

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
7-14	1-3
15-18	4, 5

Solve each literal equation for the indicated variable.

- $A = lw$, for w
- $V = Bh$, for h
- $I = \frac{V}{R}$, for V
- $P = \frac{w}{t}$, for w
- $3T = 5r - s$, for r
- $d = 3b + \frac{1}{3}a$, for a
- $-3x + 5y = 15$, for x
- $-2A + 6B = C$, for B

SCIENCE The density D of an object is the amount of mass m is contained in one unit of volume v . The density of an object is given by the formula $D = \frac{m}{v}$. Solve this literal equation for m .

- Solve the equation $D = \frac{m}{v}$ for m .
- Find the mass of 800 cubic centimeters of steel if steel has a density of 7.80 grams per cubic centimeter. (*Hint:* The volume is measured in cubic centimeters.)

SCIENCE The acceleration a of a moving object is given by the formula $a = \frac{f - o}{t}$, where f is the final speed of an object, o is the original speed of the object, and t is the time.

- Solve the equation $a = \frac{f - o}{t}$ for f .
- Find the final speed of a car with an original speed of 30 miles per hour, an acceleration rate of 2 miles per hour per second, and a time of 10 seconds.

H.O.T. Problems

- CHALLENGE** To convert a temperature in degrees Celsius C to a temperature in degrees Fahrenheit F , you can use the formula $F = \frac{9}{5}C + 32$. Solve this literal equation for C and use your equation to find the temperature in degrees Celsius if the temperature in degrees Fahrenheit is 80. Round to the nearest tenth of a degree if necessary.
- WRITING IN MATH** Explain the steps you would take to solve the literal equation $3x + 2y = 6$ for y . Then explain the steps you would use to solve the equation $3x + 2y = 6$ for x . Your answer should include both literal equations solved for each variable.

ISTEP+ PRACTICE 7.2.4

- A rectangle with a length of 8.4 yards has a perimeter of P yards. Which equation could be used to find the width w of the rectangle?
 - $w = P - 8.4$
 - $w = 2P - 8.4$
 - $w = \frac{P}{2} - 16.8$
 - $w = \frac{P - 16.8}{2}$
- Which of the following equations is NOT equivalent to the formula for the volume of a rectangular prism $V = \ell wh$?
 - $\ell = \frac{V}{wh}$
 - $h = V - \ell w$
 - $w = \frac{V}{\ell h}$
 - $h = \frac{V}{\ell w}$



Operations with Decimals

MAIN IDEA

Solve problems that involve operations with decimals.

IN Academic Standards

7.1.7 Solve problems that involve multiplication and division with integers, fractions, decimals and combinations of the four operations.

IN Math Online

glencoe.com

▶ GET READY for the Lesson



NUTS Nina is buying bags of nuts. The nuts costs \$1.25 for each pound. The average weight of each bag is 2.6 pounds.

1. The expression 1.25×2.6 can be used to find the total price of each bag. Estimate the product of 1.25 and 2.6.
2. Multiply 125 by 26.
3. **MAKE A CONJECTURE** about how you can use your answers from Exercises 1 and 2 to find the product of 1.25 and 2.6.
4. What is the total cost of one bag?
5. Use your conjecture from Exercise 3 to find 5.2×2.7 . Explain each step.

To multiply by a decimal, multiply as with whole numbers. To decide where to place the decimal point, count the number of decimal places in each factor. The product has the same number of decimal places.

EXAMPLES Multiply Decimals

Multiply.

1 1.3×0.9 **Estimate** $1 \times 1 = 1$

$$\begin{array}{r}
 1.3 \quad \leftarrow 1 \text{ decimal place} \\
 \times 0.9 \quad \leftarrow 1 \text{ decimal place} \\
 \hline
 1.17 \quad \leftarrow 2 \text{ decimal places}
 \end{array}$$

Compare to the estimate. Since $1.17 \approx 1$, the answer is reasonable.

2 0.054×1.6 **Estimate** $0 \times 2 = 0$

$$\begin{array}{r}
 0.054 \quad \leftarrow 3 \text{ decimal places} \\
 \times 1.6 \quad \leftarrow 1 \text{ decimal place} \\
 \hline
 324 \\
 540 \\
 \hline
 0.0864
 \end{array}$$

Annex a zero on the left so the answer has four decimal places.

Compare to the estimate. Since $0.0864 \approx 0$, the answer is reasonable.

When dividing by decimals, change the divisor into a whole number. To do this, multiply both the divisor and the dividend by the same power of 10. Then divide as with whole numbers.

EXAMPLES

Divide Decimals

Divide.

3 $25.8 \div 2$ **Estimate** $26 \div 2 = 13$

The divisor, 2, is already a whole number, so you do not need to move the decimal point. Divide as with whole numbers.

$$\begin{array}{r} 12.9 \\ 2 \overline{)25.8} \\ \underline{-2} \\ 5 \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$$

Place the decimal point in the quotient directly above the decimal point in the dividend.

4 $199.68 \div 9.6$ **Estimate** $200 \div 10 = 20$

$$\begin{array}{r} 20.8 \\ 9.6 \overline{)199.68} \\ \underline{-192} \\ 768 \\ \underline{-768} \\ 0 \end{array}$$

Compare to the estimate. Since $12.9 \approx 13$, the answer is reasonable.

Move each decimal point one place to the right.

Compare to the estimate. Since $20.8 \approx 20$, the answer is reasonable.

EXAMPLE

Evaluate an Expression

5 ALGEBRA Evaluate $3.5x$ if $x = 4.5$.

$3.5x = 3.5 \times 4.5$ Replace x with 4.5.

$$\begin{array}{r} 3.5 \leftarrow \text{one decimal place} \\ \times 4.5 \leftarrow \text{one decimal place} \\ \hline 175 \\ + 1400 \\ \hline 15.75 \end{array}$$

The product has two decimal places.



CHECK Your Understanding

Examples 1, 2
(p. IN14)

Multiply or divide.

1. $\begin{array}{r} 0.3 \\ \times 0.9 \\ \hline \end{array}$

2. $\begin{array}{r} 0.45 \\ \times 0.12 \\ \hline \end{array}$

3. 0.003×4.82

4. 3.06×0.9

Examples 3, 4
(p. IN15)

5. $0.3 \overline{)9.81}$

6. $3.2 \overline{)5.76}$

7. $0.34 \div 0.2$

8. $14.4 \div 0.12$

ALGEBRA Evaluate each expression if $a = 2.41$.

Example 5
(p. IN15)

9. $0.6a - 1.016$

10. $3.8 + 5.3a$

11. $\frac{10a}{0.5}$

12. $a \div 0.02 - 35.4$

13. **JOBS** Antonia earns \$10.75 per hour. What are her total weekly earnings if she works 34.5 hours? Round to the nearest cent.

Practice and Problem Solving

HOMWORK HELP

For Exercises	See Examples
14–29	1–4
30–37	5

Multiply or divide.

14. 1.8×4.3 15. 1.21×0.35 16. 0.0023×0.28 17. 6.007×1.48
18. 0.44×0.5 19. 38.3×29.1 20. 0.017×5.3 21. 6.05×0.73
22. $0.22 \overline{)0.0132}$ 23. $0.04 \overline{)0.0084}$ 24. $3.18 \overline{)0.636}$ 25. $19.2 \overline{)4.416}$
26. $20.24 \div 2.3$ 27. $2.475 \div 0.03$ 28. $4.6848 \div 0.366$ 29. $97.812 \div 1.1$

ALGEBRA Evaluate each expression if $x = 1.07$, $y = 3.1$, and $z = 0.4$. Round to the nearest tenth.

30. $xy + z$ 31. $x \times 6.023 - z$ 32. $3.25y + x$ 33. xyz
34. $\frac{xy}{z}$ 35. $\frac{yz}{x}$ 36. $\frac{x+y}{z}$ 37. $\frac{x+y-z}{z}$

38. **MEASUREMENT** Neal bought 6.75 yards of fleece fabric to make blankets for a charity. He needs 1.35 yards of fabric for each blanket. How many blankets can Neal make with the fabric he bought?

39. **MEASUREMENT** A meter is approximately equal to about 39.37 inches. How many inches are in 3.3 meters? Round to the nearest tenth.

COMMUNICATION For Exercises 40–42, use the table that shows the most used method to communicate with friends.

Most Used Method To Communicate	
Communication Method	Portion of Responses
cell phone	0.27
E-mail	0.12
between class	0.11
text message	0.3
web site	0.1
other	0.05
home phone	0.03
mail	0.02

40. How many times more respondents use cell phones rather than E-mail? Round to the nearest tenth.
41. How many times more respondents communicate between classes than by home phone? Round to the nearest tenth.
42. How many times more respondents use either cell phones or text messages than Web sites? Round to the nearest tenth.

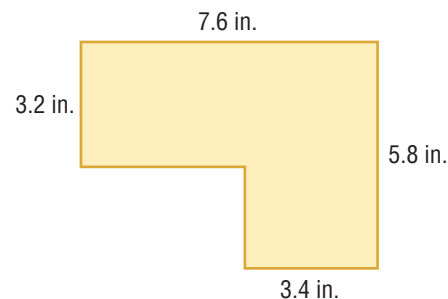
43. **OLYMPICS** In the 2008 Olympics, LaShawn Merritt of the U.S. ran the 400-meter run in 43.75 seconds. To the nearest hundredth, find his speed in meters per second.

44. **GROCERY SHOPPING** Potatoes cost \$1.47 per pound, and carrots cost \$1.99 per pound. Mrs. Rolloson bought 4.65 pounds of potatoes and 1.7 pounds of carrots. How much did she pay for the potatoes and carrots? Round to the nearest hundredth.



MEASUREMENT Refer to the figure shown at the right.

45. Describe two different methods you could use to find the area of the figure.
46. Find the area of the figure using each method. What do you notice?



STATISTICS Find the mean for each set of data.

47. 12.8, 14.6, 15.1, 16.7
48. 4.06, 5.17, 4.87, 5.11, 5.09

49. **MOUNTAINS** Find the average height of the mountains shown in the table.

World's Tallest Mountains (mi)				
5.49	5.35	5.33	5.29	5.27

50. **MEASUREMENT** The thickness of each type of coin is shown in the table. How much thicker is a stack of a dollar's worth of nickels than a dollar's worth of quarters?

Coin	Thickness (mm)
penny	1.55
nickel	1.95
dime	1.35
quarter	1.75



H.O.T. Problems

51. **CHALLENGE** Find two positive decimals x and y that make the following statement true. Then find two positive decimals x and y that make the statement false.

$$\text{If } x < 1 \text{ and } y < 1, \text{ then } x \div y < 1.$$

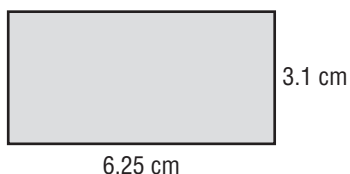
52. **OPEN ENDED** Write a multiplication problem in which the product is between 0.05 and 0.5.
53. **NUMBER SENSE** Place the decimal point in the answer to make it correct. Explain your reasoning.

$$0.0458 \times 9.0194 = 41308852$$

54. **WRITING IN MATH** Write a real-world problem that involves dividing a decimal by a decimal.

ISTEP+ PRACTICE 7.1.7

55. What is the area of the rectangle?



- A 9.35 cm^2 C 19.375 cm^2
 B 18.7 cm^2 D 193.75 cm^2

56. Callie is 4.05 feet tall. Her brother, Lance, is 5.67 feet tall. How many times as tall as Callie is Lance?

- F 0.7
 G 1.2
 H 1.4
 J 2.8



Slope and Similar Triangles

MAIN IDEA

Relate the slope of a line to similar triangles.

IN Academic Standards

7.2.5 Find the slope of a line from its graph and relate the slope of a line to similar triangles.

New Vocabulary

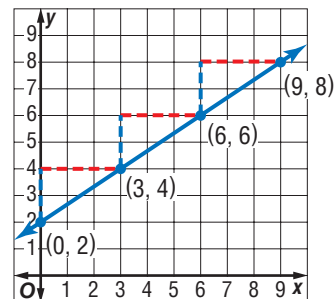
congruent triangles
similar triangles

IN Math Online

glencoe.com

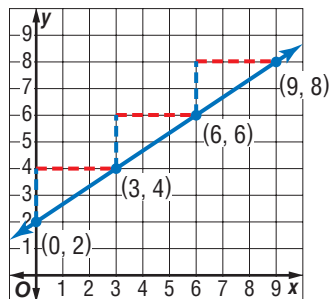
▶ GET READY for the Lesson

- Refer to the graph at the right.
- 1. Find the slope of the line.
- 2. What geometric figure is formed by connecting the vertices $(0, 2)$, $(0, 4)$, and $(3, 4)$?
- 3. What geometric figure is formed by connecting the vertices $(6, 6)$, $(6, 8)$, and $(9, 8)$?
- 4. How do the two figures you identified in Exercises 3 and 4 relate to each other?
- 5. What geometric figure is formed by connecting the vertices $(0, 2)$, $(0, 6)$, and $(6, 6)$?
- 6. How do the two figures you identified in Exercises 3 and 5 relate to each other?



In Lesson 6–3, you learned to find the slope of a line from its graph. In this lesson, you will extend this concept to study the triangles that can be formed in relationship to the slope of a line.

The triangles you identified in Exercises 3 and 4 above are congruent triangles. **Congruent triangles** have the same size and the same shape. The corresponding side lengths of congruent triangles are equal.



Each triangle has a vertical side length of 2 units and a horizontal side length of 3 units.

