$\square$ Indiana Academic Standards
7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions. Preparation for 8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.
$\square$ Key Vocabulary probability (p. 460) sample space (p. 465)
Real-World Link
Sports Before a football game, there is a coin toss to determine which team gets the ball first. Probability tells you that each team has a $50 \%$ chance of winning the coin toss.

Probability


## GET READY for Chapter 9

Diagnose Readfness You have two options for checking Prerequisite Skills.

## Option 2

## Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

## Quiz

Multiply. (Prior Grade)

1. $7 \times 15$
2. $24 \times 6$
3. $13 \times 4$
4. $8 \times 21$
5. $5 \times 32$
6. $30 \times 8$
7. $7 \times 6 \times 5$
8. $8 \times 7 \times 6$
9. $6 \times 5 \times 4 \times 3$
10. $4 \times 3 \times 2 \times 1$
11. $10 \times 9 \times 8 \times 7$
12. $11 \times 10 \times 9$
13. JOBS If you earn $\$ 9$ an hour and work 5 hours each day for 7 days, how much have you earned?
(Prior Grade)

Write each fraction in simplest form.
Write simplified if the fraction is
already in simplest form. (Lesson 4-4)
14. $\frac{8}{12}$
15. $\frac{3}{18}$
16. $\frac{4}{9}$
17. $\frac{5}{15}$
18. SLEEP If the average adult gets 8 hours of sleep, what fraction of the day, in simplest form, is spent asleep? (Lesson 4-4)

## Review

Example 1 Multiply $7 \times 6 \times 5 \times 4$.

$$
\begin{array}{rlrl}
7 \times 6 \times 5 \times 4 & =42 \times 5 \times 4 & & \text { Multiply from } \\
& =210 \times 4 & \text { left to right. } \\
& =840 &
\end{array}
$$

Example 2 Write $\frac{21}{28}$ in simplest form.


Divide the numerator and
denominator by the GCF, 7 .

Example 3 Find the value of $\frac{6 \times 5 \times 4}{3 \times 2 \times 1}$.

$$
\begin{aligned}
& \frac{6 \times 5 \times 4}{3 \times 2 \times 1}=\frac{120}{6} \longleftarrow \text { Multiply the numerator. } \\
&=20 \text { Multiply the denominator. } \\
& \text { Simplify. }
\end{aligned}
$$

## 9-1 Simple Events

## MAIN IDEA

Find the probability of a simple event.

## IN Academic Standards

### 7.4.5 Understand that when all outcomes of

 an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.Preparation for
8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.

## New Vocabulary

outcome
simple event
probability
random
complementary event

## IN Math Online

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- Concepts In Motion
- Extra Examples
- Personal Tutor
- Self-Check Quiz


## $\triangle$ GET READY for the Lesson

FOOD A cheesecake has four slices of each type as shown.

1. What fraction of the cheesecake is chocolate? Write in simplest form.
2. Suppose your friend gives you the first piece of cheesecake without asking which type you prefer. Are your chances of getting original the same as getting raspberry?


An outcome is any one of the possible results of an action. A simple event is one outcome or a collection of outcomes. For example, getting a piece of chocolate cheesecake is a simple event. The chance of that event happening is called its probability.

## Probability

Words If all outcomes are equally likely, the probability of a simple event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.
Symbols $\quad P$ (event) $=\frac{\text { number of favorable outcomes }}{\text { number of possible outcomes }}$

## EXAMPLE Find Probability

1] What is the probability of rolling an even number on a number cube marked with $1,2,3,4,5$, and 6 on its faces?

$$
\begin{aligned}
P(\text { even number }) & =\frac{\text { even numbers possible }}{\text { total numbers possible }} \\
& =\frac{3}{6} \text { or } \frac{1}{2}
\end{aligned}
$$



The probability of rolling an even number is $\frac{1}{2}, 0.5$, or $50 \%$.

## CHECK Your Progress

Use the number cube above to find each probability. Write as a fraction in simplest form.
a. $P$ (odd number)
b. $P(5$ or 6$)$
c. $P$ (prime number)

Outcomes occur at random if each outcome occurs by chance. For example, rolling a number on a number cube occurs at random.


Real World Link
Founded in 1842, the New York Philharmonic is the oldest symphony orchestra in the United States. They have performed over 14,000 concerts.
Source: New York Philharmonic

## Real-WOTA EXAMPLE

(2) TALENT COMPETITION Simone and her three friends were deciding how to pick the song they will sing for their school's talent show. They decide to roll a number cube. The person with the lowest number chooses the song. If her friends rolled a 6,5 , and 2 , what is the probability that Simone will get to choose the song?
The possible outcomes of rolling a number cube are $1,2,3,4,5$, and 6 . In order for Simone to be able to choose the song, she will need to roll a 1.

Let $P(A)$ be the probability that Simone chooses the song.
$P(A)=\frac{\text { number of favorable outcomes }}{\text { number of possible outcomes }}$
$=\frac{1}{6} \quad$ There are 6 possible outcomes, and 1 of them is favorable.
The probability that Simone will choose the song is $\frac{1}{6}$, or about $17 \%$.

## CHECK Your Progress

MUSIC The table shows the numbers of brass instrument players in the New York Philharmonic. Suppose one brass instrument player is randomly selected to be a featured performer. Find the probability of each event. Write

| New York Philharmonic <br> Brass Instrument Players |  |
| :--- | :---: |
| Horn | 6 |
| Trombone | 4 |
| Trumpet | 3 |
| Tuba | 1 |

Source: New York Philharmonic as a fraction in simplest form.
d. $P$ (trumpet)
e. $P$ (brass)
f. $P$ (flute)
g. $P$ (horn or tuba)

The probability that an event will happen can be any number from 0 to 1 , including 0 and 1 , as shown on the number line below. Notice that probabilities can be written as fractions, decimals, or percents.

| Impossible | As likely to <br> happen as not |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Unlikely |  | Likely | Certain |
| 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
| 0 | 0.25 | 0.5 | 0.75 | 1 |
| $0 \%$ | $25 \%$ | $50 \%$ | $75 \%$ | $100 \%$ |

Either Simone will go first or she will not go first. These two events are complementary events. The sum of the probabilities of an event and its complement is 1 or $100 \%$. In symbols, $P(A)+P(\operatorname{not} A)=1$.

## Study Tip

Complement of an Event The probability that event $A$ will not occur is noted as $P($ not $A)$. Since $P(A)+$ $P($ not $A)=1, P($ not $A)$ $=1-P(A) . P($ not $A)$ is read as the probability of the complement of $A$.

## EXAMPLE Complementary Events

(3) TALENT COMPETITION Refer to Example 2. Find the probability that Simone will not choose the song.
The probability that Simone will not choose the song is the complement of the probability that Simone will choose the song.

$$
\begin{array}{rlrl}
P(A)+P(\operatorname{not} A) & =1 & & \text { Definition of complementary events } \\
\frac{1}{6}+P(\operatorname{not} A) & =1 & & \text { Replace } P(A) \text { with } \frac{1}{6} . \\
-\frac{1}{6} & -\frac{1}{6} & & \text { Subtract } \frac{1}{6} \text { from each side. } \\
\hline P(\operatorname{not} A) & =\frac{5}{6} & 1-\frac{1}{6} \text { is } \frac{6}{6}-\frac{1}{6} \text { or } \frac{5}{6}
\end{array}
$$

The probability that Simone will not choose the song is $\frac{5}{6}$, or about $83 \%$.

## CHECK Your Progress

SCH00L Ramón's teacher uses a spinner similar to the one shown at the right to determine the order in which each group will make their presentation. Use the spinner to find each probability. Write as a fraction in simplest form.

h. $P$ (not group 4)
i. $P($ not group 1 or group 3$)$

## Your Understanding

Example 1 Use the spinner to find each probability. Write as a (p. 460) fraction in simplest form.

1. $P(M)$
2. $P(Q$ or $R)$
3. $P$ (vowel)

Examples 2, 3 MARBLES Robert has a bag that contains 7 blue, (pp. 461-462) 5 purple, 12 red, and 6 orange marbles. Find
 each probability if he draws one marble at random from the bag. Write as a fraction in simplest form.
4. $P$ (purple)
5. $P$ (red or orange)
6. $P$ (green)
7. $P$ (not blue)
8. $P$ (not red or orange)
9. $P$ (not yellow)

Example 3 (p. 462)
10. SURVEYS Shanté asked her classmates how many pets they own. The responses are in the table. If a student in her class is selected at random, what is the probability that the student does not own 3 or more pets?

| Number <br> of Pets | Response |
| :---: | :---: |
| None | 6 |
| $1-2$ | 15 |
| 3 or more | 4 |

## Practice and Problem Solving

| HOMEWORK HELP |  |
| :---: | :---: |
| For <br> Exercises | See <br> Examples |
| $11-14$ | 1 |
| $17-22$ | 2 |
| $15-16$, | 3 |
| $23-26$ |  |

A set of 20 cards is numbered $1,2,3, \ldots, 20$. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.
11. $P(1)$
12. $P(3$ or 13$)$
14. $P$ (even number)
13. $P$ (multiple of 3 )
16. $P$ (not a factor of 10 )

RAFFLE The table shows those students in seventh grade who entered in the school drawing to win lunch with the principal. Suppose that only one student is randomly selected to win. Find the probability of each event. Write as a fraction in simplest form.

| Lunch Raffle |  |
| :--- | :---: |
| girls | 25 |
| boys | 15 |
| Room 10 | 10 |
| Room 11 | 16 |
| Room 12 | 14 |

17. $P$ (girl)
18. $P$ (boy)
19. $P$ (Room 12)
20. $P$ (girl or boy)
21. $P$ (not Room 10)
22. SOUP A cupboard contains 20 soup cans. Seven are tomato, 4 are cream of mushroom, 5 are chicken, and 4 are vegetable. If one can is chosen at random from the cupboard, what is the probability that it is neither cream of mushroom nor vegetable soup? Write as a percent.
23. VIDEOS In a drawing, one name is randomly chosen from a jar of 75 names to receive free video rentals for a month. If Enola entered her name 8 times, what is the probability that she is not chosen to receive the free rentals? Write as a fraction in simplest form.
24. PETS The graph shows the last 33 types of pets that were purchased at a local pet store. Based on this, what is the probability that the next pet purchased will be a dog? Write as a fraction in simplest form.
25. GAMES For a certain game, the probability of choosing a card with the number 13 is 0.008 . What is the probability of not choosing card 13 ?