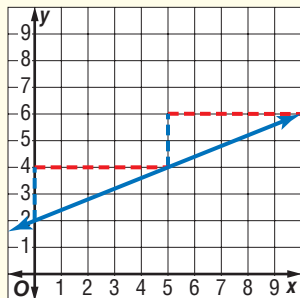
In addition, the slanted line segments between each point have equivalent lengths. You can confirm this by using a ruler.

The slope of the line above is $\frac{2}{3}$. Note that the ratio of the vertical side length to the horizontal side length of each triangle is also $\frac{2}{3}$. This demonstrates the Key Concept at the top of the next page.

Words

The simplified ratio of the vertical side length to the horizontal side length of each congruent triangle formed by the slope of a line is equivalent to the absolute value of the slope.

Example



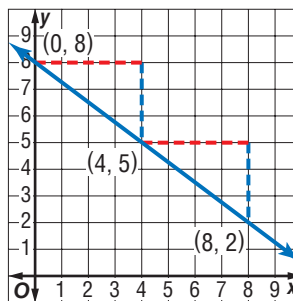
$$\text{ratio: } \frac{\text{vertical side length}}{\text{horizontal side length}} = \frac{2}{5}$$

$$\text{slope} = \frac{2}{5}$$

Note that the ratio of the side lengths is equivalent to the absolute value of the slope. Recall that the *absolute value* of a number is the distance the number is from zero. In the Key Concept box above, both the slope and the ratio were positive.

The slope of a line can sometimes be positive or negative. However, the side lengths of each triangle formed are always positive. When a line has a negative slope, the ratio of the side lengths of the triangles formed remains positive and is equal to the absolute value of the slope.

Refer to the graph below.



The slope of the line is $-\frac{3}{4}$.

The lengths of the horizontal red dotted line segments are each 4 units.

The lengths of the vertical blue dotted line segments are each 3 units.

The ratio of the vertical side length to the horizontal side length of each triangle is $\frac{3}{4}$.

Note that the ratio of the side lengths is positive. However, the slope is negative.

Since $\left|-\frac{3}{4}\right| = \frac{3}{4}$, the ratio is equivalent to the absolute value of the slope.

You can use the information from the Key Concept box above to analyze the congruent triangles formed by the slope of the line.

EXAMPLES

Analyze Congruent Triangles

Refer to the graph at the right.

- Find the length of the red dotted line segments in each of the three triangles formed.

Each red dotted line segment is three units long.

- Find the length of the blue dotted line segments in each of the three triangles formed.

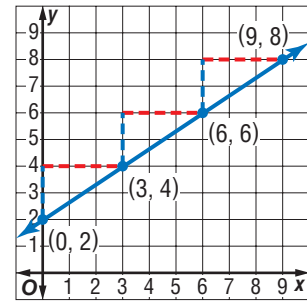
Each blue dotted line segment is two units long.

- Describe how the three triangles relate to each other.

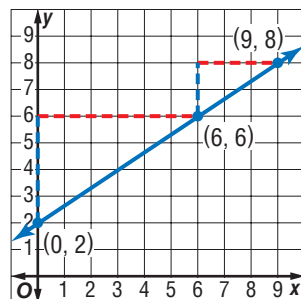
The three triangles are congruent. They have the same shape and the same size.

- Describe how the side lengths of each triangle relate to the slope of the line.

The slope of the line is $\frac{2}{3}$. The ratio of the vertical side length to the horizontal side length of each triangle is equivalent to the absolute value of the slope.



The triangles you identified in Exercises 3 and 5 at the beginning of the lesson are similar triangles. Those triangles are shown below. **Similar triangles** have the same shape but not necessarily the same size.



The ratio of the vertical side length to the horizontal side length of the larger triangle is 4 to 6, or $\frac{2}{3}$.

The ratio of the vertical side length to the horizontal side length of the smaller triangle is 2 to 3, or $\frac{2}{3}$.

The corresponding side lengths of similar triangles are *proportional*. Recall from Lesson 6-6 that two quantities are proportional if they have the same ratio. The side lengths of the similar triangles above have the same simplified ratio, $\frac{2}{3}$. Note that the slope of the line is also $\frac{2}{3}$.

Both the slope and the simplified ratio of the side lengths of the graph above are positive. When a line has a negative slope, the simplified ratio will still be positive.

The relationship between the slope of a line and the side lengths of the similar triangles formed is the same as the relationship between the slope of a line and the side lengths of the congruent triangles formed.

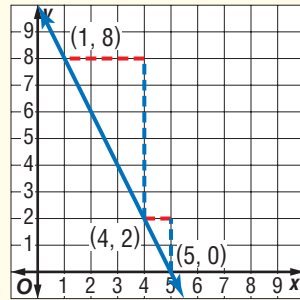
Slope and Similar Triangles

Key Concept

Words

The simplified ratio of the vertical side length to the horizontal side length of each congruent triangle formed by the slope of a line is equivalent to the absolute value of the slope.

Example



$$\text{slope} = \frac{-2}{1}, \text{ or } -2$$

Larger Triangle

$$\text{ratio: } \frac{\text{vertical side length}}{\text{horizontal side length}} = \frac{6}{3}, \text{ or } 2$$

Smaller Triangle

$$\text{ratio: } \frac{\text{vertical side length}}{\text{horizontal side length}} = \frac{2}{1}, \text{ or } 2$$

You can use the information from the Key Concept box above to analyze the similar triangles formed by the slope of the line.

EXAMPLES

Analyze Similar Triangles

Refer to the graph of the line at the right.

- 5** Find the length of the red dotted line segments in each triangle formed.

In the smaller triangle, the length of the red dotted line segment is two units. In the larger triangle, the length of the red dotted line segment is four units.

- 6** Find the length of the blue dotted line segments in each triangle formed.

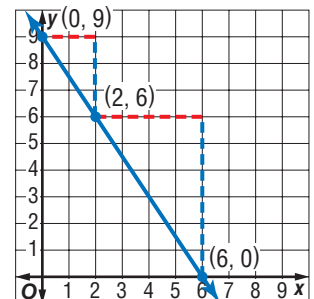
In the smaller triangle, the length of the blue dotted line segment is three units. In the larger triangle, the length of the blue dotted line segment is six units.

- 7** Describe how the triangles relate to each other.

The two triangles are similar. They have the same shape but not the same size.

- 8** Describe how each triangle relates to the slope of the line.

The slope of the line is $-\frac{3}{2}$. The simplified ratio of the vertical side length to the horizontal side length of each triangle is $\frac{3}{2}$, which is equivalent to the absolute value of the slope.



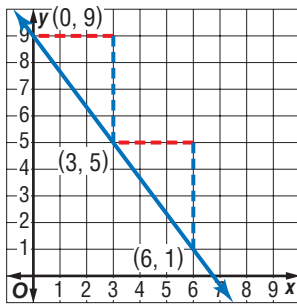
CHECK Your Understanding

Examples 1–8
(pp. IN20–IN21)

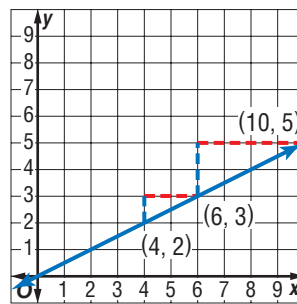
For each graph,

- Find the slope of each line.
- Find the length of the vertical and horizontal side lengths of each triangle shown.
- Find the simplified ratio of the vertical side length to the horizontal side length and explain how this ratio relates to the slope of the line.

1.



2.



Practice and Problem Solving

HOMEWORK HELP

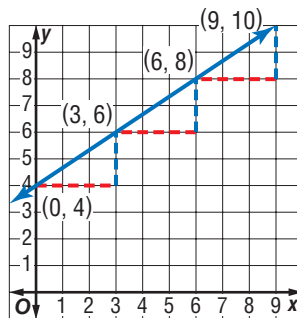
For Exercises
3–6

See Examples
1–8

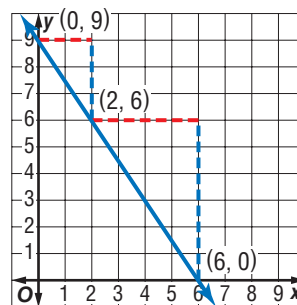
For each graph,

- Find the slope of each line.
- Find the length of the vertical and horizontal side lengths of each triangle shown.
- Find the simplified ratio of the vertical side length to the horizontal side length and explain how this ratio relates to the slope of the line.

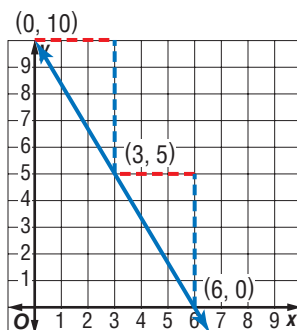
3.



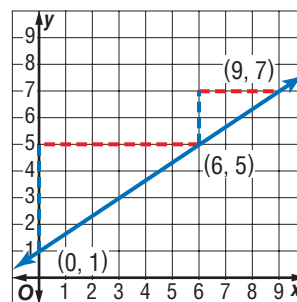
4.



5.

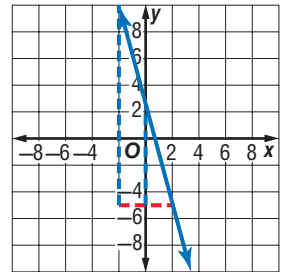


6.



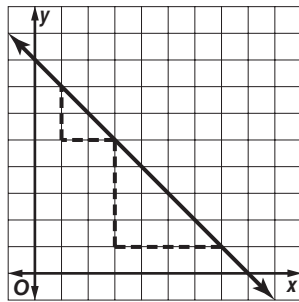
H.O.T. Problems

7. **OPEN ENDED** Draw the graph of a line with a positive slope. Then draw the triangles formed by the slope of the line and demonstrate that the simplified ratio of the vertical side length to the horizontal side length of each triangle is equivalent to the slope.
8. **REASONING** The ratio of the vertical side length to the horizontal side length of each triangle formed by the slope of a line is $\frac{1}{5}$. Find two possible slopes for the line. Justify your response.
9. **REASONING** If two lines have the same slope, then they are parallel. The graph of line a has a slope of $-\frac{3}{2}$. For line b , the ratio of the vertical side length to the horizontal side length of each triangle formed by the slope is $\frac{3}{2}$. Does this automatically imply that lines a and b are parallel? Explain.
10. **CHALLENGE** The slope of a line is -3.5 . What is the simplified ratio of the vertical side length to the horizontal side length of each triangle formed? Justify your response.
11. **WRITING IN MATH** Write a few sentences explaining how the slope of a line is related to similar triangles.

**ISTEP+ PRACTICE**

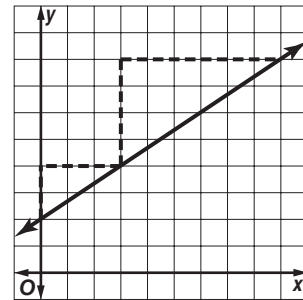
7.2.5

12. Which of the following statements is NOT true concerning the graph below?



- A The simplified ratio of the vertical side length to the horizontal side length of each triangle is 1.
- B The slope of the line is 1.
- C The slope of the line is -1 .
- D The smaller triangle and the larger triangle shown are similar.

13. Which statement is TRUE concerning the slope of the line below?



- F It is equivalent to the simplified ratio of the vertical side length to the horizontal side length of each triangle shown.
- G It is equivalent to $\frac{3}{2}$.
- H It is equivalent to the simplified ratio of the horizontal side length to the vertical side length of each triangle shown.
- J It is equivalent to $-\frac{2}{3}$.



Graphs of Linear Equations

MAIN IDEA

Draw a line given its slope and one point on the line or two points on the line.

IN Academic Standards

7.2.6 Draw the graph of a line given its slope and one point on the line or two points on the line.

IN Math Online

glencoe.com

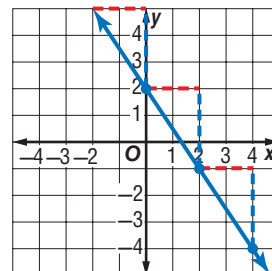
▶ GET READY for the Lesson

Refer to the graph at the right.

1. Find the slope of the line.
2. Complete the statement below by filling in the blanks with whole numbers.

From the point $(0, 2)$, count ____ units down and ____ units to the right to arrive at the point $(2, -1)$.

3. How do the numbers you found in Exercise 2 relate to the slope of the line you found in Exercise 1?
4. To arrive at the point $(2, -1)$ from the point $(0, 2)$, you need to count *down* and to the right. How is counting *down* represented in the value of the slope?



In Lesson 6-3, you found the slope of a line from a table of values and from a graph. In this lesson, you will extend this concept to graph a line given its slope and a point on the line. Begin by graphing the point. Then use the slope to find additional points on the line. Connect the points with a solid line.

EXAMPLES Draw Lines Given Slope and Point

Draw a line that has the given slope and passes through the indicated point.

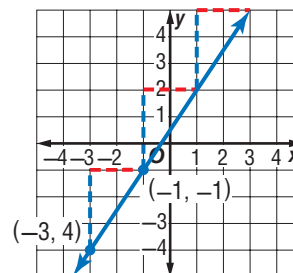
- 1** slope: $\frac{3}{2}$, point: $(-3, -4)$

STEP 1 Graph the point $(-3, -4)$.

STEP 2 Use the slope to find a second point on the line. The slope is $\frac{3}{2}$. The numerator 3 represents the vertical change. The denominator 2 represents the horizontal change. So, from $(-3, -4)$, count 3 units up and 2 units to the right. Graph the point $(-1, -1)$.

STEP 3 From $(-1, -1)$, count 3 units up and 2 units to the right. Graph the point $(1, 2)$.

STEP 4 Continue this process to find additional points on the line. Connect the points with a solid line.



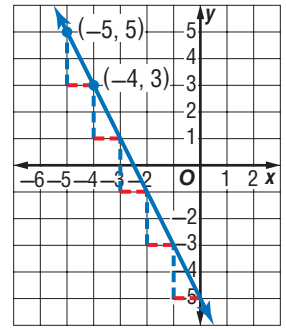
2 slope: -2 , point: $(-5, 5)$

STEP 1 Graph the point $(-5, 5)$.

STEP 2 Use the slope to find a second point on the line. The slope is -2 . As a fraction, this is $\frac{-2}{1}$. The numerator -2 represents the vertical change. The denominator 1 represents the horizontal change. So, from $(-5, 5)$, count 2 units down and 1 unit to the right. Graph the point $(-4, 3)$.

STEP 3 From $(-4, 3)$, count 2 units down and 1 unit to the right. Graph the point $(-3, 1)$.

STEP 4 Continue this process to find additional points on the line. Connect the points with a solid line.



You can also draw a line given two points on the line by graphing each point and then drawing a line that connects the points.

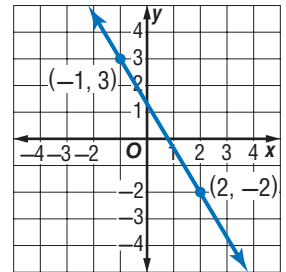
EXAMPLE Draw a Line Given Two Points

3 Draw a line that passes through the points $(-1, 3)$ and $(2, -2)$. Then find the slope of the line as a ratio in simplest form.

Graph each point. Using a straightedge, connect the two points with a solid line.

Find the slope of the line.

$$\begin{aligned}\text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{-2 - 3}{2 - (-1)} && \text{Use } (-1, 3) \text{ and } (2, -2). \\ &= \frac{-5}{3}, \text{ or } -\frac{5}{3} && \text{Simplify. The slope is } -\frac{5}{3}.\end{aligned}$$



You can also use an equation to find one or more points on a line and the slope of a line.

EXAMPLE Find Points and Slope

4 ALGEBRA Find a point on the line $y = 4x - 1$. Then find the slope of the line.

$$y = 4x - 1 \quad \text{Write the equation.}$$

$$y = 4(1) - 1 \quad \text{Replace } x \text{ with any value, such as } 1.$$

$$y = 3 \quad \text{Simplify. The } y\text{-value is } 3.$$

One point on the line is $(1, 3)$.

To find the slope, find another point on the line using the same method as above. Another point on the line is $(2, 7)$.

The slope between the points $(1, 3)$ and $(2, 7)$ is $\frac{7-3}{2-1}$, or 4.



Real-World EXAMPLE

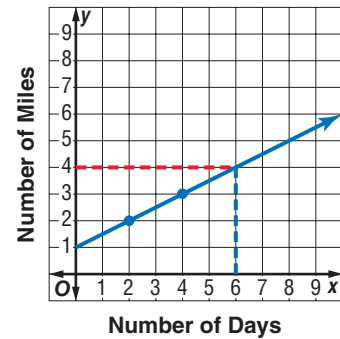
- 5 RUNNING** Alexa is training for a race. She ran 2 miles on the second day of training. She ran 3 miles on the fourth day of training. The points $(2, 2)$ and $(4, 3)$ represent this situation where each x -coordinate represents the number of days of training and each y -coordinate represents the number of miles ran. Graph the line that passes through these points. Then use your line to find the number of miles Alexa will run on the sixth day of training if this pattern continues.

Graph each point. Using a straightedge, connect the two points with a solid line as shown.

Find the point on the line that has an x -coordinate of 6. The point is $(6, 4)$.

So, on the sixth day of training, Alexa will run 4 miles.

Alexa's Race Training



CHECK Your Understanding

Examples 1, 2
(pp. IN24–IN25)

Draw a line that has the given slope and passes through the indicated point.

1. slope: $\frac{3}{4}$, point $(0, 0)$ 2. slope: $\frac{1}{2}$, point $(-1, 2)$ 3. slope: -2 , point $(3, -4)$

Example 3
(p. IN25)

Draw a line that has passes through each pair of points. Then find the slope of the line as a ratio in simplest form.

4. $(3, 4)$ and $(-1, -1)$ 5. $(1, 2)$ and $(4, 4)$ 6. $(-3, 5)$ and $(2, -1)$

Example 4
(p. IN25)

ALGEBRA For each equation, find a point on the line. Then find the slope of the line.

7. $y = x + 1$ 8. $y = \frac{1}{4}x + 3$ 9. $y = -\frac{2}{3}x - 2$

ALGEBRA For each equation, find two points on the line.

10. $y = x - 5$ 11. $y = \frac{3}{4}x + 1$ 12. $y = -\frac{4}{5}x$

Example 5
(p. IN26)

13. **JOBS** During the summer, Harrison mows lawns in the neighborhood. In the third week of the summer, he mowed 2 lawns. In the sixth week of the summer, he mowed 4 lawns. The points $(3, 2)$ and $(6, 4)$ represent this situation where each x -coordinate represents the number of weeks and each y -coordinate represents the number of lawns mowed. Graph the line that passes through these points. Then use your line to find the number of lawns Harrison will mow during the ninth week of the summer if this pattern continues.

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
14–21	1, 2
22–29	3
32–43	4
30, 31	5

Draw a line that has the given slope and passes through the indicated point.

14. slope: 3, point (1, 4) 15. slope: $-\frac{3}{2}$, point (–3, 2)
16. slope: $\frac{1}{4}$, point (0, –1) 17. slope: –2, point (0, 0)
18. slope: 1, point (2, –3) 19. slope: $-\frac{3}{2}$, point (0, 8)
20. slope: $\frac{2}{5}$, point (–2, –9) 21. slope: –1, point (–3, 4)

Draw a line that passes through each pair of points. Then find the slope of the line as a ratio in simplest form.

22. (0, 1) and (–3, –1) 23. (2, –5) and (0, 0)
24. (2, 2) and (4, 0) 25. (–1, 4) and (3, –2)
26. (–1, 1) and (3, 2) 27. (0, 0) and (2, 1)
28. (–2, –1) and (2, 0) 29. (0, 3) and (3, –4)

30. **SCHOOL** Elisa is selling bracelets to raise funds for the German Club. On the second day, she sold 5 bracelets. On the fourth day, she sold ten bracelets. The points (2, 5) and (4, 10) represent this situation where each x -coordinate represents the number of days and each y -coordinate represents the number of bracelets sold. Graph the line that passes through these points. Then use your line to find the number of bracelets Elisa will sell on the 8th day if this pattern continues.



31. **TEXT MESSAGES** Prim sent 5 text messages for \$1. Later, he sent 10 text messages for \$2. The points (5, 1) and (10, 2) represent this situation where each x -coordinate represents the number of text messages and each y -coordinate represents the cost in dollars. Graph the line that passes through these points. Then use your line to find the cost in dollars if Prim sends 20 text messages.



ALGEBRA For each equation, find a point on the line and the slope of the line.

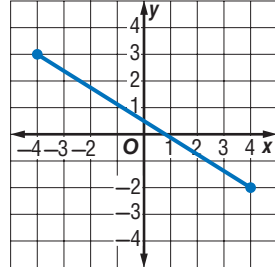
32. $y = 2x + 1$ 33. $y = \frac{3}{4}x$ 34. $y = -\frac{1}{2}x + 2$
35. $y = x$ 36. $y = 3x$ 37. $y = \frac{1}{3}x + 1$

ALGEBRA For each equation, find two points on the line.

38. $y = 4x$ 39. $y = \frac{1}{2}x + 5$ 40. $y = -x + 4$
41. $y = \frac{2}{3}x$ 42. $y = 3x - 1$ 43. $y = 2x$

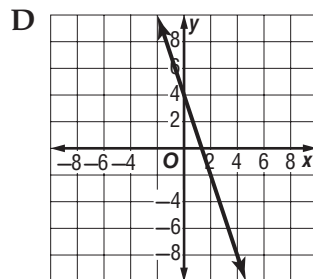
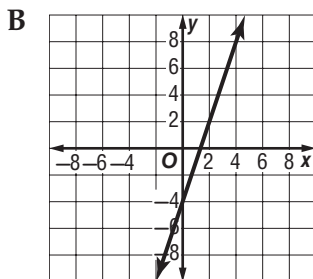
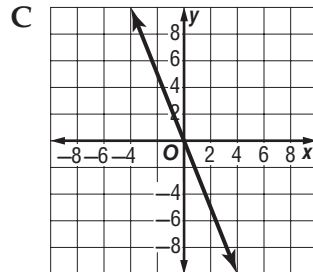
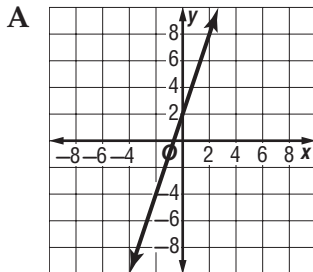
H.O.T. Problems

44. **OPEN ENDED** A line passes through the points $(-3, 1)$ and $(2, 4)$. Find three other points on this line.
45. **ALGEBRA** A line passes through the points (a, b) and $(a + 3, b + 2)$. What is the slope of this line?
46. **REASONING** A line has a slope of 1.5 and passes through the point $(-3, 1)$. Describe the steps you would take to graph this line beginning at the point $(-3, 1)$.
47. **CHALLENGE** The *midpoint* of a line segment is the point on the line that marks the halfway point between the segment's two endpoints. The line segment at the right has endpoints $(4, -2)$ and $(-4, 3)$. Use the slope of this line segment to find the midpoint of this line segment. Explain your method.
48. **WRITING IN MATH** Write a few sentences explaining how to draw the graph of a line given its slope and a point on the line.



ISTEP+ PRACTICE 7.2.6

49. A line has a slope of 3 and passes through the point $(3, 5)$. Which of the following is the correct graph of the line?



50. What is the correct **FIRST** step in graphing the line that passes through the point $(-1, 4)$ and that has a slope of $\frac{2}{3}$?
- F Graph the point $(0, 0)$.
- G Graph a point that is 2 units up and 3 units to the right of the origin.
- H Graph the point $(-1, 4)$.
- J Graph a point that is 3 units up and 2 units to the right of the point $(-1, 4)$.

7

Direct Variation

MAIN IDEA

Solve problems that involve direct variation.

IN Academic Standards

7.2.7 Identify situations that involve proportional relationships, draw graphs representing these situations and recognize that these situations are described by a linear function in the form $y = mx$ where the unit rate m is the slope of the line.

New Vocabulary

direct variation
constant of variation

IN Math Online

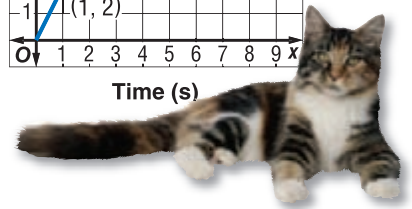
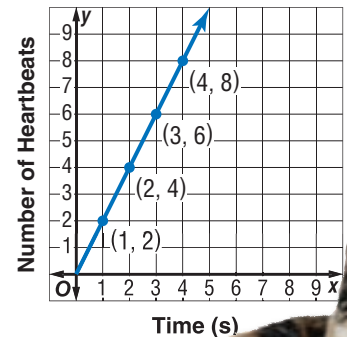
glencoe.com

▶ GET READY for the Lesson

CATS The graph at the right shows the average number of heartbeats for an adult housecat.

1. Find the slope of the line.
2. Describe the relationship between the x -value and the y -value of each point on the line.
3. Write an equation that gives the value of y for each value of x on the line.
4. How does the equation you wrote in Exercise 2 show the slope of the line?
5. What is the value of y when the x -value is zero?

Average Heartbeat of a Housecat



The graph above demonstrates a proportional relationship. Recall that two quantities are *proportional* if they have a constant rate or ratio. In the graph above, the constant rate is the slope of the line, 2.

When two quantities are proportional, their relationship is a **direct variation**. The graph of a direct variation is a straight line that passes through the origin.

Direct Variation

Key Concept

Words

A direct variation is a relationship in which the ratio of y to x is a constant, k . We say y varies directly with x .

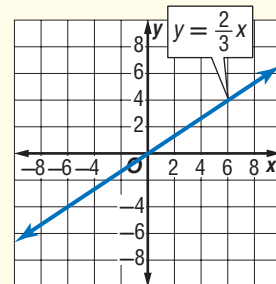
Symbols

$$y = kx, \text{ where } k \neq 0$$

Example

$$y = \frac{2}{3}x$$

Graph



In the equation $y = kx$, k is called the **constant of variation**. Note that k is the slope of the line. Often, the slope of a line is noted by the variable m . So, $y = mx$ and $y = kx$ each represent a direct variation relationship.

Not all relationships with a constant rate of change are proportional. Likewise, not all linear functions are direct variations.

EXAMPLES Identify Direct Variation

Find the slope of each linear function. Then determine whether each linear function is a direct variation. If so, state the constant of variation and write the direct variation equation. If not, explain why not.

1	Miles, x	25	50	75	100
	Gallons, y	1	2	3	4

Find the slope.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{2 - 1}{50 - 25} = \frac{1}{25}$$

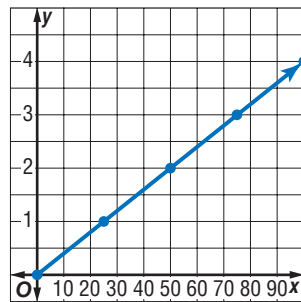
Use two points, such as (25, 1) and (50, 2).

The slope is $\frac{1}{25}$.

Determine if the linear function is a direct variation. Each y -value is obtained by multiplying each x -value by $\frac{1}{25}$. The linear function is a direct variation since the ratio of each y -value to each x -value is constant, $\frac{1}{25}$.

The constant of variation is $\frac{1}{25}$. So, the direct variation equation is $y = \frac{1}{25}x$. The slope is also $\frac{1}{25}$.