26. WEATHER The forecast for tomorrow says that there is a $37 \%$ chance of rain. Describe the complementary event and predict its probability.
27. ANNOUNCEMENTS There are 90 students in the seventh grade. Fifty-two of those students are girls. If one student will be chosen at random to read the morning announcements for the week, what is the probability that the student is a boy?
28. REASONING A leap year has 366 days and occurs in non-century years that are evenly divisible by 4 . The extra day is added as February 29th.
Determine whether each probability is 0 or 1 . Explain your reasoning.
a. $P$ (there will be 29 days in February in 2032)
b. $P$ (there will be 29 days in February in 2058)
29. CHALLENGE A bag contains 6 red marbles, 4 blue marbles, and 8 green marbles. How many marbles of each color should be added so that the total number of marbles is 27 , but the probability of randomly selecting one marble of each color remains unchanged? Explain your reasoning.
30. Which One Doesn't Belong? Identify the pair of probabilities that does not represent probabilities of complementary events. Explain your reasoning.

$$
\begin{array}{l|l|}
\frac{3}{5}, \frac{2}{5} & 0.625, \frac{3}{8}
\end{array} \frac{6}{8}, \frac{1}{4} \quad 0.33,0.44
$$

34. WRITINGIN MATH Marissa has 5 black T-shirts, 2 purple T-shirts, and 1 orange T-shirt. Without calculating, determine whether each of the following probabilities is reasonable if she randomly selects one T-shirt. Explain your reasoning.
a. $\quad P($ black T-shirt $)=\frac{1}{3}$
b. $\quad P($ orange T -shirt $)=\frac{4}{5}$
c. $P($ purple T-shirt $)=\frac{1}{4}$

## ISTEP+ PRACTICE

### 7.4.5, 8.4.7

35. A bag contains 8 blue marbles, 15 red marbles, 10 yellow marbles, and 3 brown marbles. If a marble is randomly selected, what is the probability that it will be brown?
A 0.27
C $0.08 \overline{3}$
B $11 \%$
D $\frac{3}{8}$
36. What is the probability of the spinner landing on a number less than 3 ?
F 25\%
H 50\%
G 37.5\%
J 75\%

## Spiral Review

37. RAIN The scatter plot shows the relationship between rainfall and lawn growth. Why might the graph be misleading? (Lesson 8-9)
38. PARKS A researcher randomly selected 100 households near a city park. Of these, $26 \%$ said they visit the park daily. If there are 500 households near the park, about how many visit it daily? (Lesson 8-8)

## GET READY for the Next Lesson

PREREQUISITE SKILL Write each fraction in simplest form. (Lesson 4-4)
39. $\frac{2}{6}$
40. $\frac{6}{8}$
41. $\frac{15}{30}$
42. $\frac{6}{16}$
43. $\frac{18}{32}$

## 9-2 <br> Sample Spaces

## MAIN IDEA Find sample spaces and probabilities. IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.
Preparation for
8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.

## New Vocabulary

sample space
tree diagram

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## MIN Lab



Here is a probability game for two players.

- Place two green marbles into Bag A. Place one green and one red marble into Bag B.
- Without looking, player 1 chooses a marble from each bag. If both marbles are the same color, player 1 wins a point. If the marbles are different colors, player 2 wins a point. Record your results and place the marbles back in the bag.
- Player 2 then pulls a marble from each bag and records the results. Continue alternating turns until each player has pulled from the bag 10 times. The player with the most points wins.

1. Make a conjecture. Do you think this is a fair game? Explain.
2. Now, play the game. Who won? What was the final score?

The set of all of the possible outcomes in a probability experiment is called the sample space. A table or grid can be used to list the outcomes in the sample space.

## EXAMPLE Find the Sample Space

(1) ICE CREAM A vendor sells vanilla and chocolate ice cream. Customers can choose from a waffle or sugar cone and either hot fudge or caramel topping. Find the sample space for all possible orders of one scoop of ice cream in a cone with one topping.
Make a table that shows all of the possible outcomes.

| Outcomes |  |  |
| :--- | :--- | :--- |
| Vanilla | Waffle | Hot Fudge |
| Vanilla | Waffle | Caramel |
| Vanilla | Sugar | Hot Fudge |
| Vanilla | Sugar | Caramel |
| Chocolate | Waffle | Hot Fudge |
| Chocolate | Waffle | Caramel |
| Chocolate | Sugar | Hot Fudge |
| Chocolate | Sugar | Caramel |

## CHECK Your Progress

a. PETS The animal shelter has both male and female Labradors in yellow, brown, or black. Find the sample space for all possible Labradors available at the shelter.

A tree diagram can also be used to display the sample space.

## ISTEP+ EXAMPLE 7.4.5, 8.4.7

A car comes in silver, red, or purple as a convertible or hardtop. Which list shows all possible color-top outcomes?
A

| Outcomes |  |
| :--- | :--- |
| silver | convertible |
| silver | hardtop |
| red | convertible |
| red | hardtop |
| purple | convertible |
| purple | hardtop |

C

| Outcomes |  |
| :--- | :--- |
| silver | convertible |
| red | hardtop |
| purple | convertible |
| silver | hardtop |
| purple | convertible |

B

| Outcomes |  |
| :--- | :--- |
| silver | convertible |
| red | hardtop |
| purple | convertible |

D

| Outcomes |  |
| :--- | :--- |
| silver | convertible |
| red | hardtop |
| purple | convertible |
| silver | hardtop |

## Test-Taking Tip

Educated Guess Find out if there is a penalty for incorrect answers. If there is no penalty, making an educated guess can only increase your score, or at worst, leave your score the same.

## Read the Item

The car comes in 3 colors: silver, red, or purple, and 2 tops: convertible or hardtop. Find all of the color-top combinations.

## Solve the Item

Make a tree diagram
to show the sample space.

There are 6 different color-top combinations.
The answer is A .


## CHECK Your Progress

b. For dessert, you can have apple or cherry pie with or without ice cream. Which list shows all the possible outcomes of choosing pie and ice cream?
F

| Outcomes |  |
| :--- | :--- |
| apple | with ice cream |
| cherry | without ice cream |

H

| Outcomes |  |
| :--- | :--- |
| apple | without ice cream |
| cherry | with ice cream |

G

| Outcomes |  |
| :--- | :--- |
| apple | with ice cream |
| apple | without ice cream |
| cherry | with ice cream |
| cherry | without ice cream |

J

| Outcomes |  |
| :--- | :--- |
| apple | with ice cream |
| cherry | with ice cream |
| apple | without ice cream |

## EXAMPLE Find Probability

## Study Tip

Fair Game A fair game is one in which each player has an equal chance of winning. This game is a fair game.

GAMES Refer to the Mini Lab at the start of this lesson. Find the sample space. Then find the probability that player 2 wins.

There are 4 equally likely outcomes with 2 favoring each player. So, the probability that player 2 wins is $\frac{2}{4}$, or $\frac{1}{2}$.

## CHECK Your Progress

c. GAMES Marcos tosses three coins. If all three coins show up heads, Marcos wins. Otherwise, Kara wins. Find the sample space. Then find the probability that Marcos wins.

Examples 1, 2 For each situation, find the sample space using a table or tree diagram. (pp. 465-466)

Example 2 (p. 466)

1. A number cube is rolled twice.
2. A pair of brown or black sandals are available in sizes 7,8 , or 9 .
3. MULTIPLE CHOICE Sandwiches can be made with ham or turkey on rye, white, or sourdough breads. Which list shows all the possible outcomes?
A

| Outcomes |  |
| :--- | :--- |
| ham | rye |
| turkey | white |
| ham | sourdough |
| ham | rye |
| turkey | white |
| turkey | sourdough |

C

| Outcomes |  |
| :--- | :--- |
| ham | rye |
| turkey | rye |
| ham | white |
| turkey | white |
| ham | sourdough |
| turkey | sourdough |

B

| Outcomes |  |
| :--- | :--- |
| ham | rye |
| turkey | white |
| turkey | sourdough |

D

| Outcomes |  |
| :--- | :--- |
| ham | rye |
| turkey | white |
| ham | sourdough |

Example 3 (p. 467)
4. GAMES Brianna spins a spinner with four sections of equal size twice, labeled A, B, C, and D. If letter A is spun at least once, Brianna wins. Otherwise, Odell wins. Find the probability that Odell wins.

## Practice and Problem Solving

| HOMEWORK HELP |  |
| :---: | :---: |
| For <br> Exercises | See <br> Examples |
| $5-12$ | 1,2 |
| 28 | 2 |
| $13-14$ | 3 |



Real-World Link The average family size in the United States is 2.59 people.