Check your answer by graphing the line through the points in the table.



The graph is a straight line with a slope of $\frac{1}{25}$.

The graph passes through the origin.

So, the graph is a direct variation with equation $y = \frac{1}{25}x$. The answer is correct.

2	Hours, x	2	4	6	8
	Earnings, y	36	52	68	84

Find the slope.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{52 - 36}{4 - 2} = \frac{16}{2} = 8$$

Use two points, such as (2, 36) and (4, 52).

The slope is 8.

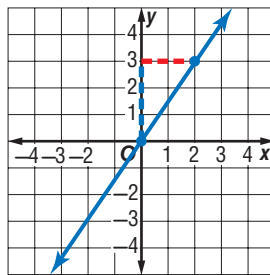
Determine if the linear function is a direct variation. Check to see if the two quantities are proportional.

$$\frac{\text{earnings, } y}{\text{hours, } x} \longrightarrow \frac{36}{2}, \text{ or } 18 \quad \frac{52}{4}, \text{ or } 13 \quad \frac{68}{6}, \text{ or } 11\frac{1}{3} \quad \frac{84}{8}, \text{ or } 10\frac{1}{2}$$

The ratios are not the same, so the function is not a direct variation.

Note that the rate of change, or slope, of the linear function in Example 2 is constant. But since the ratio of each y -value to each x -value is not constant, the relationship is not a direct variation. In a direct variation, as in Example 1, the slope of the line is the same as the constant of variation.

You can draw the graph of a direct variation. Recall from Indiana Additional Lesson 6 that you can graph a line given its slope and a point on the line. The graph of a direct variation passes through the origin, $(0, 0)$. You can use $(0, 0)$ as a point on the line. Then use the slope to find a second point on the line.



Consider the direct variation equation $y = \frac{3}{2}x$.

The graph passes through $(0, 0)$.

Graph the point $(0, 0)$.

The slope is $\frac{3}{2}$.

From $(0, 0)$, count up 3 units and 2 units to the right. Graph the point $(2, 3)$.

Connect the points with a solid line.

EXAMPLE

Graph a Direct Variation

3 Graph the direct variation function $y = -\frac{3}{5}x$.

The slope of the line is $-\frac{3}{5}$. Because the function is a direct variation, it passes through the origin, $(0, 0)$. Use the point $(0, 0)$ as a point on the line.

STEP 1 Graph the point $(0, 0)$.

STEP 2 Use the slope to find a second point on the line.

The slope is $-\frac{3}{5}$.

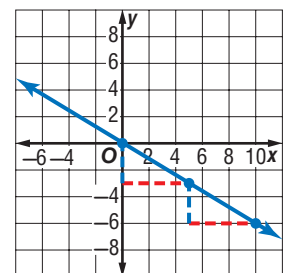
The numerator -3 represents the vertical change.

The denominator 5 represents the horizontal change.

So, from $(0, 0)$, count 3 units down and 5 units to the right. Graph the point $(-3, -5)$.

STEP 3 From $(-3, -5)$, count 3 units down and 5 units to the right. Graph the point $(10, -6)$.

STEP 4 Continue this process to find additional points on the line. Connect the points with a solid line.





Real-World EXAMPLE

Write the Direct Variation

4 BAKING The recipe at the right requires $3\frac{1}{2}$ cups of flour and makes 14 servings of turtle cake. The relationship of number of servings to number of cups of flour is directly proportional. Find the constant of variation and write the direct variation equation that gives the number of servings y for x cups of flour.



Find the constant of variation.

$$y = kx \quad \text{Write the direct variation equation.}$$

$$14 = k\left(3\frac{1}{2}\right) \quad \text{Replace } y \text{ with } 14 \text{ and } x \text{ with } 3\frac{1}{2}.$$

$$\frac{14}{3\frac{1}{2}} = \frac{k\left(3\frac{1}{2}\right)}{3\frac{1}{2}} \quad \text{Divide both sides by } 3\frac{1}{2}.$$

$$4 = k \quad \text{Simplify.}$$

The constant of variation is 4.

Write the direct variation equation.

$$y = kx \quad \text{Write the direct variation equation.}$$

$$y = 4x \quad \text{Replace } k \text{ with } 4.$$

The direct variation equation is $y = 4x$.

Check $y = kx$ Write the direct variation equation.

$$y = 4\left(3\frac{1}{2}\right) \quad \text{Replace } k \text{ with } 4 \text{ and } x \text{ with } 3\frac{1}{2}.$$

$$y = 14 \quad \text{Multiply.}$$

Since 14 is the correct number of servings that can be made from $3\frac{1}{2}$ cups of flour, the equation $y = 4x$ is correct. ✓



CHECK Your Understanding

Examples 1, 2
(p. IN30)

Find the slope of each linear function. Then determine whether each linear function is a direct variation. If so, state the constant of variation and write the direct variation equation.

Days, x	5	10	15	20
Height, y	12.5	25	37.5	50

Time, x	4	6	8	10
Distance, y	12	16	20	24

Example 3
(p. IN31)

Graph each direct variation function. State the constant of variation.

3. $y = -2x$

4. $y = \frac{2}{3}x$

Example 4
(p. IN32)

5. **GROCERIES** A grocery store sells 6 oranges for \$2. The relationship of cost to number of oranges is directly proportional. Find the constant of variation and write the direct variation equation that gives the cost y of x number of oranges.

Practice and Problem Solving

HOMESCHOOL HELP

For Exercises	See Examples
6–9	1, 2
10–13	3
14, 15	4

Find the slope of each linear function. Then determine whether each linear function is a direct variation. If so, state the constant of variation and write the direct variation equation. If not, explain why not.

6.

Hours, x	2	3	4	5
Miles, y	116	174	232	290

7.

Price (\$), x	10	15	20	25
Tax (\$), y	0.70	1.05	1.40	1.75

8.

Minutes, x	200	400	600	800
Cost (\$), y	65	115	165	215

9.

Pictures, x	5	6	7	8
Profile, y	20	24	28	32

Graph each direct variation function. State the constant of variation.

10. $y = -\frac{3}{4}x$ 11. $y = \frac{2}{3}x$ 12. $y = \frac{1}{3}x$ 13. $y = -x$

14. **ELECTRONICS** The width of a wide-screen television screen is directly proportional to its height. The width of a screen is 57.6 centimeters and its height is 32 centimeters. Find the constant of variation and write the direct variation equation that gives the width y given the height x .
15. **MEASUREMENT** An object that weighs 70 pounds on Mars weighs 210 pounds on Earth. The relationship of weight on Mars to weight on Earth is directly proportional. Find the constant of variation and write the direct variation equation that gives the weight on Mars y given the weight on Earth x .

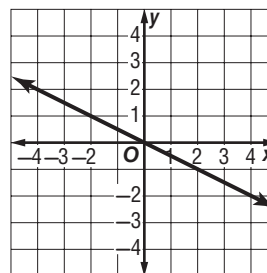
H.O.T. Problems

16. **OPEN ENDED** Write a linear equation that represents a direct variation. Identify the constant of variation and state three points that satisfy your equation.
17. **REASONING** The formula $d = rt$ gives the distance d traveled by an object with a rate r for a time t . Suppose a car is traveling at a rate of 50 miles per hour. Explain how the equation $d = 50t$ represents a direct variation relationship.
18. **WRITING IN MATH** Determine whether the statement below is *always*, *sometimes*, or *never* true. Explain your reasoning.

A relationship that has a constant rate of change is a proportional relationship.

ISTEP+ PRACTICE 7.2.7

19. Which equation represents the relationship shown at the right?
- A $y = -2x$ C $y = -\frac{1}{2}x + 1$
 B $y = -\frac{1}{2}x$ D $y = 2x$





Choose an Appropriate Display

MAIN IDEA

Choose the most appropriate display for a situation and justify the choice.

IN Academic Standards

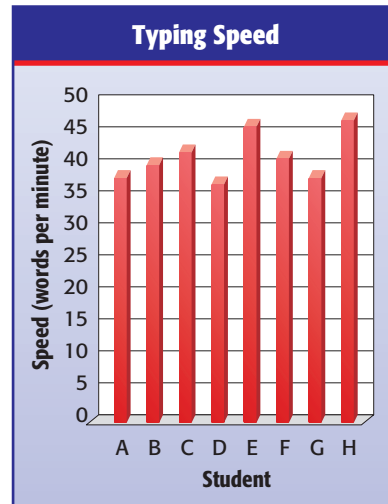
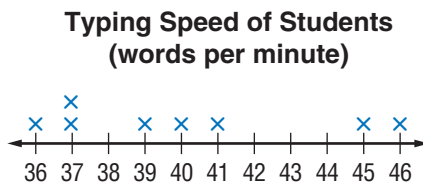
7.4.1 Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs. Justify the choice of data display.

IN Math Online

glencoe.com

▶ GET READY for the Lesson

TYPING The displays show the typing speed of eight students in Mr. Terrell's typing class.



1. Which display allows you to find student A's typing speed? Justify your choice.
2. In which display is it easier to find the number of students that type 45 words per minute? Justify your choice.

Data can often be displayed in several different ways. The display you choose depends on your data and what you want to show.

Statistical Displays

Concept Summary

Type of Display	Best Used to
Bar Graph	show the number of items in specific categories
Circle Graph	compare parts of the data to the whole
Histogram	show frequency of data divided into equal intervals
Line Graph	show change over a period of time
Line Plot	show how many times each number occurs in the data

EXAMPLE

Choose an Appropriate Display

- 1 **SHOPPING** Choose an appropriate display to show the sales of a particular brand of clothing compared to the total sales of all brands sold at the store. Justify your choice.

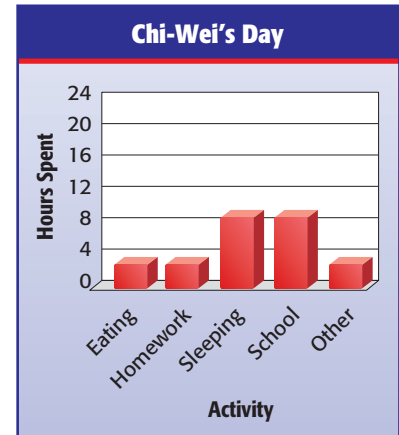
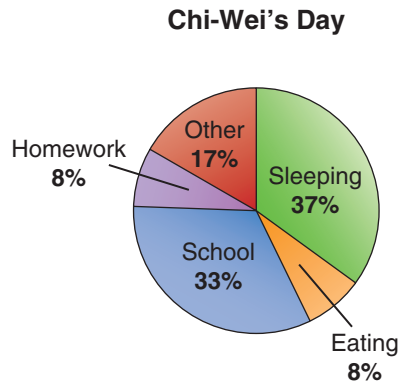
Since the display will show the parts of a whole, a circle graph would be an appropriate display to represent this data.



Real-World EXAMPLE

Choose an Appropriate Display

- 2 TIME** Which display allows you to easily estimate the number of hours that Chi-Wei typically spends on each activity throughout the day? Justify your choice.



The bar graph allows you to estimate the number of hours that Chi-Wei typically spends on each activity. The circle graph gives you the percentage of the entire day that Chi-Wei spends on each activity. Because you know that there are 24 hours in a day, you can use the circle graph to determine the number of hours spent, but this is not *easily* determined from the display itself.

EXAMPLE

Construct an Appropriate Display

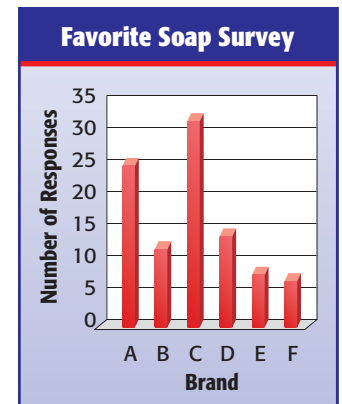
- 3 MARKETING** A market researcher conducted a survey to compare different brands of soap. The table shows the number of first-choice responses for each brand. Construct an appropriate type of display to compare the number of responses for each brand of soap. Justify your choice.

Favorite Soap Survey			
Brand	Responses	Brand	Responses
A	24	D	13
B	11	E	7
C	31	F	6

These data show the number of responses for each brand. So, a bar graph would be the best display to compare the responses.

STEP 1 Draw and label horizontal and vertical axes. Add a title.

STEP 2 Draw a bar to represent the number of responses for each brand.



Study Tip

Appropriate Displays
A circle graph could also be used to display the data. A circle graph would show the portion of responses of the whole, 92 responses, but it would not show the number of responses for each brand.

CHECK Your Understanding

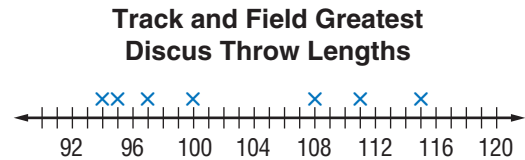
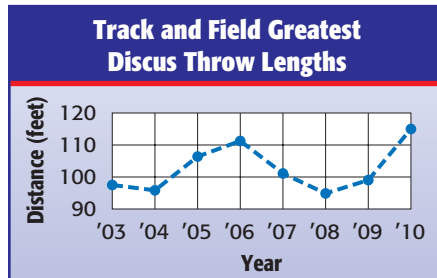
Example 1
(p. IN34)

Choose an appropriate type of display for data gathered about each situation. Justify your choice.

- the favorite television show of the seventh-grade students from a list of 5 television shows

Example 2
(p. IN35)

- the winning times over the past 20 years of the Boston marathon
- TRACK AND FIELD** Which display makes it easier to see how many times the discus was thrown over 101 feet? Justify your choice.



Example 3
(p. IN35)

- TESTS** Choose and construct an appropriate display for the data in the table below. Justify your choice.

78	89	94	75	87	91	93	86	97	97	92
65	98	86	72	85	90	83	74	88	81	77

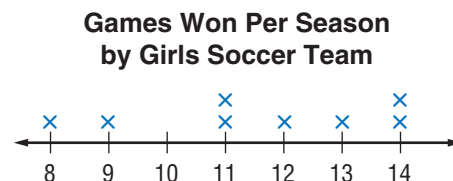
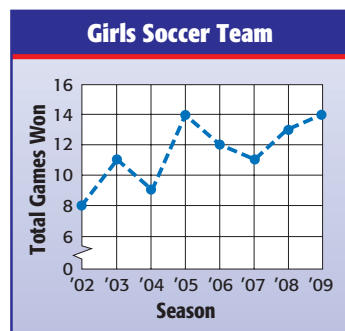
Practice and Problem Solving

HOMEWORK HELP

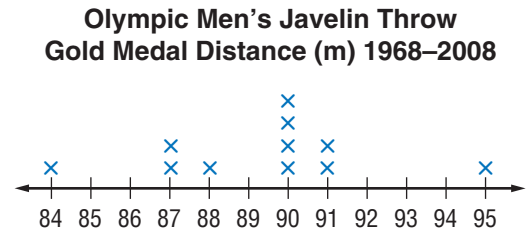
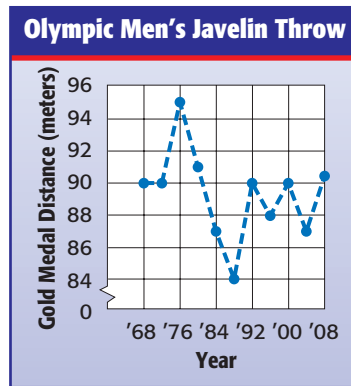
For Exercises	See Examples
5–8	1
9, 10	2
11, 12	3

Choose an appropriate type of display for data gathered about each situation. Justify your choice.

- the number of cell phone subscribers for the past 10 years
- the prices of six different brands of athletic shoes
- Juliana's height on January 1st of each year for the past 5 years
- the portion of your day spent doing various activities
- SOCCER** Which display allows you to see whether the team's record of wins has steadily improved since 2002? Justify your choice.



10. **OLYMPICS** Which display makes it easier to see how many times the winning distance of the javelin throw was 90 meters? Explain.



Choose and construct an appropriate type of display for each situation.

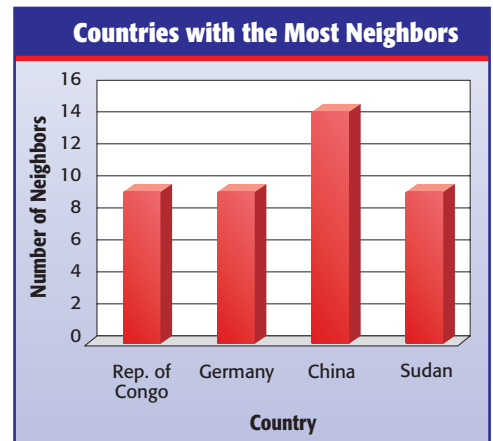
11. **Ocean Areas**

Ocean	Area (sq. mi)
Arctic	5,427,000
Atlantic	29,637,900
Indian	26,469,900
Pacific	60,060,700
Southern	7,848,300

12. **Average Height of Females**

Age (yr)	Height (in.)
10	56.4
11	59.6
12	61.4
13	62.6
14	63.7
15	63.8

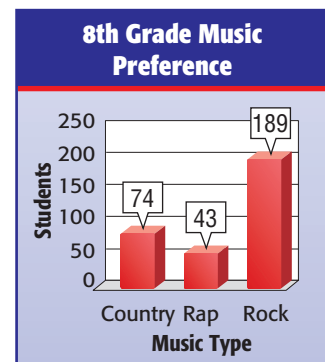
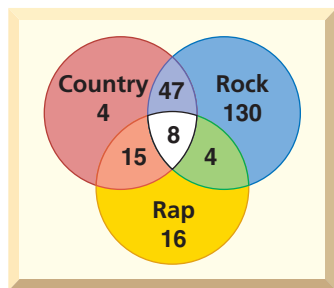
13. **GEOGRAPHY** Display the data in the bar graph using another type of display. Compare the advantages of each display.
14. **RESEARCH** Use the Internet or another source to find a set of data that is displayed in a bar graph, line graph, stem-and-leaf plot, or line plot. Was the most appropriate type of display used? What other ways might these same data be displayed?



Source: *Top 10 of Everything*

15. **MUSIC** Which display is most appropriate to determine the number of students who like only country music? Justify your response.

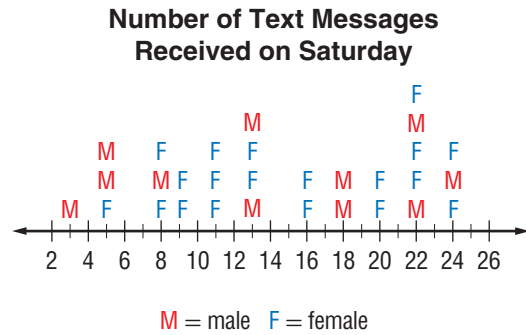
8th Grade Music Preference



H.O.T. Problems

16. **OPEN ENDED** Give an example of data that could be represented using a histogram.
17. **REASONING** Which type(s) of display allows you to easily find the mode of the data? Explain your reasoning.
18. **REASONING** Which type(s) of displays addressed in this lesson do *not* show the individual data values?

CHALLENGE Refer to the line plot at the right that shows the number of text messages selected students received on Saturday.

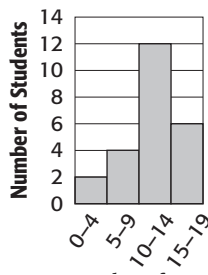


19. Display the data in another type of display.
20. Write a paragraph comparing and contrasting the advantages and disadvantages of each type of display.
21. **WRITING IN MATH** Write a paragraph explaining when it is best to use each of the following types of displays: bar graph, line graph, circle graph, line plot, and histogram.

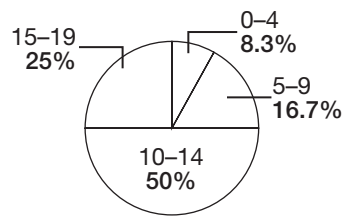
ISTEP+ PRACTICE 7.4.1

22. Guido polled 24 classmates to find out the average number of hours each spends online each week. Which of the following displays is appropriate for this situation AND shows the individual student responses?

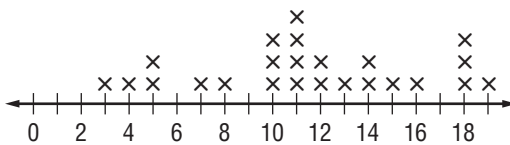
A Number of Hours Spent Online Each Week



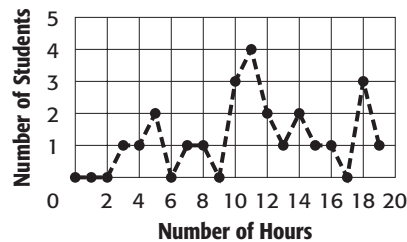
C Number of Hours Spent Online Each Week



B Number of Hours Spent Online Each Week



D Number of Hours Spent Online Each Week



Analyze Questions in Surveys

MAIN IDEA

Analyze ways in which the wording of questions can influence survey results.

IN Academic Standards

7.4.4 Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.

New Vocabulary

valid survey results

IN Math Online

glencoe.com

▶ GET READY for the Lesson

SURVEYS Mary Anne wanted to determine the favorite type of pet of the students in her science class. Mary Anne asked the students in her class to choose one of the pet types listed in the table as their favorite. Of the 24 students surveyed, 8 responded that their favorite pet is a dog. Six students preferred a cat, four preferred a reptile, and two preferred a rabbit.



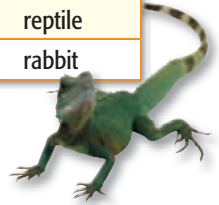
What is Your Favorite Type of Pet?

cat

dog

reptile

rabbit



1. How many students did not choose any of the pets listed in the table?
2. Mary Anne stated that those students who did not choose any of the pets listed in the table did not like pets. What is wrong with her statement?
3. How could Mary Anne have reworded her question to determine the pet preferences of everyone in the class?

The way that questions are worded in surveys can often influence the actual survey results. This can happen when a survey question includes information that describes how others feel about the question being asked.

EXAMPLES Analyze Wording of Survey Questions

Analyze each of the following survey questions. Describe how the wording of the question can influence the survey results.

- 1 Do you prefer thrilling action movies or boring documentaries?

Action movies are described as thrilling which could influence responses to prefer them over documentaries which are described as boring.

The question also limits the responses to only two choices. The question does not lend itself to responses from people who do not like either type of movie or who prefer another type of movie.

- 2 Most employees working at Sam's Supply Store love their jobs. How do you feel about working at Sam's Supply Store?

By stating that most employees love their jobs, the question does not encourage other responses. An employee who is not satisfied with their job may not be honest with their response.

Valid survey results occur when they are not influenced by other factors, such as the wording of the survey questions.

EXAMPLES Rewrite a Survey Question

- 3** Rewrite the survey question from Example 1 so that the survey results might be more valid.

Do you prefer thrilling action movies or boring documentaries?

The survey question “Do you like movies? If so, what kind of movies do you enjoy?” does not influence the survey results. People who do not enjoy movies can answer the question. People who do enjoy movies can also answer the question with their preferred type(s) of movie(s).

- 4** Rewrite the survey question from Example 2 so that the survey results might be more valid.

Most employees working at Sam’s Supply Store love their jobs. How do you feel about working at Sam’s Supply Store?

You could rewrite the survey question by simply deleting the first sentence. The question “How do you feel about working at Sam’s Supply Store?” does not influence the survey results. It is an open-ended question.

Another possible question could be “Do you enjoy working at Sam’s Supply Store?” This second question encourages responses of *yes* or *no*, which is more of a closed-ended question.

Either question is appropriate.

Study Tip

Open-Ended vs. Closed-Ended Questions Open-ended questions typically do not have one definite answer. Closed-ended questions have a certain number of answer choices. One of the choices may be “Other.”

CHECK Your Understanding

Examples 1, 2
(p. IN39)

Analyze each of the following survey questions. Describe how the wording of the question could influence the survey results.

1. Which is your favorite season: summer or fall?
2. Didn’t you think that the book was too long?
3. Most students prefer to text message than e-mail. How do you prefer to communicate?
4. Do you play a sport or work an after-school job?

Examples 3, 4
(p. IN40)

Rewrite each of the following survey questions so that the survey results might be more valid.

5. My favorite subject in school is French. What is your favorite subject?
6. Most of the students at Jefferson Middle School prefer pepperoni pizza. Do you like pepperoni pizza?
7. Would you rather travel to Italy or Germany?

Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
8–13	1, 2
14–19	3, 4

Analyze each of the following survey questions. Describe how the wording of the question could influence the survey results.

- Do you walk or ride your bicycle to school?
- Most Americans think that we should recycle more. Do you think that we should recycle more?
- Would you rather join the Debate Team or the Drama Club?
- Most students at Winslow High School plan to go to college. Do you?
- Would you rather go downhill skiing or cross-country skiing?
- Don't you think that cats make better pets than dogs?

Rewrite each of the following survey questions so that the survey results might be more valid.

- I like the color yellow. Don't you?
- Do you prefer basketball or football?
- Aren't roses the prettiest of flowers?
- Do you prefer roller coasters or water rides?
- Most teenagers like to listen to rock music. Do you like to listen to rock music?
- Would you rather go camping or swimming during your summer vacation?

H.O.T. Problems

- CHALLENGE** Write an open-ended survey question. Then write a closed-ended survey question about the same topic. Describe the similarities and differences in the kinds of responses you might get from each question.
- WRITING IN MATH** Write a survey question that is worded to specifically influence a survey's results. Then explain how your question might affect the survey results.

ISTEP+ PRACTICE 7.4.4

- Mei-Ling asked the survey question:
Do you prefer vanilla or chocolate ice cream? All of the following are possible reasons for why the survey results might not be valid EXCEPT for which one?
 - Some people may not like ice cream.
 - Some people might like vanilla and chocolate ice cream equally.
 - Some people might like strawberry ice cream the best.
 - Most people prefer chocolate ice cream.
- How could the following survey question be reworded so that the results would be more valid?
Didn't you think the new comedy movie was hilarious?
 - I laughed through the whole movie. Didn't you?
 - Did you think the new comedy movie was funny?
 - Wasn't the new comedy movie funnier than the last one?
 - I thought the new comedy movie was hilarious. Did you?



Parallel Lines and Transversals

MAIN IDEA

Analyze the relationships of angles formed by two parallel lines and a transversal.

IN Academic Standards

7.3.1 Identify and use basic properties of angles formed by transversals intersecting pairs of parallel lines.

New Vocabulary

- parallel lines
- transversal
- alternate interior angles
- alternate exterior angles
- corresponding angles

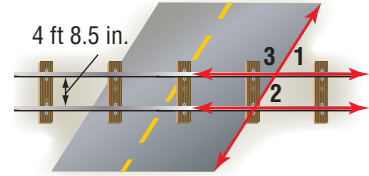
IN Math Online

glencoe.com

- Personal Tutor
- Self-Check Quiz
- Extra Examples

▶ GET READY for the Lesson

In the United States, the standard distance between rails on a railroad track is 4 feet 8.5 inches. The diagram at the right shows a road crossing over railroad tracks.



1. Measure angles 1 and 2. Record the measures.
2. Make a conjecture about the measure of angle 3. Then measure the angle to verify your conjecture.