Source: U.S. Census Bureau
For each situation, find the sample space using a table or tree diagram.
5. tossing a coin, and spinning the spinner from the choices at the right
6. choosing left or right and either boots, gym shoes, or dress shoes
7. tossing a coin and rolling a number cube
8. picking a number from 1 to 5 and choosing
 the color red, white, or blue
9. choosing a purple, green, black, or silver mountain bike having 10, 18, 21, or 24 speeds
10. choosing a letter from the word SPACE and choosing a consonant from the word MATH
11. CLOTHES Jerry can buy a school T-shirt with either short sleeves or long sleeves in either gray or white and in small, medium, or large. Find the sample space for all possible T-shirts he can buy.
12. FOOD Three-course dinners can be made from the menu shown. Find the sample space for a dinner consisting of an appetizer, entrée, and dessert.

| Appetizers | Entrees | Desserts |
| :---: | :--- | :---: |
| Soup | Steak <br> Salad <br> Chicken <br> Fish | Carrot Cake <br> Cheesecake |

For each game, find the sample space. Then find the indicated probability.
13. Elba tosses a quarter, a dime, and a nickel. If tails comes up at least twice, Steve wins. Otherwise Elba wins. Find P(Elba wins).
14. Ming rolls a number cube, tosses a coin, and chooses a card from two cards marked A and B. If an even number and heads appears, Ming wins, no matter which card is chosen. Otherwise Lashonda wins. Find $P$ (Ming wins).
$\therefore$ FAMILIES Mr. and Mrs. Romero are expecting triplets. Suppose the chance of each child being a boy is $50 \%$ and of being a girl is $50 \%$. Find each probability.
15. $P$ (all three children will be boys)
16. $P$ (at least one boy and one girl)
17. $P$ (two boys and one girl)
18. $P$ (at least two girls)
19. $P$ (the first two born are boys and the last born is a girl)
20. GAMES The following is a game for two players. Find the probability that each player wins.

- Three counters are labeled according to the table at the right.

|  | Side 1 | Side 2 |
| :--- | :--- | :--- |
| Counter 1 | red | blue |
| Counter 2 | red | yellow |
| Counter 3 | blue | yellow |

- Toss the three counters.
- If exactly 2 counters match, Player 1 scores a point. Otherwise, Player 2 scores a point.

UNIFORMS For Exercises 21 and 22, use the information below.
The University of Oregon's football team has many different uniforms. The coach can choose from four colors of jerseys and pants: green, yellow, white, and black. There are three helmet options: green, white, and yellow. Also, there are four colors of socks and two colors of shoes.
21. How many jersey/pant combinations are there?
22. If the coach picks a jersey/pant
 combination at random, what is the probability he will pick a yellow jersey with green pants?
23. RESEARCH Use the Internet or another resource to find the number of

- ISTEP+

Extra Practice, pp. 691, 712 jerseys and pants your favorite college or professional sports team has as part of its uniform. How many jersey/pants combinations are there for the team you chose?
24. CHALLENGE Refer to Exercise 20. A fair game is one in which each player has an equal chance of winning. Adjust the scoring of the game so that it is fair.
25. SELECT A TOOL Mei wants to determine the probability of guessing correctly on two true-false questions on her history test. Which of the following tools might Mei use to determine the probability of answering both questions correctly by guessing? Justify your selection(s). Then use the tool(s) to solve the problem.

26. FIND THE ERROR Rhonda and Elise are finding all the possible unique outcomes of rolling an even number on a number cube and landing on A or B on the spinner shown. Who is correct? Explain your reasoning.

| Rhonda | Elise |  |  |
| :---: | :---: | :---: | :---: |
| Outcomes  <br> Number <br> Cube  <br> 2  | Spinner |  |  |
| 2 | A |  |  |
| 4 | B | Numbercomes <br> Cube | Spinner |
| 4 | A | 2 | A |
| 6 | B | 2 | B |
| 6 | C |  |  |


27. WRITINGIN MATH Describe a game between two players using one coin in which each player has an equal chance of winning.

## ISTEP+ PRACTICE

28. Mr. Zajac will choose one student from each of the two groups below to present their history reports to the class.

| Group 1 |
| :--- |
| Julia |
| Antoine |
| Greg |

Which set shows all the possible choices?
A \{(Julia, Keith), (Antoine, Keith), (Greg, Keith)\}
B \{(Julia, Antoine), (Antoine, Greg), (Isabel, Keith)\}
C \{(Julia, Keith), (Antoine, Keith), (Greg, Keith), (Julia, Isabel), (Antoine, Isabel), (Greg, Isabel)\}
D \{(Isabel, Antoine), (Keith, Greg), (Julia, Isabel), (Keith, Antoine)\}

## Spiral Review

PROBABILITY A spinner is equally likely to stop on each of its regions numbered 1 to 20 . Find each probability as a fraction in simplest form.
(Lesson 9-1)
29. a prime number
30. $\operatorname{GCF}(12,18)$
32. not a multiple of 4
33. factor of 10 or 6
31. multiple of 2 or 3
34. not an even number

UTILITIES For Exercises 35 and 36, use the graph that shows the prices for natural gas charged by an Illinois natural gas supplier. (Lesson 8-6)
35. What trend seems to be revealed in the line graph?
36. What problems might there be in using this information to predict future prices of natural gas?

37. SAVINGS Amare invests $\$ 3,400$ in a savings account that pays 3.5 \% simple interest. After four years, how much money will be in his account?

Find each number. Round to the nearest tenth if necessary. (Lesson 7-1)
38. $43 \%$ of 266
39. $17 \%$ of 92
40. $2.5 \%$ of 44

## GET READY for the Next Lesson

## PREREQUISITE SKILL Multiply.

41. $7 \cdot 22$
42. $11 \cdot 16$
43. $23 \cdot 20$
44. $131 \cdot 4$

## $9-3$ <br> The Fundamental Counting Principle

## MAIN IDEA

Use multiplication to count outcomes and find probabilities.

## IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.
Preparation for
8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models. Also addresses P.4.1.

## New Vocabulary

Fundamental Counting Principle

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- Reading in the Content Area


## GET READY for the Lesson

SALES The Shoe Warehouse sells sandals in different colors and styles.

1. According to the table, how many colors of sandals are available?
2. How many styles are available?
3. Find the product of the two numbers you found in Exercises 1 and 2.

4. Draw a tree diagram to find the number of different color and style combinations. How does the number of outcomes compare to the product you found above?

In the activity above, you discovered that multiplication, instead of a tree diagram, can be used to find the number of possible outcomes in a sample space. This is called the Fundamental Counting Principle.

## Fundamental Counting Principle

## Key Concept

Words If event $M$ has $m$ possible outcomes and event $N$ has $n$ possible outcomes, then event $M$ followed by event $N$ has $m \times n$ possible outcomes.

## EXAMPLE Find the Number of Outcomes

(1) Find the total number of outcomes when a coin is tossed and a number cube is rolled.

|  | number <br> cube | total |  |
| :---: | :---: | :---: | :---: |
| $\downarrow$ | $\downarrow$ |  | $\downarrow$ |
|  |  | $\downarrow$ |  |
| 2 |  | 6 |  |
|  |  |  |  |

There are 12 different outcomes.
Check Draw a tree diagram to show the sample space.

## CHECK Your Progress

a. Find the total number of outcomes when choosing from bike helmets that come in three colors and two styles.

The Fundamental Counting Principle can be used to find the number of possible outcomes and solve probability problems in more complex problems, when there are more than two events.

## Reallworid EXAMPLE

## Study Tip

Jean size In mens' jeans, the size is labeled waist $x$ length. So, a $32 \times 34$ is a 32-inch waist with a 34-inch length.
(2) JEANS The Jean Shop sells young men's jeans in different sizes, styles, and lengths, as shown in the table. Find the number of jeans available. Then find the probability of selecting a size $32 \times 34$ slim fit.


There are 45 different types of jeans to choose from. Out of the 45 possible outcomes, only one is favorable. So, the probability of randomly selecting a $32 \times 34$ slim fit is $\frac{1}{45}$.

## CHECK Your Progress

b. JEANS If the Jean Shop adds relaxed fit jeans to its selection, find the number of available jeans. Then find the probability of randomly selecting a $36 \times 30$ relaxed fit pair of jeans.

## Your Understanding

Example 1 Use the Fundamental Counting Principle to find the (p. 471) total number of outcomes in each situation.

1. tossing a quarter, a dime, and a nickel
2. choosing scrambled, sunny-side up, or poached eggs with bacon or sausage and milk or orange juice
3. choosing a number on a number cube and picking a marble from the bag at the right

Example 2
4. CLOTHES Beth has 3 sweaters, 4 blouses, and 6 skirts that coordinate. Find the number of different outfits consisting of a sweater, blouse, and skirt that are possible.
 Then find the probability of randomly selecting a particular sweater-blouse-skirt outfit.

## Practice and Problem Solving

| HOMEWORK HELP |  |
| :---: | :---: |
| For <br> Exercises | See <br> Examples |
| $5-10$ | 1 |
| $11-12$ | 2 |



Real-World Link
A popular orchard in Wapato, Washington, grows 7 varieties of apples including Granny Smith and Fuji. Source: All About Apples

Extra Practice, pp. 691, 712

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.
5. choosing a bagel with one type of cream cheese from the list shown in the table
6. choosing a number from 1 to 20 and a color from 7 colors
7. picking a month of the year and a day of the week
8. choosing from a comedy, horror, or action movie

| Bagels | Cream <br> Cheese |
| :--- | :--- |
| plain | plain |
| blueberry | chive |
| cinnamon |  |
| raisin |  |
| garlic |  |$\quad$| sun-dried |
| :---: |
| tomato | each shown in four theaters

9. rolling a number cube and tossing two coins
10. choosing iced tea in regular, raspberry, lemon, or peach flavors; sweetened or unsweetened; and in a glass or a plastic container
11. ROADS Two roads, Broadway and State, connect the towns of Eastland and Harping. Three roads, Park, Fairview, and Main, connect the towns of Harping and Johnstown. Find the number of possible routes from Eastland to Johnstown that pass through Harping. Then find the probability that State and Fairview will be used if a route is selected at random.
-12. APPLES An orchard makes apple nut bread, apple pumpkin nut bread, and apple buttermilk bread using 6 different varieties of apples, including Fuji. Find the number of possible bread choices. Then find the probability of selecting a Fuji apple buttermilk bread if a customer buys a loaf of bread at random.
12. GAMES Find the number of possible outcomes if five number cubes are rolled at one time during a board game.
13. PASSWORDS Find the number of possible choices for a 2-digit password that is greater than 19. Then find the number of possible choices for a 4-digit Personal Identification Number (PIN) if the digits cannot be repeated.
14. T-SHIRTS A store advertises that they have a different T-shirt for each day of the year. The store offers 32 different T-shirt designs and 11 choices of color. Is the advertisement true? Explain.
15. ANALYZE TABLES The table shows cell phone options offered by a wireless phone company. If a phone with one payment plan and one accessory is given away at random, predict the probability that it will be Brand B and

| Phone <br> Brands | Payment <br> Plans | Accessories |
| :--- | :--- | :--- |
| Brand A | Individual | Leather Case |
| Brand B | Family | Car Mount |
| Brand C | Business <br> Government | Headset <br> Travel Charger | have a headset. Explain your reasoning.