Lines in a plane that never intersect are **parallel lines**. When two or more parallel lines are intersected by a third line, this line is called a **transversal**. Angles formed when two parallel lines are intersected by a transversal have special relationships. Those relationships are described below.

Special Relationships

Key Concept

If a pair of parallel lines is intersected by a transversal, the following pairs of angles are congruent.

Alternate interior angles are on opposite sides of the transversal and inside the parallel lines.

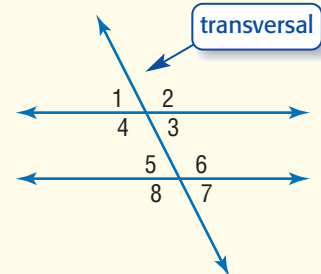
$$\angle 3 \cong \angle 5, \angle 4 \cong \angle 6$$

Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.

$$\angle 1 \cong \angle 7, \angle 2 \cong \angle 8$$

Corresponding angles are in the same position on the parallel lines in relation to the transversal.

$$\angle 1 \cong \angle 5, \angle 2 \cong \angle 6, \angle 3 \cong \angle 7, \angle 4 \cong \angle 8$$



You can use these special relationships to find measures of angles.

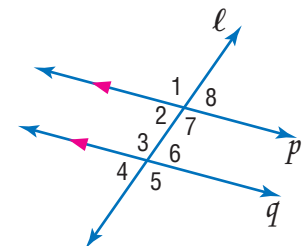
EXAMPLE Find Measures of Angles

In the figure at the right, $p \parallel q$ and $m\angle 3 = 95^\circ$. Find $m\angle 7$.

1 $m\angle 7$

$\angle 3$ and $\angle 7$ are alternate interior angles.

Alternate interior angles are congruent, so $m\angle 7 = m\angle 3 = 95^\circ$.

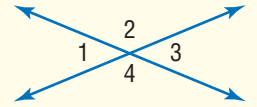


Vertical Angles and Adjacent Angles

Key Concept

Vertical angles are opposite angles formed by the intersection of two lines. Vertical angles are congruent.

$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$



Two angles that have the same vertex, share a common side, and do not overlap are *adjacent angles*. Adjacent angles are supplementary.

$\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 1$

Reading Math

Congruent Angles

- Angle 1 is congruent to angle 2: $\angle 1 \cong \angle 2$.
- The measure of $\angle 1$ is equal to the measure of $\angle 2$: $m\angle 1 = m\angle 2$.

EXAMPLES

Use Vertical Angles and Adjacent Angles

In the figure at the right, $m\angle 1 = 70^\circ$. Find each measure.

2 $m\angle 2$

$\angle 1$ and $\angle 2$ are vertical angles, so they are congruent.

$$m\angle 2 = m\angle 1 = 70^\circ$$

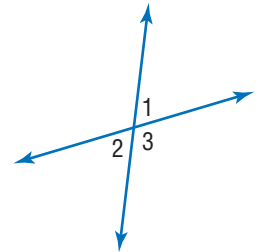
3 $m\angle 3$

$\angle 1$ and $\angle 3$ are adjacent angles, so they are supplementary.

$$m\angle 1 + m\angle 3 = 180 \quad \text{Definition of supplementary angles}$$

$$70 + m\angle 3 = 180 \quad \text{Replace } m\angle 1 \text{ with } 70.$$

$$m\angle 3 = 110^\circ \quad \text{Subtract } 70 \text{ from each side.}$$



EXAMPLES

Use Angle Relationships

ALGEBRA In the figure at the right, $r \parallel s$.

4 Find the value of x .

The angles with measures $2x^\circ$ and 110° are vertical angles, so they are congruent.

$$2x = 110 \quad \text{Congruent angles have equal measures.}$$

$$\frac{2x}{2} = \frac{110}{2} \quad \text{Divide each side by 2.}$$

$$x = 55 \quad \text{Simplify.}$$

5 Find the value of y .

The angles with measures $2x^\circ$ and $(y + 7)^\circ$ are alternate interior angles, so they are congruent.

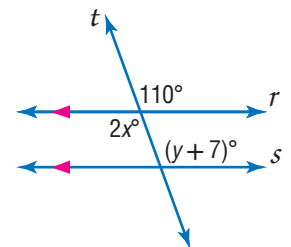
$$2x = y + 7 \quad \text{Congruent angles have equal measures.}$$

$$2(55) = y + 7 \quad \text{Replace } x \text{ with } 55.$$

$$110 = y + 7 \quad \text{Multiply.}$$

$$110 - 7 = y + 7 - 7 \quad \text{Subtract } 7 \text{ from each side.}$$

$$103 = y \quad \text{Simplify.}$$

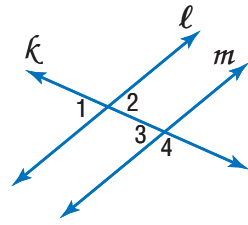


CHECK Your Understanding

Examples 1–3
(pp. IN42–IN43)

In the figure at the right, $\ell \parallel m$ and k is a transversal.
If $m\angle 1 = 56^\circ$, find each measure.

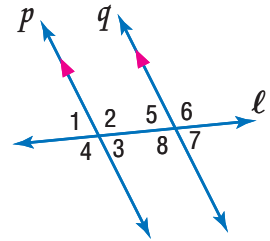
1. $m\angle 2$ 2. $m\angle 3$ 3. $m\angle 4$



4. **GEOMETRY** Refer to the figure above. Classify angles 1 and 3 as *alternate interior angles*, *alternate exterior angles*, or *corresponding angles*.

In the figure at the right, $p \parallel q$ and ℓ is a transversal.
If $m\angle 8 = 120^\circ$, find each measure.

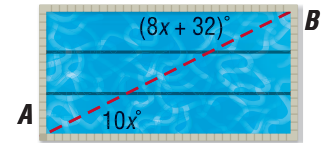
5. $m\angle 1$ 6. $m\angle 3$ 7. $m\angle 5$



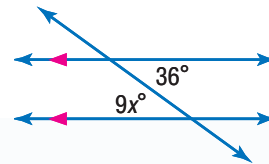
8. **GEOMETRY** Refer to the figure for Exercises 5–7. Classify angles 4 and 6 as *alternate interior angles*, *alternate exterior angles*, or *corresponding angles*.

Example 4
(p. IN43)

9. **SWIMMING** A swimmer crosses the lanes in a pool and swims from point A to point B, as shown in the figure. What is the value of x ?



10. **ALGEBRA** Find the value of x in the figure at the right.



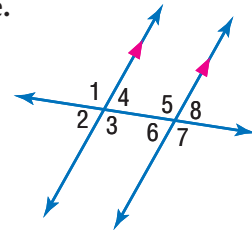
Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
11–18	1–3
19–26	4

In the figure at the right, if $m\angle 5 = 108^\circ$, find each measure.

11. $m\angle 1$ 12. $m\angle 3$
13. $m\angle 6$ 14. $m\angle 7$

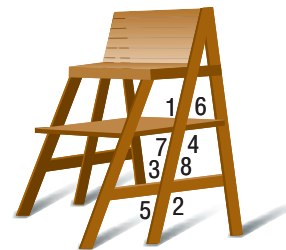


In the figure at the right, if $m\angle 2 = 74^\circ$, find each measure.

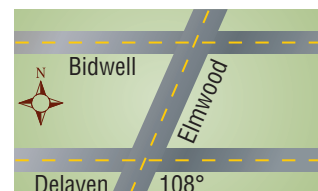
15. $m\angle 8$ 16. $m\angle 6$
17. $m\angle 4$ 18. $m\angle 1$

FURNITURE For Exercises 19–21, refer to the chair at the right where $m\angle 4 = 106^\circ$.

19. Find $m\angle 6$ and $m\angle 3$.
20. Find $m\angle 1$ and $m\angle 2$.
21. Find $m\angle 5$ and $m\angle 4$.



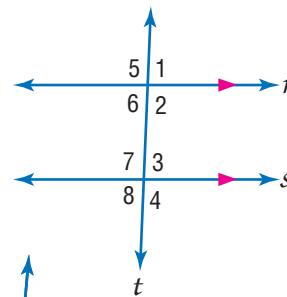
22. **DRIVING** Ambulances can't safely make turns of less than 70° . The angle at the southeast corner of Delavan and Elmwood is 108° . Can an ambulance safely turn the northeast corner of Bidwell and Elmwood? Explain your reasoning.



In the figure at the right, $m\angle 7 = 96^\circ$.
Find each measure.

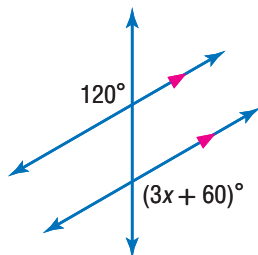
23. $m\angle 2$
25. $m\angle 4$

24. $m\angle 5$
26. $m\angle 8$

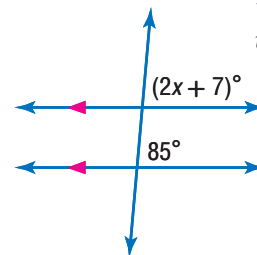


ALGEBRA Find the value of x in each figure.

27.

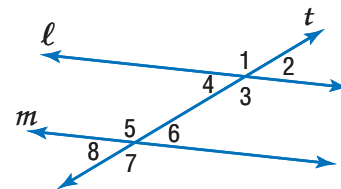


28.



ALGEBRA In the figure at the right, $m \parallel \ell$ and t is a transversal. Find the value of x for each of the following.

29. $m\angle 2 = 2x + 3$ and $m\angle 4 = 4x - 7$
30. $m\angle 8 = 4x - 32$ and $m\angle 5 = 5x + 50$



31. **FLAGS** The flag at the right is the national flag of Bosnia. If $m\angle 1 = 135^\circ$, what is $m\angle 2$? Explain how you found your answer.



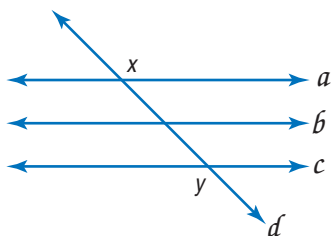
32. **ALGEBRA** A transversal intersects two parallel lines and forms adjacent angles 5 and 6. If $m\angle 5 = (7x - 11)^\circ$ and $m\angle 6 = (3x + 1)^\circ$, find the measures of the angles.

H.O.T. Problems

33. **CHALLENGE** Suppose two parallel lines are cut by a transversal. How are the interior angles on the same side of the transversal related?
34. **OPEN ENDED** Draw a pair of adjacent, supplementary angles. Label the angle measures.
35. **WRITING IN MATH** Summarize the angle relationships that are formed by parallel lines and a transversal. Describe which angles are congruent.

ISTEP+ PRACTICE 7.3.1

36. **CONSTRUCTED RESPONSE** In the figure below, $a \parallel b \parallel c$ and d is a transversal.



What is the value of angle y if $m\angle x$ is 135° ?

37. A transversal intersects two parallel lines and forms adjacent angles 1 and 2. If $m\angle 1$ is $(2x + 5)^\circ$ and $m\angle 2$ is $3x^\circ$, what is the measure of angle 1?
A 35°
B 75°
C 105°
D 135°



Real-World Link
The area of Bosnia is 19,741 square miles. Until 1992, this country was a part of Yugoslavia.
Source: Time for Kids Almanac

Rotations

MAIN IDEA

Identify and graph rotations on a coordinate plane.

IN Academic Standards

7.3.1 Identify, describe and use transformations (translations, rotations, reflections and simple compositions of these transformations) to solve problems.

New Vocabulary

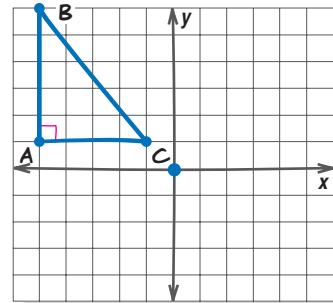
rotation
rotational symmetry
angle of rotation

IN Math Online

glencoe.com

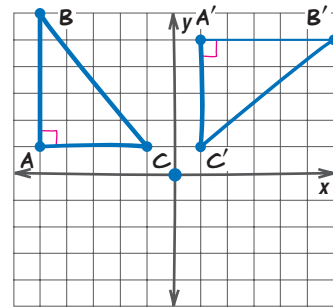
MINI Lab

STEP 1 Draw and label triangle ABC with vertices $A(-5, 1)$, $B(-5, 6)$, and $C(-1, 1)$.



STEP 2 Attach a piece of tracing paper to the coordinate plane with a fastener. Then trace the triangle and the x - and y -axis.

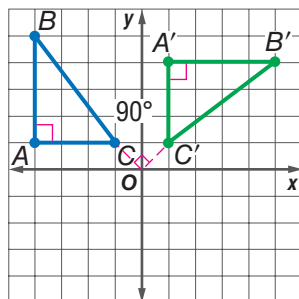
STEP 3 Turn the tracing paper clockwise so that the original y -axis is on top of the original x -axis.



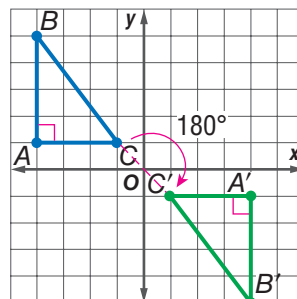
- Describe the transformation that occurred from triangle ABC to triangle $A'B'C'$.
- What are the coordinates of triangle $A'B'C'$?
- Measure the line segment connecting point C and the origin. Then measure the segment connecting point C' and the origin. What do you notice?
- Measure the angle formed by the segments in Exercise 3. What is this angle measure?

A **rotation** occurs when a figure is rotated around a point, such as the origin. A rotation does not change the size or shape of the figure. The three different rotations shown below are clockwise around the origin.