17. CHALLENGE Determine the number of possible outcomes for tossing one coin, two coins, and three coins. Then determine the number of possible outcomes for tossing $n$ coins. Describe the strategy you used.
18. Which One Doesn't Belong? Identify the choices for event $M$ and $N$ that do not result in the same number of outcomes as the other two. Explain your reasoning.

19. WRITING IN MATH Explain when you might choose to use the Fundamental Counting Principle to find the number of possible outcomes and when you might choose to use a tree diagram.

## ISTEP+ PRACTICE <br> 7.4.5, 8.4.7

20. A bakery offers white, chocolate, or yellow cakes with white or chocolate icing. There are also 24 designs that can be applied to a cake. If all orders are equally likely, what is the probability that a customer will order a white cake with white icing in a specific design?
A $\frac{1}{30}$
C $\frac{1}{120}$
B $\frac{1}{64}$
D $\frac{1}{144}$
21. SHORT RESPONSE Hat World sells 9 different styles of hats in several different colors for 2 different sports teams. If the company makes 108 kinds of hats, how many different colors do they make?

## Spiral Review

22. SCHOOL Horacio can choose from 2 geography, 3 history, and 2 statistics classes. Find the sample space for all possible schedules. (Lesson 9-2)

PROBABILITY Find the probability that the spinner shown at the right will stop on each of the following. Write as a fraction in simplest form. (Lesson 9-1)
23. a vowel
24. red

Order each set of numbers from least to greatest. (Lesson 6-8)

25. $27 \%, \frac{1}{5}, 0.22,20.1$
26. $\frac{19}{20}, 88 \%, 0.85, \frac{3}{4}$

## GET READY for the Next Lesson

## PREREQUISITE SKILL Multiply.

27. $3 \cdot 2 \cdot 1$
28. $9 \cdot 8 \cdot 7$
29. $5 \cdot 4 \cdot 3 \cdot 2$
30. $7 \cdot 6 \cdot 5 \cdot 4$

## 9-4 <br> Permutations

## MAIN IDEA <br> Find the number of permutations of a set of objects and find probabilities. <br> IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.
Preparation for 8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.

## New Vocabulary

permutation

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## MIN Lab

How many different ways are there to arrange your first 3 classes if they are math, science, and language arts?
STEPT Write math, science, and language arts on the index cards.


STEP2 Find and record all arrangements of classes by changing the order of the index cards.

1. When you first started to make your list, how many choices did you have for your first class?
2. Once your first class was selected, how many choices did you have for the second class? then, the third class?

A permutation is an arrangement, or listing, of objects in which order is important. In the example above, the arrangement science, math, language arts is a permutation of math, science, language arts because the order of the classes is different. You can use the Fundamental Counting Principle to find the number of possible permutations.

## EXAMPLE Find a Permutation

(1) SCHEDULES Find the number of possible arrangements of classes in the Mini Lab above using the Fundamental Counting Principle.

There are $\mathbf{3}$ choices for the first class.
There are $\mathbf{2}$ choices that remain for the second class.
There is 1 choice that remains for the third class.
$3 \bullet 2 \quad$ • $2=6 \longleftarrow$ The number of permutations of 3 classes
There are 6 possible arrangements, or permutations, of the 3 classes.

## CHECK Your Progress

a. VOLLEYBALL In how many ways can the starting six players of a volleyball team stand in a row for a picture?

You can use a permutation to find the probability of an event.

## Rea-Wora Example Find Probability

2 SWIMMING The finals of the Northwest Swimming League features 8 swimmers. If each swimmer has an equally likely chance of finishing in the top two, what is the probability that Yumii will be in first place and Paquita in second place?

| Northwest <br> League Finalists |  |
| :--- | :--- |
| Octavia | Eden |
| Natasha | Paquita |
| Calista | Samantha |
| Yumii | Lorena |

There are $\mathbf{8}$ choices for first place.


There are 56 possible arrangements, or permutations, of the 2 places.
Since there is only one way of having Yumii come in first and Paquita second, the probability of this event is $\frac{1}{56}$.

## CHECK Your Progress

b. LETTERS Two different letters are randomly selected from the letters in the word math. What is the probability that the first letter selected is $m$ and the second letter is $h$ ?

## 6 CaETK Your Understanding

Example 1 (p. 475)

1. AMUSEMENT PARKS Seven friends are waiting to ride the new roller coaster. In how many ways can they board the ride, once it is their turn?
2. COMMITTEES In how many ways can a president, vice president, and secretary be randomly selected from a class of 25 students?

Example 2 (p. 476)
3. DVDs You have five seasons of your favorite TV show on DVD. If you randomly select two of them from a shelf, what is the probability that you will select season one first and
 season two second?
4. PASSWORDS A password consists of four letters, of which none are repeated. What is the probability that a person could guess the entire password by randomly selecting the four letters?
HOMEWORK HELP

| For <br> Exercises | See <br> Examples |
| :---: | :---: |
| $5-8$ | 1 |
| $9-12$ | 2 |


5. CONTESTS In the Battle of the Bands contest, in how many ways can the four participating bands be ordered?
6. CODES A garage door code has 5 digits. If no digit is repeated, how many codes are possible?
7. LETTERS How many permutations are possible of the letters in the word friend?
8. NUMBERS How many different 3-digit numbers can be formed using the digits $9,3,4,7$, and 6 ? Assume no number can be used more than once.
9. CAPTAINS The members of the Evergreen Junior High Quiz Bowl team are listed at the right. If a captain and an assistant captain are chosen at random, what is the probability that Walter is selected as captain and Mi-Ling as co-captain?
10. BASEBALL Adriano, Julián, and three of their friends will sit in a row of five seats at a baseball

| Evergreen Junior High <br> Quiz Bowl Team |  |
| :--- | :--- |
| Jamil | Luanda |
| Savannah | Mi-Ling |
| Tucker | Booker |
| Ferdinand | Nina |
| Walter | Meghan | game. If each friend is equally likely to sit in any seat, what is the probability that Adriano will sit in the first seat and Julián will sit in the second seat?

11. GAMES Alex, Aiden, Dexter, and Dion are playing a video game. If they each have an equally likely chance of getting the highest score, what is the probability that Dion will get the highest score and Alex the second highest?
12. BLOCKS A child has wooden blocks with the letters $G, R, T, I$, and $E$. Find the probability that the child randomly arranges the letters in the order TIGER.

Real-World Link In 2006, Stephen Clarke set the record for the fastest time to carve a face into a pumpkin. It took him 24.03 seconds.

Source: Guinness World Records


$\because:$ $\vdots 13$. SELECT A TOOL Refer to the information in the table. There are four ribbons passed out to participants in the Perfect Pumpkin competition. Which of the following tools might you use to find the number of ways a ribbon can be awarded to a pumpkin in the Perfect Pumpkin competition? Justify your selection(s) to solve the problem.

| Pumpkin Festival |  |
| :---: | :---: |
| Competition | Number of <br> Participants |
| Perfect Pumpkin | 24 |
| Biggest Pumpkin | 17 |
| Heaviest Pumpkin | 15 |
| Pumpkin Carving | 5 |

14. PHOTOGRAPHY A family discovered they can stand in a row for their portrait in 720 different ways. How many members are in the family?
15. STUDENT ID Hamilton Middle School assigns a four-digit identification number to each student. The number is made from the digits $1,2,3$ and 4 , and no digit is repeated. If assigned randomly, what is the probability that an ID number will end with a 3 ?

## H.O.T. Problems,

16. CHALLENGE There are 1,320 ways for three students to win first, second, and third place during a debate match. How many students are there on the debate team? Explain your reasoning.
17. WRITINGIN MATH Describe a real-world situation that has 6 permutations. Justify your answer.

## ISTEP+ PRACTICE

7.4.5, 8.4.7
18. The five finalists in a random drawing are shown. Find the probability that Sean is awarded first prize and Teresa is awarded second prize.
A $\frac{1}{5}$
B $\frac{1}{10}$
C $\frac{2}{5}$
D $\frac{1}{20}$

| Finalists |
| :---: |
| Cesar |
| Teresa |
| Sean |
| Nikita |
| Alfonso |

19. A baseball coach is deciding on the batting order for his nine starting players with the pitcher batting last. How many batting orders are possible?
F 8
G 72
H 40,320
J 362,880

## Spiral Review

20. BREAKFAST Find the total number of outcomes if you can choose from 8 kinds of bagels, 3 toppings, and 4 beverages. (Lesson 9-3)
21. LUNCH Make a tree diagram showing different ways to make a sandwich with turkey, ham, or salami and either cheddar or Swiss cheese. (Lesson 9-2)
22. PROBABILITY What is the probability of rolling a number greater than four on a number cube? (Lesson 9-1)

Find each product. Write in simplest form. (Lesson 5-5)
23. $\frac{4}{5} \times 2 \frac{1}{3}$
24. $11 \frac{1}{8} \times \frac{1}{2}$
25. $4 \frac{5}{6} \times \frac{7}{8}$

## GET READY for the Next Lesson

PREREQUISITE SKILL Find each value. (Lesson 1-4)
26. $\frac{5 \cdot 4}{2 \cdot 1}$
27. $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$
28. $\frac{5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2}$
29. $\frac{10 \cdot 9 \cdot 8 \cdot 7}{8 \cdot 7 \cdot 6 \cdot 5}$

A number cube is rolled. Find each probability. Write as a fraction in simplest form. (Lesson 9-1)

1. $P$ (an odd number)
2. $P$ (a number not greater than 4$)$
3. $P$ (a number less than 6 )
4. $P$ (a multiple of 2 )
5. BOOKS Brett owns 5 science-fiction books, 3 biographies, and 12 mysteries. If he randomly picks a book to read, what is the probability that he will not pick a science-fiction book? Write as a percent.

For each situation, find the sample space using a table or tree diagram. (Lesson 9-2)
6. Two coins are tossed.
7. The spinner shown is spun, and a digit is randomly selected from the number 803.

8. MULTIPLE CHOICE At a diner, a customer can choose from eggs or pancakes as an entrée and from ham or sausage as a side. Which set shows all the possible choices of one entrée and one side? (Lesson 9-2)

A \{(eggs, pancakes), (ham, sausage) $\}$
B \{(eggs, ham), (eggs, sausage), (pancakes, ham), (pancakes, sausage)\}
C \{(eggs, ham), (eggs, pancakes), (sausage, pancakes)\}
D $\{($ eggs, ham), (pancakes, sausage) $\}$

9. GAMES Abbey rolls a number cube and chooses a card from among cards marked $A$, $B$, and $C$. If an odd number and a vowel turn up, Abbey wins. Otherwise, Benny wins. Find the sample space. Then find the probability that Benny wins. (Lesson 9-2)

For Exercises 10 and 11, use the Fundamental Counting Principle to find the total number of possible outcomes in each situation. (Lesson 9-3)
10. A customer chooses a paper color, size, and binding style for some copies.

| Color | Size | Binding |
| :---: | :---: | :---: |
| white | $8.5^{\prime \prime} \times 11^{\prime \prime}$ | paper clip |
| yellow | $8.5^{\prime \prime} \times 14^{\prime \prime}$ | binder clip |
| green | $8.5^{\prime \prime} \times 17^{\prime \prime}$ | staple |

11. A number cube is rolled and three coins are tossed.
12. CARS A certain car model comes in the colors in the table and either automatic or manual transmission. Find the probability

| Exterior | Interior |
| :---: | :---: |
| Black | Gray |
| White | Tan |
| Red |  |
| Silver |  | that a randomly selected car will have a black exterior, tan interior, and manual transmission if all combinations are equally likely. (Lesson 9-3)

13. COMPETITION How many ways can 6 swimmers come in first, second, or third place?
14. MULTIPLE CHOICE Noriko packed five different sweaters for her weekend vacation. If she randomly selects one sweater to wear each day, what is the probability that she will select the brown sweater on Friday, the orange sweater on Saturday, and the pink sweater on Sunday? She will not wear each sweater more than once. (Lesson 9-4)
F $\frac{1}{60}$
G $\frac{1}{120}$
H $\frac{3}{5}$
J $\frac{1}{5}$

## 9-5 <br> Combinations

## MAIN IDEA

Find the number of combinations of a set of objects and find probabilities.

## IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.
Preparation for 8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.

## New Vocabulary

combination

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## $\triangle$ GET READY for the Lesson

FOOD Mr. Rius is making a salad for a party. He has tomatoes, green peppers, cucumbers, and radishes. He starts with lettuce and then decides to add three of the above ingredients to his salad.

1. Use the first letter of each vegetable to list all of the permutations of the ingredients added to the lettuce. How many are there?
2. Cross out any arrangement that contains the same letters as another one in the list. How many are there now?
3. Explain the difference between the two lists.

An arrangement, or listing, of objects in which order is not important is called a combination. In the activity above, choosing cucumbers and radishes is the same as choosing radishes and cucumbers.

Permutations and combinations are related. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways the smaller set can be arranged.

## EXAMPLE Find the Number of Combinations

(1) FOOD Terrence's Pizza Parlor is offering the special shown in the table. How many different two-topping pizzas are possible?
This is a combination problem because the order of the toppings on the pizza is not important.


## METHOD 1

## Make a list.

Use the first letter of each topping to list all of the permutations of the toppings taken two at a time. Then cross out the pizzas that are the same as another one.

| $p, s$ | $s, p$ | $g, p$ | $0, p$ | $m, p$ |
| :--- | :--- | :--- | :--- | :--- |
| $p, o$ | $s, o$ | $g, s$ | $0, s$ | $m, s$ |
| $p, m$ | $s, m$ | $g, 0$ | $o, m$ | $m, 0$ |
| $p, g$ | $s, g$ | $g, m$ | $o, g$ | $m, g$ |

So, there are 10 different two-topping pizzas.

## METHOD 2 Use a permutation.

Step 1 Find the number of permutations of the entire set. $5 \cdot 4=20$ A permutation of 5 toppings, taken 2 at time

Step 2 Find the number of permutations of the smaller set. $2 \cdot 1=2$ Number of ways to arrange 2 toppings

Step 3 Find the number of combinations.
$\frac{20}{2}$ or 10 Divide the number of permutations of the entire set by the number of permutations of each smaller set.

So, there are 10 different two-topping pizzas.

## CHOOSE Your Method



Real-World Link
The Last Man Standing One-on-One Basketball Tournament is held in New York. The top 16 players from preliminary competition play in a championship tournament in Madison Square Garden.
a. FOOD How many different three-topping pizzas are possible if Terrence adds ham and anchovies to the topping choices?

## ReaFWorf EXAMPLES

(2) BASKETBALL A one-on-one basketball tournament has six players from the regional finals. Each player will play every opponent once. The 2 players with the best records will then play in the championship. How many one-on-one games will be played?

Find the number of ways 2 players can be chosen from a group of 6 .

There are 15 games plus 1 final game to determine the overall winner. So, there will be 16 games played.

Check Make a diagram in which each person is represented by a point. Draw a line segment between each pair of points to represent all the games. This produces 15 line segments, or 15 games. Then add the final game to make
 a total of 16 games.

If the players in Example 2 are selected at random, what is the probability that player 3 will play player 6 in the first game? There are 15 possible first games and only one favorable outcome. So, the probability is $\frac{1}{15}$.

## CHECK Your Progress

b. BASKETBALL What is the probability that player 3 will play against player 12 in the first game if 12 people play in the tournament?

## Your Understanding

Example 1 (pp. 480-481)

1. STICKERS In how many ways can you pick 2 stickers from a package of 7 ?
2. CRAFT FAIR In how many ways can 3 out of 10 students be chosen to present their projects at a craft fair?

## Examples 2, 3 PAINTING For Exercises 3 and 4, use the information below.

Jade is going to paint her room two different colors from among white, gray, sage, or yellow.
3. How many combinations of two paint colors are there?
4. Find the probability that two colors chosen randomly will be white and sage.

## Practice and Problem Solving



| For <br> Exercises | See <br> Examples |
| :---: | :---: |
| $5-8$ | 1,2 |
| $9-10$ | 3 |

5. VOLUNTEERS In how many ways can you select 4 volunteers out of 10 ?
6. ART In how many ways can four drawings out of 15 be chosen for display?
7. INTERNET Of 12 Web sites, in how many ways can you choose to visit 6 ?
8. SPORTS On an 8-member volleyball team, how many different 6-player starting teams are possible?
9. STUDENT COUNCIL The students listed are members of Student Council. Three will be chosen at random to form a committee. Find the probability that the three students chosen will be Placido, Maddie, and Akira.
10. FOOD At a hot dog stand, customers can select three toppings from among chili, onions, cheese, mustard, or relish. What is the probability that three toppings

| 0 | Roster |
| :---: | :--- |
| 0 | Leon |
|  | Placido |
| - | Maddie |
|  | Adrahan |
|  | Matt |
| 0 | Akira |
| 0 |  | selected at random will include onions, mustard, and relish?

11. MUSIC Marissa practiced the five pieces listed at the right for a recital. Find the number of different ways that three pieces will be randomly chosen for her to play. Then find the probability that all three were composed by Beethoven.

| Recital Piece | Composer |
| :--- | :--- |
| Fur Elise | Ludwig van Beethoven |
| First Piano Sonata | Sergei Rachmaninoff |
| The Four Seasons | Antonio Vivaldi |
| Cappricio | Ludwig van Beethoven |
| Moonlight Sonata | Ludwig van Beethoven |

Tell whether each problem represents a permutation or a combination. Then solve the problem.
12. Six children remain in a game of musical chairs. If two chairs are removed, how many different groups of four children can remain?
13. How many ways can first and second chair positions be awarded in a band that has 10 flute players?
14. How many ways can 12 books be stacked in a single pile?
15. CHALLENGE How many people were at a party if each person shook hands exactly once with every other person and there were 105 handshakes?
16. FIND THE ERROR Ling and Daniela are calculating the number of ways that a 4-member committee can be chosen from an 8-member club. Who is correct? Explain your reasoning.