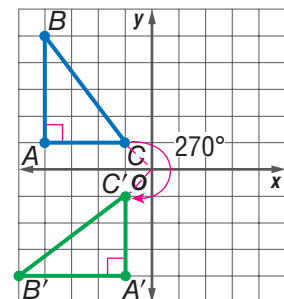
90° Rotation



180° Rotation



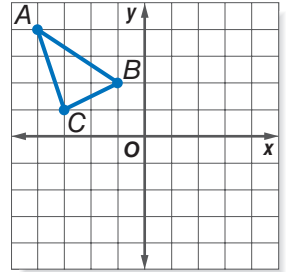
270° Rotation



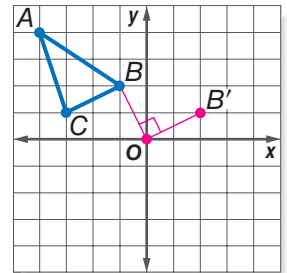
EXAMPLE**Rotate a Figure About the Origin**

- 1 Triangle ABC has vertices $A(-4, 4)$, $B(-1, 2)$, and $C(-3, 1)$. Graph the figure and its rotated image after a clockwise rotation of 90° about the origin. Then give the coordinates of the vertices for $\triangle A'B'C'$.

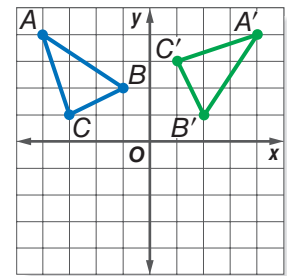
STEP 1 Graph $\triangle ABC$ on a coordinate plane.



STEP 2 Sketch segment \overline{BO} connecting point B to the origin. Sketch another segment, $\overline{B'O}$ so that the angle between point B , O , and B' measures 90° and the segment is congruent to \overline{BO} .



STEP 3 Repeat Step 2 for points A and C . Then connect the vertices to form $\triangle A'B'C'$.



So, the coordinates of the vertices of $\triangle A'B'C'$ are $A'(4, 4)$, $B'(2, 1)$, and $C'(1, 3)$.

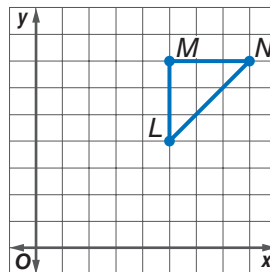
Reading Math

Segment Notation \overline{BO} means the line segment connecting points B and O .

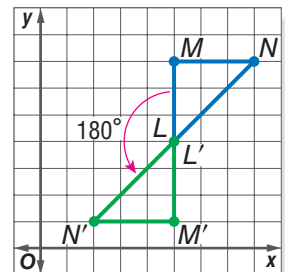
EXAMPLE**Rotate a Figure About a Point**

- 2 Triangle LMN has vertices $L(5, 4)$, $M(5, 7)$, and $N(8, 7)$. Graph the figure and its rotated image after a counterclockwise rotation of 180° about vertex L . Then give the coordinates of the vertices for $\triangle L'M'N'$.

STEP 1 Graph the original triangle.



STEP 2 Graph the rotated image.



So, the coordinates of the vertices of $\triangle L'M'N'$ are $L'(5, 4)$, $M'(5, 1)$, and $N'(2, 1)$.

A figure can have **rotational symmetry** if the figure can be rotated a certain number of degrees about its center and still look like the original. The **angle of rotation** is the degree measure of the angle through which the figure is rotated.

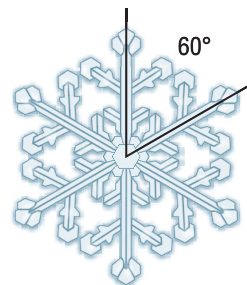


Real-World EXAMPLE

Determine Rotational Symmetry

3 SNOW Determine whether the snowflake has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.

Since the snowflake can be rotated and still look like it does in its original position, the snowflake has rotational symmetry. The snowflake will match itself after being rotated 60° , 120° , 180° , 240° , 300° , and 360° .

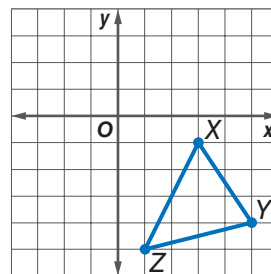


CHECK Your Understanding

Examples 1–2
(p. IN46)

Graph $\triangle XYZ$ and its rotated image after each rotation. Then give the coordinates of the vertices for $\triangle X'Y'Z'$.

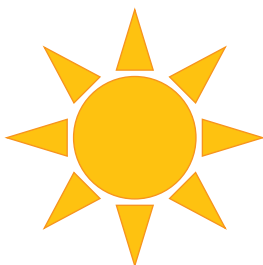
- 180° clockwise about the origin
- 270° counterclockwise about vertex X
- 90° counterclockwise about the origin
- 270° clockwise about vertex Y
- 180° counterclockwise about vertex Z
- 90° clockwise about the origin



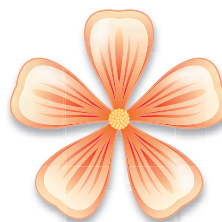
Example 3
(p. IN48)

Determine whether each figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.

7.



8.

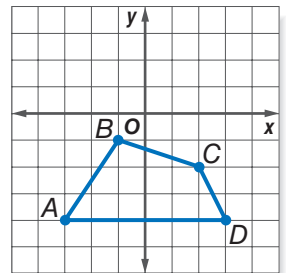


Practice and Problem Solving

HOMESCHOOL HELP

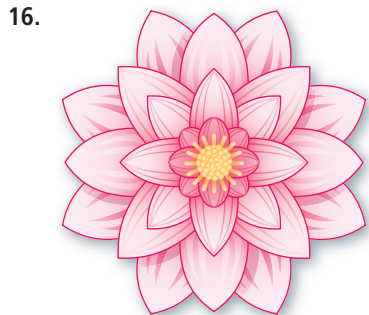
For Exercises	See Examples
9–14	1–2
15–18	3

Graph quadrilateral $ABCD$ and its rotated image after each rotation. Then give the coordinates of the vertices for quadrilateral $A'B'C'D'$.



9. 90° counterclockwise about the origin
10. 90° clockwise about vertex A
11. 180° counterclockwise about vertex D
12. 270° clockwise about the origin
13. 90° clockwise about the origin
14. 180° clockwise about vertex B

Determine whether each figure has rotational symmetry. Write *yes* or *no*. If *yes*, name its angle(s) of rotation.

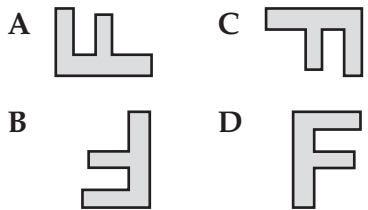


H.O.T. Problems

19. **OPEN ENDED** Sketch a figure that has rotational symmetry. Describe the angle(s) of rotation.
20. **WRITING IN MATH** Describe what information is needed to rotate a figure.

ISTEP+ PRACTICE 7.3.2

21. Which figure shows the letter F after a rotation of 270° clockwise?



22. Triangle XYZ has vertices $X(2, -2)$, $Y(5, 0)$, and $Z(3, -4)$. What are the coordinates of point Y' after a rotation of 180° ?

- F $(0, -5)$ H $(0, 5)$
 G $(-5, 0)$ J $(5, 0)$

Nets of Cylinders and Cones

MAIN IDEA

Draw nets for cylinders and cones.

IN Academic Standards

7.3.2 Draw two-dimensional patterns (nets) for three-dimensional objects, such as right prisms, pyramids, cylinders and cones.

New Vocabulary

central angle
slant height

IN Math Online

glencoe.com

▶ GET READY for the Lesson

Refer to the figures at the right.

1. How many bases does a cylinder have?
2. Describe the shape of the base(s) of a cylinder.
3. How many bases does a cone have?
4. Describe the shape of the base(s) of a cone.



cylinder



cone

In Extend 11-6, you drew nets for three-dimensional figures such as prisms and pyramids. Recall that a net is a two-dimensional figure that helps you see the faces that make up the surface of a figure. In this activity, you will draw nets for cylinders and cones.

You can use a cylindrical container with a lid to help you draw the net of a cylinder.

EXAMPLE Draw a Net of a Cylinder

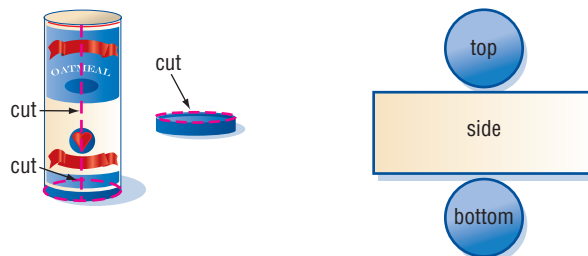
- 1 Draw a net of a cylinder using a cylindrical container with a lid.

STEP 1 Use an empty cylinder-shaped container that has a lid. Measure and record the height of the container.

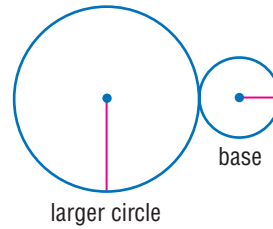
STEP 2 Then label the lid and bottom face using a blue marker. Label the curved side using a red marker.



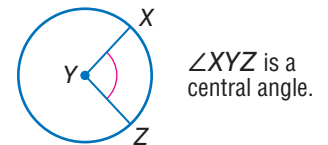
STEP 3 Take off the lid of the container and make 2 cuts as shown. Next, cut off the sides of the lid. Finally, lay the lid, the curved side, and the bottom flat to form the net of the container.



To draw a net of a cone, you will need a compass. The base of a cone is a circle. The lateral surface of a cone is part of a larger circle. So that the edges match, the circumference of the base is equal to part of the circumference of the larger circle.



To draw the lateral surface of the cone from its partial circumference, you need to know the measure of its central angle. A **central angle** of a circle is an angle whose vertex is the center of the circle.

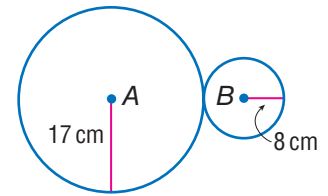


The activity below shows you how to draw the net of a cone given its radius and slant height. The **slant height** of a cone is the height of the cone's lateral surface. The slant height is also the radius of the larger circle.

EXAMPLE Draw a Net of a Cone

2 Draw a net of a cone with a radius of 8 centimeters and a slant height of 17 centimeters.

STEP 1 Use a compass to draw two circles slightly touching, one with a radius of 17 centimeters and one with a radius of 8 centimeters.



STEP 2 Think: What is the ratio of the circumference of B to the circumference of A? Let x represent this ratio.

$$x = \frac{16\pi}{34\pi} \quad \begin{array}{l} \text{The circumference of B is } 16\pi. \\ \text{The circumference of A is } 34\pi. \end{array}$$

$$x = \frac{16\cancel{\pi}}{34\cancel{\pi}} \quad \text{Cancel the common factor } \pi.$$

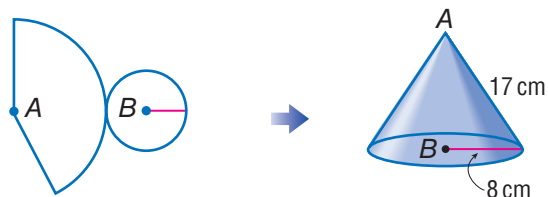
$$x \approx 0.47 \quad \text{Simplify.}$$

You need 0.47 of the circumference of A.

STEP 3 Find the size of the central angle to be cut from A.

$$0.47 \cdot 360^\circ \approx 170^\circ$$

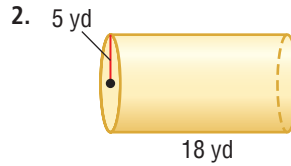
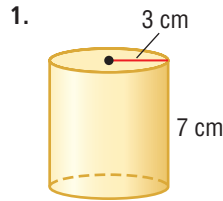
Cut a central angle of 170° from circle A.



CHECK Your Understanding

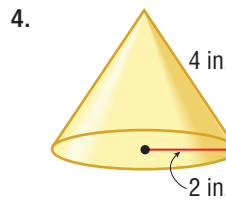
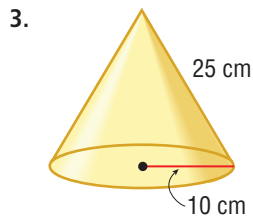
Example 1
(p. IN50)

Draw the net for each cylinder. Label the dimensions on the net.



Example 2
(p. IN51)

Find the central angle of each cone. Then draw the net of the cone. Label the dimensions on the net.

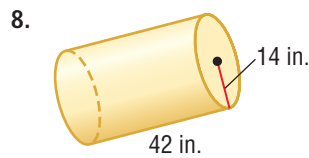
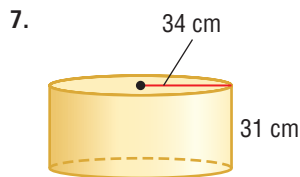
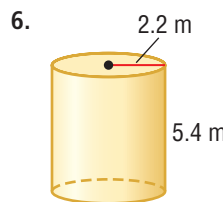
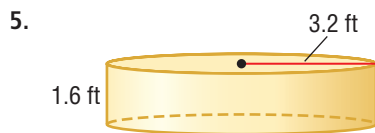


Practice and Problem Solving

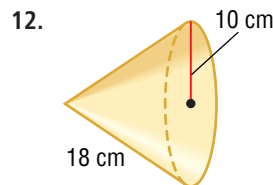
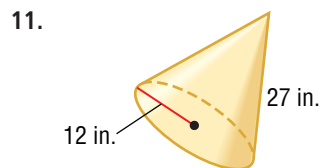
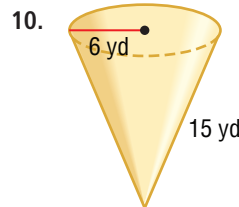
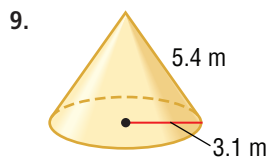
HOMEWORK HELP

For Exercises	See Examples
5–8	1
9–12	2

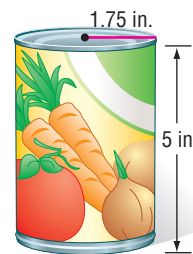
Draw the net for each cylinder. Label the dimensions on the net.