17. WRITINGIN MATH Write about a real-world situation that can be solved using a combination. Then solve the problem.

## ISTEP+ PRACTICE

7.4.5, 8.4.7
18. Three cheerleaders will be randomly selected to represent the squad at a game. If there are 12 cheerleaders, find the probability that the three members chosen are Kameko, Lynn, and Tory.
A $\frac{1}{220}$
C $\frac{1}{4}$
B $\frac{3}{110}$
D $\frac{1}{3}$
19. Four students are to be chosen from a roster of 9 students to attend a science camp. In how many ways can these 4 students be chosen?
F 5
H 126
G 36
J 3,024

## spiral Review

20. TRACK Six sprinters are entered in a 100-meter dash. In how many different ways can the race be completed? Assume there are no ties. (Lesson 9-4)
21. T-SHIRTS A clothing company makes T-shirts with the choices as shown in the table. How many different T-shirts are possible? (Lesson 9-3)

Estimate. (Lesson 5-1)
22. $\frac{1}{10}+\frac{7}{8}$
23. $\frac{5}{12}-\frac{1}{9}$

| Size | Color | Style | Material |
| :---: | :--- | :--- | :--- |
| S | Orange | Crew | Cotton |
| M | Pink | V-Neck | Polyester |
| L | Lime Green | Tank-Top |  |
| XL |  |  |  |

## GET READY for the Next Lesson

PREREQUISITE SKILL For Exercises 24 and 25, use the graph. (Lesson 8-5)
24. How many students were surveyed?
25. Find the probability that a student's favorite picnic game is sack racing.

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## 9-6 <br> Problem-Solving Investigation

## MAIN IDEA: Solve problems by acting it out.

P.1.3 Apply and adapt a variety of appropriate strategies to solve problems. P.5.1 Create and use representations to organize, record, and communicate mathematical ideas. Also addresses P.3.1, P.3.2.

## P.S.I. TERTM +

## e-Mail: ACT IT OUT

EDDIE: I've been practicing free throws every day after school. Now I can make an average of 3 out of every 4 free throws I try. I wonder how many times I usually make two free throws in a row.

YOUR MISSION: Act it out to determine the probability that Eddie makes two free throws in a row.


| Understand | You know that Eddie makes an average of 3 out of every 4 free throws. You could have Eddie actually make free throws, but that requires a basketball hoop. You could also act it out with a spinner. |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Plan | Spin a spinner, numbered 1 to 4 , two times. If the spinner lands on 1,2 , or 3 , he makes the free throw. If the spinner lands on 4 , he doesn't make it. Repeat the experiment 10 times. |  |  |  |  |  |  |  |  |  |  |
| Solve | Spin the spinner and make a table of the results. |  |  |  |  |  |  |  |  |  |  |
|  | Trials | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | First Spin | 4 | 1 | 4 | 3 | 1 | 2 | 2 | 1 | 3 | 2 |
|  | Second Spin | 2 | 3 | 3 | 2 | 1 | 4 | 1 | 4 | 3 | 3 |
|  | The highlighted columns show that six out of the 10 trials resulted in two free throws in a row. So, the probability is $60 \%$. |  |  |  |  |  |  |  |  |  |  |
| Check | Repeat the experiment several times to see whether the results agree. |  |  |  |  |  |  |  |  |  |  |

## Analyze The Strategy

1. Explain whether the results of the experiment would be the same if it were repeated.
2. WRITINGIN MATH Write a problem that can be solved by acting it out. Then solve the problem by acting it out.

## Mised Problem Soluing

For Exercises 3-6, use the act it out strategy.
3. TESTS Determine whether using a spinner with four equal sections is a good way to answer a 5 -question multiple-choice quiz if each question has choices A, B, C, and D. Justify your answer.
4. BOOKS There are 6 students in a book club. Two of them order books, and the delivery comes to the classroom teacher. However, the teacher cannot remember which 2 students ordered the books. Is it a good idea for the teacher to randomly pass out the books to any two students? Explain. What is the probability that the teacher will give the correct books to the students?
5. RUNNING Six runners are entered in a race. Assuming no ties, in how many different ways can first and second places be awarded?
6. MOVIES In how many different ways can four friends sit in a row of four seats at the movies if two of the friends insist on sitting next to each other?
Use any strategy to solve Exercises 7-14. Some strategies are shown below.

7. FESTIVALS The Student Council will have a booth set up at the town festival. They surveyed 160 students to find out their preference for the booth. The results are shown below. Is 35,65 , or 95 a reasonable answer for the number of students who would prefer a dunking booth? Explain.

Town Festival Survey Results

8. TRANSPORTATION There are 4 students waiting at a subway stop. How many different ways can they board the subway?
9. ALGEBRA The pattern below is known as Pascal's Triangle. Would 1, 6, 10, 10, 6, 1 be a reasonable conjecture for the numbers in the 6th row? Justify your answer.

10. CHESS A chess tournament is held, and 32 students participate. The tournament will be single-elimination, which means if a player loses one match, he or she will be eliminated. How many total games will be played in the tournament?
11. SCHOOL Suppose rolling an even number on a number cube corresponds to an answer of true and rolling an odd number corresponds to an answer of false. Determine whether rolling this number cube is a good way to answer a 5-question true-false quiz. Justify your answer.
12. CABLE A cable company is running a special for new customers. For the first 3 months they will get a discount of $20 \%$ off their regular bill. Would $\$ 50, \$ 60$, or $\$ 70$ be a reasonable estimate for the first bill if their regular bill is $\$ 95$ ? Explain.
13. MONEY If you save $\$ 1.25$ every week in a piggy bank, how much money will you have after 3 years?
14. MUSIC Liseta has an equal number of jazz, country, rap, pop, and R\&B songs on her MP3 player. She listens to her MP3 player on random mode on both Wednesday and Thursday nights. What is the probability Liseta will hear a rap song first on Wednesday night?

## 9-1 <br> Theoretical and Experimental Probability

## MAIN IDEA

Find and compare experimental and theoretical probabilities.

IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.
Preparation for
8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models. Also
addresses P.3.4, P.4.1.

## New Vocabulary

theoretical probability experimental probability

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## MIN Lab

Follow the steps to determine how many times doubles are expected to turn up when two number cubes are rolled.


Step 1 Use the table to help you find the expected number of times doubles should turn up when rolling two number cubes 36 times. The top row represents one number cube, and the left column represents the other number cube.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 |
| 2 | 2,1 | 2,2 | 2,3 | 2,4 | 2,5 | 2,6 |
| 3 | 3,1 | 3,2 | 3,3 | 3,4 | 3,5 | 3,6 |
| 4 | 4,1 | 4,2 | 4,3 | 4,4 | 4,5 | 4,6 |
| 5 | 5,1 | 5,2 | 5,3 | 5,4 | 5,5 | 5,6 |
| 6 | 6,1 | 6,2 | 6,3 | 6,4 | 6,5 | 6,6 |

Step 2 Roll two number cubes 36 times. Record the number of times doubles turn up.

1. Compare the number of times you expected to roll doubles with the number of times you actually rolled doubles.
2. Write the probability of rolling doubles out of 36 rolls using the number of times you expected to roll doubles from Step 1. Then write the probability of rolling doubles out of 36 rolls using the number of times you actually rolled doubles from Step 2.

In the Mini Lab above, you found both the theoretical probability and the experimental probability of rolling doubles using two number cubes. Theoretical probability is based on what should happen when conducting a probability experiment. This is the probability you have been using since Lesson 9-1. Experimental probability is based on what actually occurred during such an experiment.

## Theoretical Probability

$\frac{6}{36} \leftarrow 6$ rolls should occur

## Experimental Probability

$$
\frac{n}{36} \leftarrow n \text { rolls actually occurred }
$$

The theoretical probability and the experimental probability of an event may or may not be the same. As the number of times an experiment is conducted increases, the theoretical probability and the experimental probability should become closer in value.

## EXAMPLE Experimental Probability

(1) Two number cubes are rolled 75 times, and a sum of 9 is rolled 10 times. What is the experimental probability of rolling a sum of 9 ?
$P(9)=\frac{\text { number of times a sum of } 9 \text { occurs }}{\text { total number of rolls }}$

$$
=\frac{10}{75} \text { or } \frac{2}{15}
$$

The experimental probability of rolling a sum of 9 is $\frac{2}{15}$.

## CHECK Your Progress

a. In the above experiment, what is the experimental probability of rolling a sum that is not 9 ?
b. In the above experiment, what is the experimental probability of rolling a sum that is not six if a sum of six was rolled 18 times?

## Stualy Tip

Trials A trial is one experiment in a series of successive experiments.

## EXAMPLES Experimental and Theoretical Probability

(2) The graph shows the results of an experiment in which a spinner with 3 equal sections is spun sixty times. Find the experimental probability of spinning red for this experiment.

The graph indicates that the spinner landed on red 24 times, blue 15 times, and green 21 times.

$$
\begin{aligned}
P(\text { red }) & =\frac{\text { number of times red occurs }}{\text { total number of spins }} \\
& =\frac{24}{60} \text { or } \frac{2}{5}
\end{aligned}
$$



The experimental probability of spinning red was $\frac{2}{5}$.
(3) Compare the experimental probability you found in Example 2 to its theoretical probability.
The spinner has three possible outcomes, red, blue, or green. So, the theoretical probability of spinning red is $\frac{1}{3}$. Since $\frac{2}{5} \approx \frac{1}{3}$, the experimental probability is close to the theoretical probability.

## CHECK Your Progress

c. Refer to Example 2. If the spinner was spun 3 more times and landed on green each time, find the experimental probability of spinning green for this experiment.
d. Compare the experimental probability you found in Exercise c to its theoretical probability.

Theoretical and experimental probability can be used to make predictions about future events.

## Rea-World EXAMPLES Predict Future Events

Real-World Career. . How Does a Media Buyer Use Math? A media buyer looks at previous sales to make predictions on how many items to buy in the future.

## IN Math Online

For more information, go to glencoe.com.

MEDIA A media buyer examines last year's DVD sales to decide how many DVDs of each type to buy this year. Last year's sales are shown in the table. What is the probability that a person buys a comedy DVD?

There were 2,000 DVDs sold and 580 chose

| DVDs Sold |  |
| :--- | :---: |
| Type | Number |
| action | 670 |
| comedy | 580 |
| drama | 450 |
| horror | 300 | comedy. So, the probability is $\frac{580}{2,000}$ or $\frac{29}{100}$.

(5) Suppose the media buyer expects to sell 5,000 DVDs this year. How many drama DVDs should she buy?

$$
\begin{aligned}
\frac{450}{2,000} & =\frac{x}{5,000} & & \text { Write a proportion. } \\
450 \cdot 5,000 & =2,000 \cdot x & & \text { Find the cross products. } \\
2,250,000 & =2,000 x & & \text { Multiply. } \\
1,125 & =x & & \text { Divide each side by 2,000. }
\end{aligned}
$$

She should buy about 1,125 drama DVDs.

## CHECK Your Progress

e. What is the probability that a person buys a horror DVD?
f. If the media buyer expects to sell 3,000 DVDs this year, about how many action movies should she buy?

## Your Understanding

Examples 1-3 For Exercises 1-3, a coin is tossed 50 times, and it lands on heads 28 times.
(p. 487)

1. Find the experimental probability of the coin landing heads.
2. Find the theoretical probability of the coin landing heads.
3. Compare the probabilities in Exercises 1 and 2.

FOOD For Exercises 4 and 5, use the table showing the types of muffins that customers bought one morning from their local bakery.
Example 4 (p. 488)
4. What is the probability that a customer buys a blueberry muffin?

| Muffin | Number <br> of People |
| :--- | :---: |
| blueberry | 22 |
| poppyseed | 17 |
| banana | 11 |

5. If 100 customers buy muffins tomorrow, about how many would you expect to buy a banana muffin?

## Practice and Probjem Solving

| HOMEWORK HELP |  |
| :---: | :---: |
| For <br> Exercises | See <br> Examples |
| $6-7$ | $1-3$ |
| $8-9$ | 4 |
| $10-11$ | 5 |

For Exercises 6 and 7, a number cube is rolled 20 times and lands on 1 two times and on 5 four times.
6. Find the experimental probability of landing on 5. Compare the experimental probability to the theoretical probability.
7. Find the experimental probability of not landing on 1 . Compare the experimental probability to the theoretical probability.

Z00 For Exercises 8-11, use the graph of a survey of 70 zoo visitors who were asked to name their favorite animal exhibit.
8. What is the probability that the elephant exhibit is someone's favorite?
9. What is the probability that the bear exhibit is someone's favorite?
10. Suppose 540 people visit the zoo. Predict how many people will choose the monkey exhibit as their favorite.

What is your Favorite Animal Exhibit?

| What is your Favorite Animal Exhibit? |  |  |
| :---: | :---: | :---: |
| Exhibit | Tally | Frequency |
| bears | W I | 6 |
| elephants | W W W W II | 17 |
| monkeys | WX WW W W I | 21 |
| penguins | WXY III | 13 |
| snakes | W W W III | 13 |

11. Suppose 720 people visit the zoo. Predict how many people will choose the penguin exhibit as their favorite.

For Exercises 12-14, a spinner with three equal-sized sections marked $A, B$, and $C$ is spun 100 times.
12. What is the theoretical probability of landing on A ?
13. The results of the experiment are shown in the table.

| Section | Frequency |
| :---: | :---: |
| A | 24 |
| B | 50 |
| C | 26 | What is the experimental probability of landing on A? of landing on C ?

14. Make a drawing of what the spinner might look like based on its experimental probabilities. Explain your reasoning.

Extra Practice, pp. 692, 712

GIFTS For Exercises 15-17, use the graph at the right.
15. What is the probability that a mother will receive a gift of flowers or plants? Write the probability as a fraction.
16. Out of 400 mothers that receive gifts, predict how many will receive flowers or plants.
17. Out of 750 mothers that receive gifts, is it reasonable to expect 250 mothers to receive jewelry? Why or why not?

## Most Popular Mother's Day Gifts



Source: Carlton Cards
H.O.T. Problems
18. CHALLENGE The experimental probability of a coin landing on heads is $\frac{7}{12}$. If the coin landed on tails 30 times, find the number of tosses.
19. REASONING Twenty sharpened pencils are placed in a box containing an unknown number of unsharpened pencils. Suppose 15 pencils are taken out at random, of which five are sharpened. Based on this, is it reasonable to assume that the number of unsharpened pencils was 40? Explain your reasoning.
20. WRITNGIN MATH Compare and contrast experimental probability and theoretical probability.

## ISTEP+ PRACTICE

7.4.5, 8.4.7
21. The frequency table shows Mitch's record for the last thirty par-3 holes he has played.

| Mitch's Golf Results |  |
| :---: | :---: |
| Score | Number of Holes |
| 2 | 4 |
| 3 | 14 |
| 4 | 9 |
| 5 | 3 |

Based on this record, what is the probability that Mitch will score a 2 or 3 on the next par- 3 hole?
A $\frac{7}{9}$
C $\frac{3}{10}$
B $\frac{3}{5}$
D $\frac{9}{50}$
22. J.R. tossed a coin 100 times and graphed the results.


Based on this information, what is the experimental probability of tossing tails on the next toss?
F $\frac{1}{5}$
H $\frac{3}{5}$
G $\frac{2}{3}$
J $\frac{4}{5}$

## Spiral Review

23. SPINNERS In how many ways could the colors in the spinner shown be arranged so that red and blue remain in the same places? (Lesson 9-6)
24. BASEBALL How many ways can a baseball coach select four starting pitchers from a pitching staff of eight? (Lesson 9-5)

25. CLOTHES A pair of jeans comes in 4 different styles, 3 different colors, and 5 different sizes. How many unique outcomes are possible? (Lesson 9-3)

## GET READY for the Next Lesson

PREREQUISITE SKILL Write each fraction as a percent. (Lesson 6-9)
26. $\frac{4}{5}$
27. $\frac{11}{20}$
28. $\frac{7}{8}$
29. $\frac{39}{50}$

## $=$ $9-7$ <br> Probability Lab Simulations

## MAIN IDEA

Investigate experimental probability by conducting a simulation.

## IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions. Preparation for 8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models. Also addresses P.3.1.

A simulation is a way of acting out a problem situation. Simulations often use models to act out events that would be difficult or impractical to perform. In this lab, you will simulate purchasing a box of cereal and getting one of four possible prizes inside.

## AGIVIIY

STEPT Place four different colored cubes into a paper bag.

STEP 2 Without looking, draw a cube from the bag, record its color, and then place the cube back in the bag.

STEP3 Repeat steps 1 and 2 until you have drawn a cube from
 the bag a total of four times.

## Analyze the Results

1. Based on your results, predict the probability of getting each prize.
2. What is the theoretical probability of getting each prize?
3. How do your probabilities in Exercises 1 and 2 compare?
4. MAKE A PREDICTION Predict the probability of selecting all four prizes in four boxes of cereal.
5. Repeat the simulation above 20 times. Use this data to predict the probability of selecting all four prizes in four boxes of cereal.
6. Calculate the experimental probability described in Exercise 5 using the combined data of five different groups. How does this probability compare with your prediction?
7. Describe a simulation that could be used to predict the probability of taking a five question true/false test and getting all five questions correct by guessing. Choose from among two-sided counters, number cubes, coins, or spinners as your model.
8. COLLECT THE DATA Conduct 50 trials of the experiment you described in Exercise 7. Then calculate the experimental probability of getting all five questions correct by guessing.

## 9-8 <br> Compound Events

## MAIN IDEA

Find the probability of independent and dependent events.

## IN Academic Standards

7.4.5 Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions. Preparation for 8.4.6 Describe and apply the addition rule for probabilities for simple events that are mutually exclusive and for simple events that are not.

## New Vocabulary

compound event independent events dependent events disjoint events

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- Personal Tutor
- Self-Check Quiz


## $\triangle$ GET READY for the Lesson

MONEY Reginald has 3 state quarters, one from Colorado, one from Montana, and one from Washington.

1. If Reginald picks one quarter without looking, what is the probability it is from Colorado?
2. Suppose he tosses the coin. What is the probability it lands heads up?

3. Make a tree diagram to find the probability of choosing a Colorado quarter that lands heads up.
4. How are the answers to Exercises 1, 2, and 3 related?

In the example above, choosing a quarter and tossing heads is a compound event. A compound event consists of two or more simple events. Since choosing a quarter does not affect tossing heads, the two events are called independent events. The outcome of one event does not affect the outcome of the other event.

## EXAMPLE Independent Events

1) A coin is tossed, and the spinner shown is spun. Find the probability of tossing heads and spinning a consonant.
List the sample space. Use H for heads and T for tails.

H, A
H, B
H, C
T, A
T, B
T, C
$P(\mathrm{H}$ and a consonant $)=\frac{\text { number of times heads and a consonant occurs }}{\text { number of possible outcomes }}$
$P(\mathrm{H}$ and a consonant $)=\frac{2}{6}$ or $\frac{1}{3}$
So, the probability is $\frac{1}{3}$ or about $33 \%$.

## CHECK Your Progress

A number cube is rolled, and the spinner in Example 1 is spun. Find each probability.
a. $P(4$ and a consonant $)$
b. $P($ odd and a B)

The probability in Example 1 can also be found by multiplying the probabilities of each event. $P(\mathrm{H})=\frac{1}{2}$ and $P$ (consonant) $=\frac{1}{3}$ and so $P(\mathrm{H}$ and C$)=\frac{1}{2} \cdot \frac{1}{3}$ or $\frac{1}{6}$. This leads to the following.