Find the central angle of each cone. Then draw the net of the cone. Label the dimensions on the net.

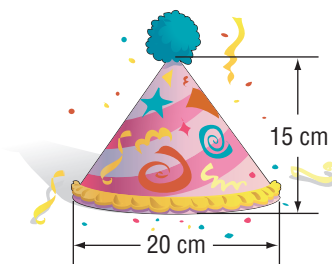


CONTAINERS For Exercises 13–15, refer to the cylindrical can of vegetables shown at the right.



13. Draw the net of this cylinder. Label the dimensions on the net.
14. Find the area of each circular base. Then find the area of the lateral surface. Use 3.14 for π . Round to the nearest hundredth, if necessary.
15. What is the total area, including both circular bases and the lateral surface, of the cylinder?

PARTY HATS For Exercises 16–18, refer to the conical party hat shown at the right.



16. Draw the net of this cone. Label the dimensions on the net.
17. What is the central angle of the cone?
18. Find the area of the lateral surface by setting up and solving the following proportion, where r is the slant height, y is the central angle, and x is the area of the lateral surface of the cone. Use 3.14 for π . Round to the nearest hundredth, if necessary.

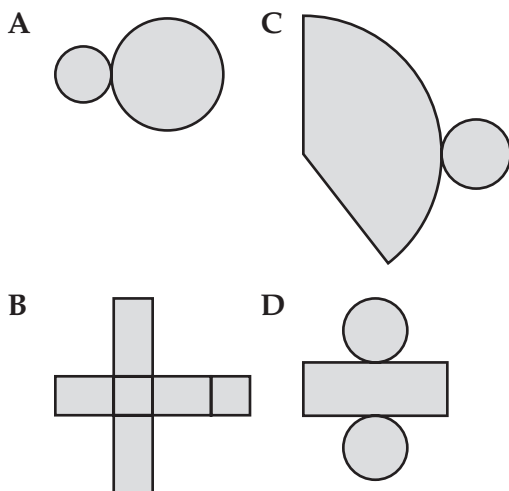
$$\frac{\text{area of the lateral surface } (x)}{\text{area of whole circle } (\pi r^2)} = \frac{\text{central angle of cone } (y)}{\text{number of degrees in whole circle } (360)}$$

H.O.T. Problems

19. **CHALLENGE** The radius of a cone is tripled. The height of the cone remains unchanged. Describe how the net of the cone is affected.
20. **WRITING IN MATH** Describe the similarities and differences between the net of a rectangular prism and the net of a cylinder.

ISTEP+ PRACTICE 7.3.3

21. Which of the following is the correct net of a cone?



22. Which of the following BEST describes the two-dimensional figures that make up the net of a cylinder with radius r and height h ?

- F a square with a side length of r units
- G one rectangle with a width of h units and a length of $r\pi$ units
- H two congruent circles each with a radius of r units
- J one rectangle with a width of h units and a length of $2r\pi$ units and two congruent circles each with a radius of r units

MAIN IDEA

Identify, compare, and order irrational numbers. Evaluate expressions with irrational numbers.

IN Academic Standards

7.1.6 Identify, write, rename, compare and order rational and common irrational numbers and plot them on a number line.

7.2.3 Evaluate numerical expressions and simplify algebraic expressions involving rational and irrational numbers.

IN Math Online

glencoe.com

▶ GET READY for the Lesson

NUMBER SENSE Consider the two sets of numbers in the table at the right.

Set A	Set B
5.87	$\sqrt{2}$
$\frac{3}{4}$	π
22%	4.58369...
-1	$\sqrt{5}$
$0.\bar{4}$	1.39142...

- How does the decimal 5.87 in Set A compare with the decimal 1.39142... in Set B?
- How does the decimal $0.\bar{4}$ in Set A compare with the decimal 1.39142... in Set B?
- Can any of the numbers in Set A be written as fractions? Justify your response.
- Can any of the numbers in Set B be written as fractions? Justify your response.

Recall from Lesson 4-9 that a *rational number* is a number that can be expressed as a fraction. Fractions, terminating and repeating decimals, percents, and integers are all rational numbers. The numbers in Set A above are all examples of rational numbers.

Recall from Lesson 12-2 that an *irrational number* cannot be expressed as the quotient of two integers. A fraction is the quotient of two integers. In other words, an irrational number cannot be expressed as a fraction where the numerator and denominator are both integers. The numbers in Set B above are all examples of irrational numbers.

EXAMPLES Identify Numbers

State whether each number is *rational* or *irrational*. Justify your response.

- $\frac{7}{9}$ All fractions are rational numbers. So, $\frac{7}{9}$ is rational.
- $\sqrt{3}$ This number cannot be written as the quotient of two integers. So, $\sqrt{3}$ is irrational.
- 0.787787778... This number is neither a terminating nor a repeating decimal. It cannot be written as the quotient of two integers. So, 0.787787778... is irrational.

You can compare and order rational and irrational numbers by plotting the numbers on a number line or by writing each number as a decimal.

Study Tip

Irrational Numbers Even though $\sqrt{15}$ is not a terminating decimal, you can still approximate its graph on a number line.

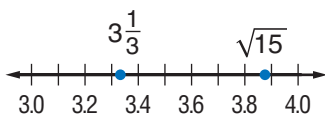
EXAMPLES Compare and Order Numbers

- 4** Replace \bullet with $<$, $>$, or $=$ to make $3\frac{1}{3} \bullet \sqrt{15}$ a true sentence.

Express each number as a decimal. Then compare the decimals.

$$3\frac{1}{3} = 3.\bar{3}, \text{ or } 3.3333333\dots$$

$$\sqrt{15} \approx 3.872983346$$



Since $3.\bar{3} < 3.872983346\dots$, $3\frac{1}{3} < \sqrt{15}$.

- 5** Order $8\frac{4}{5}$, $\sqrt{64}$, $8.\bar{3}$, $\sqrt{76}$ from least to greatest.

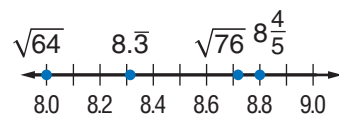
Express each number as a decimal. Then order the decimals.

$$8\frac{4}{5} = 8.8$$

$$\sqrt{64} = 8$$

$$8.\bar{3} = 8.333333333\dots$$

$$\sqrt{76} \approx 8.717797887$$



From least to greatest, the order is $\sqrt{64}$, $8.\bar{3}$, $\sqrt{76}$, and $8\frac{4}{5}$.

You can evaluate expressions involving rational and irrational numbers.

EXAMPLE Evaluate Expressions

- 6** Evaluate the expression $4y + 3\sqrt{x}$ if $x = 20$ and $y = 0.\bar{5}$. Round any irrational numbers to the nearest hundredth.

$$4y + 3\sqrt{x}$$

Write the expression.

$$= 4(0.\bar{5}) + 3\sqrt{20}$$

Replace x with 20 and y with $0.\bar{5}$.

$$= 2.22 + 3\sqrt{20}$$

Multiply. Round to the nearest hundredth.

$$\approx 2.22 + 3(4.47)$$

The square root of 20 is about 4.47.

$$\approx 2.22 + 13.41$$

Multiply.

$$\approx 15.63$$

Add.

You can simplify algebraic expressions involving rational and irrational numbers. Recall the Distributive Property from Lesson 1-8.

EXAMPLE Simplify Expressions

7 Simplify the algebraic expression $5\left(\frac{2}{3}a + \sqrt{b} - \pi\right)$.

$$5\left(\frac{2}{3}a + \sqrt{b} - \pi\right) \quad \text{Write the expression.}$$

$$= 5\left(\frac{2}{3}a\right) + 5(\sqrt{b}) - 5(\pi) \quad \text{Distributive Property}$$

$$\approx \frac{10}{3}a + 5\sqrt{b} - 15.7 \quad \text{Multiply. Round } \pi \text{ to } 3.14.$$

$$\approx 3\frac{1}{3}a + 5\sqrt{b} - 15.7 \quad \text{Simplify.}$$

$$\text{So, } 5\left(\frac{2}{3}a + \sqrt{b} - \pi\right) \approx 3\frac{1}{3}a + 5\sqrt{b} - 15.7.$$

Study Tip

Multiplying a Whole Number by a Square Root The product of 5 and \sqrt{b} can be written as $5 \times \sqrt{b}$, $5 \cdot \sqrt{b}$, or $5\sqrt{b}$.

CHECK Your Understanding

Examples 1–3 State whether each number is *rational* or *irrational*. Justify your response.
(p. IN49)

1. -39.2

2. $\sqrt{14}$

3. $-\frac{4}{15}$

4. $\sqrt{81}$

Example 4 Replace each \bullet with $<$, $>$, or $=$ to make a true statement. Use a number line if necessary.
(p. IN55)

5. $\pi \bullet 3\frac{1}{3}$

6. $1\frac{1}{4} \bullet \sqrt{2}$

7. $\sqrt{40} \bullet 6.25$

8. $7\frac{3}{8} \bullet 7.375$

Example 5 Order each of the following from least to greatest. Use a number line if necessary.
(p. IN55)

9. $\frac{1}{7}, \sqrt{7}, -1.7, \frac{7}{10}$

10. $0.\bar{5}, \sqrt{5}, \frac{2}{3}, \frac{3}{5}$

Example 6 Evaluate each expression if $a = 4$ and $b = \pi$. Round any irrational numbers to the nearest hundredth.
(p. IN55)

11. $-6b + \sqrt{a}$

12. $b\sqrt{7} - 2a$

Example 7 Simplify each algebraic expression.
(p. IN56)

13. $10(r + \sqrt{s} + t)$

14. $j(k + \sqrt{7})$

Practice and Problem Solving

HOMEWORK HELP

For Exercises

See Examples

15–26

1–3

27–38

4, 5

39–46

6, 7

State whether each number is *rational* or *irrational*. Justify your response.

15. $\sqrt{9}$

16. -1.05

17. $\sqrt{41}$

18. 42.875

19. $\sqrt{27}$

20. $\sqrt{25}$

21. $23.44444\dots$

22. $3.910742382\dots$

23. $6.\bar{7}1$

24. $\sqrt{30}$

25. $\sqrt{144}$

26. π

Replace each \bullet with $<$, $>$, or $=$ to make a true statement. Use a number line if necessary.

27. $6\frac{1}{6} \bullet 6.16$ 28. $4\frac{1}{4} \bullet \sqrt{15}$ 29. $\sqrt{121} \bullet 11$ 30. $5\frac{2}{9} \bullet 5.22$
 31. $\sqrt{48} \bullet 7\frac{2}{3}$ 32. $\pi \bullet \sqrt{\pi}$ 33. $3\frac{3}{4} \bullet \sqrt{13}$ 34. $\sqrt{400} \bullet 20$

Order each of the following from least to greatest. Use a number line if necessary.

35. $\frac{1}{10}, \sqrt{10}, 0.15, \frac{1}{15}$ 36. $0.\bar{4}, \sqrt{4}, \frac{3}{4}, \frac{1}{4}$
 37. $\frac{1}{6}, \sqrt{6}, 5.6, \frac{5}{6}$ 38. $0.\bar{9}, \sqrt{8}, \frac{4}{5}, \frac{8}{9}$

Evaluate each expression if $g = \sqrt{2}$ and $h = -1.5$. Round any irrational numbers to the nearest hundredth.

39. $7h - g$ 40. $2g + 3h$ 41. $\sqrt{3} \times h + g$
 42. $g \times h$ 43. $5g - h$ 44. g^2

Simplify each algebraic expression.

45. $\sqrt{2}(w + y - z)$ 46. $\frac{1}{4}\left(\frac{3}{5}m + \sqrt{n}\right)$ 47. $-7(c + \sqrt{d} + 1)$
 48. $\pi(p + q)$ 49. $\sqrt{x}(\sqrt{x} + 1)$ 50. $\frac{2}{3}(\sqrt{a} + b)$

H.O.T. Problems

51. **OPEN ENDED** Find a rational number and an irrational number that are each between 4 and 5. Include the decimal approximation of the irrational number to the nearest hundredth.
52. **REASONING** The area of a square is 40 square meters. Is the length of a side of the square a rational or irrational number? Explain.

$A = 40 \text{ m}^2$

CHALLENGE Tell whether each of the following is a rational or irrational number. Justify your response.

53. $5 \times \pi$ 54. $\sqrt{8} + \sqrt{8}$ 55. $\sqrt{50} \times \sqrt{50}$ 56. $\sqrt{6} \div \sqrt{6}$

57. **WRITING IN MATH** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.

All square roots are irrational numbers.

ISTEP+ PRACTICE 7.1.6, 7.2.3

58. Which of the following expressions is the GREATEST: $\sqrt{35}$, $5\frac{4}{5}$, 5.92, or $\sqrt{36}$?
 A $\sqrt{36}$
 B $\sqrt{35}$
 C 5.92
 D $5\frac{4}{5}$
59. Which of the following expressions is equivalent to $y\sqrt{x}$ if $x = 49$ and $y = \pi$?
 F $\sqrt{7}\pi$
 G 7π
 H 14π
 J 49π