## Probability of Independent Events

 Key ConceptWords The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.
Symbols $\quad P(A$ and $B)=P(A) \cdot P(B)$

## Real-World EXAMPLE

(2) SHOPPING Brianna is buying a new outfit. She is choosing among 2 red, 1 purple, 3 pink, or 4 yellow tops. For pants, she is choosing between jeans or capris. If Brianna chooses a top and capris at random, find the probability that she chooses a yellow top and jeans.
$P$ (yellow top and jeans)

$$
\begin{aligned}
& =P(\text { yellow top }) \cdot P(\text { jeans }) \\
& =\frac{4}{10} \cdot \frac{1}{2} \quad \\
& =\frac{4 \text { out of } 10 \text { tops are yellow. }}{10} \cdot \frac{1}{2_{1}}=\frac{2}{10} \text { or } \frac{1}{5} \quad \text { Simplify. }
\end{aligned}
$$

So, the probability is $\frac{1}{5}$ or about $20 \%$.

## CHECK Your Progress

c. SHOPPING If khakis and shorts are added to Brianna's pant choices, find the probability that she chooses a red top and capris.

If the outcome of one event affects the outcome of a second event, the events are called dependent events. Just as in independent events, the probabilities of dependent events can be found by multiplying the probabilities of each event. However, now the probability of the second event depends on the fact that the first event has already occurred.

## Probability of Dependent Events

Words If two events, $A$ and $B$, are dependent, then the probability of both events occurring is the product of the probability of $A$ and the probability of $B$ after $A$ occurs.

Symbols

```
P(A and B) = P(A) •P(B following A)
```


## EXAMPLE Dependent Events

(3) There are 2 red, 5 green, and 8 yellow marbles in a jar. Martina randomly selects two marbles without replacing the first marble. What is the probability that she selects two green marbles?

Since the first marble is not replaced, the first event affects the second event. These are dependent events.
$P($ first marble is green $)=\frac{5}{15} \longleftarrow$ number of green marbles
$P($ second marble is green $)=\frac{4}{14}$
$P($ two green marbles $)=\frac{\frac{1}{8}}{\frac{8}{3}} \cdot \frac{2}{\frac{4}{7}}$ or $\frac{2}{21}$
So, the probability of selecting two green marbles is $\frac{2}{21}$, or about $9.5 \%$.

## CHECK Your Progress

d. There are 4 blueberry, 6 raisin, and 2 plain bagels in a bag. Javier randomly selects two bagels without replacing the first bagel. Find the probability that he selects a raisin bagel and then a plain bagel.

Sometimes two events cannot happen at the same time. For example, when a coin is tossed, the outcome of heads cannot happen at the same time as tails. Either heads or tails will turn up. Tossing heads and tossing tails are examples of disjoint events, or events that cannot happen at the same time. Disjoint events are also called mutually exclusive events.

## Study Tip

Disjoint Events When finding the probabilities of disjoint events, the word or is usually used.

## EXAMPLE Disjoint Events

(4) A number cube is rolled. What is the probability of rolling an odd number or a 6 ?
These are disjoint events since it is impossible to roll an odd number and a 6 at the same time.
$P($ odd number or 6$)=\frac{4}{6} \longleftarrow$ There are four favorable outcomes: $1,3,5$, or 6 .
So, the probability of rolling an odd number or a 6 is $\frac{4}{6}$, or $\frac{2}{3}$.

## CHECK Your Progress

e. Twenty-six cards are labeled, each with a letter of the alphabet, and placed in a box. A single card is randomly selected. What is the probability that the card selected will be labeled with the letter M or the letter T?

Probability The probability of two disjoint events is the sum of the two individual probabilities. The probability of two independent events is the product of the two individual probabilities.

Notice that the probability in Example 4 can also be found by adding the probabilities of each event.

## Probability of Disjoint Events

Key Concept
Words If two events, $A$ and $B$, are disjoint, then the probability that either $A$ or $B$ occurs is the sum of their probabilities.
Symbols $\quad P(A$ or $B)=P(A)+P(B)$

## Your Understanding

Example 1 A number cube is rolled, and the spinner is spun. Find (p. 492) each probability.

1. $P(5$ and E$)$
2. $P$ (2 and vowel)
3. $P$ (3 and a consonant)
4. $P$ (factor of 6 and $D)$

Example 2 (p. 493)

Example 3
(p. 494)

Example 4
Example 4
(p. 494)
5. CLOTHES Loretta has 2 pairs of black pants, 3 pairs of blue pants, and 1 pair of tan pants. She also has 4 white and 2 red shirts. If Loretta chooses a pair of pants and a shirt at random, what is the probability that she will choose a pair of black pants and a white shirt?
6. MARBLES A jar contains 12 marbles. Four are red, 3 are white, and 5 are blue. A marble is randomly selected, its color recorded, and then the marble is returned to the jar. A second marble is randomly selected. Find the probability that both marbles selected are blue.
7. The digits $0-9$ are each written on a slip of paper and placed in a hat. Two slips of paper are randomly selected, without replacing the first. What is the probability that the number 0 is drawn first and then a 7 is drawn?

A number cube is rolled. Find each probability.
8. $P(4$ or 5$)$
9. $P$ (3 or even number)
10. $P(1$ or multiple of 2$)$
11. $P(6$ or number less than 3$)$

## Practice and Problem Solving

## HOMEWORK HELP

| For <br> Exercises | See <br> Examples |
| :---: | :---: |
| $12-17$ | 1 |
| $18-19$ | 2 |
| $20-21$ | 3 |
| $22-25$ | 4 |

A coin is tossed, and a number cube is rolled. Find each probability. 12. $P$ (heads and 1 )
13. $P$ (tails and multiple of 3 )

A set of five cards is labeled 1-5. A second set of ten cards contains the following colors: 2 red, 3 purple, and 5 green. One card from each set is selected. Find each probability.
14. $P(5$ and green)
15. $P$ (odd and red)
16. $P$ (prime and purple)
17. $P$ (even and yellow)
18. MUSIC Denzel is listening to a CD that contains 12 songs. If he presses the random button on his CD player, what is the probability that the first two songs played will be the first two songs listed on the album?
19. JUICE POPS Lakita has two boxes of juice pops with an equal number of pops in each flavor. Find the probability of randomly selecting a grape juice pop from the first box and randomly selecting a juice pop from the second box that is not grape.

20. FRUIT Francesca randomly selects two pieces of fruit from a basket containing 8 oranges and 4 apples without replacing the first fruit. Find the probability that she selects two oranges.
21. SCHOOL The names of 24 students, of which 14 are girls and 10 are boys, in Mr. Santiago's science class are written on cards and placed in a jar. Mr. Santiago randomly selects two cards without replacing the first to determine which students will present their lab reports today. Find the probability that two boys are selected.

A day of the week is randomly selected. Find each probability.
22. P(Monday or Tuesday)
24. $P$ (a weekday or Saturday)
23. $P$ (a day beginning with T or Friday)
25. $P$ (Wednesday or a day with 6 letters)

A coin is tossed twice, and a letter is randomly picked from the word event. Find each probability.
26. $P$ (two heads and T )
27. $P$ (tails, not tails, consonant)
28. $P$ (heads, tails, not V)
29. $P$ (two tails and vowel)

FAMILY For Exercises 30-32, use the fact that the probability for a boy or a girl is each $\frac{1}{2}$.
30. Copy and complete the table that gives the probability that all the children in a family are boys given the number of children in the family.
31. Predict the probability that, in a family of ten children, all ten are boys.
32. Predict the probability that, in a family of $n$ children, all $n$ are boys.
33. LIGHTING Gene has set two of his lights on timers.

| Number of <br> Children | $\boldsymbol{P}$ (all boys) |
| :---: | :---: |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$ |
| 3 |  |
| 4 |  |
| 5 |  |

He always has at least one light on between the hours of 8:00 P.M. and 7:00 A.m. Light A is on $30 \%$ of the time, and Light B is on $70 \%$ of the time. What is the probability both lights are on at the same time?
34. RESEARCH The contiguous United States consists of all states excluding Alaska and Hawaii. If one of these contiguous states is chosen at random, what is the probability that it will end with the letter A or O? Write as a percent.

CHALLENGE For Exercises 35 and 36, use the spinner.
35. Use a tree diagram to construct the sample space of all the possible outcomes of three successive spins.
36. Suppose the spinner is designed so that for each spin there is a $40 \%$ probability of spinning red
 and a $20 \%$ chance of spinning blue. What is the probability of spinning two reds and then one blue?
37. WRITNGIN MATH A shelf has books A, B, and C on it. You pick a book at random, place it on a table, and then pick a second book. Explain why the probability that you picked books A and B is not $\frac{1}{9}$.

## ISTEP+ PRACTICE

### 7.4.5, 8.4.6

38. A jar contains 8 white marbles, 4 green marbles, and 2 purple marbles. If Darla picks one marble from the jar without looking, what is the probability that it will be either white or purple?
A $\frac{5}{7}$
C $\frac{2}{7}$
B $\frac{4}{7}$
D $\frac{1}{7}$
39. What is the probability of spinning a red, the number 1, and the letter A on the three spinners below?


F $\frac{1}{3}$
G $\frac{1}{32}$
H $\frac{1}{12}$
J $\frac{1}{64}$

## Spiral Review

40. PROBABILITY Ella is going to roll a number cube 30 times. How many times should she expect to roll a number greater than 2? (Lesson 9-7)

41. CHORES This weekend, Brennen needs to do laundry, mow the lawn, and clean his room. How many different ways can he do these three chores? (Lesson 9-6)

ALGEBRA Evaluate each expression if $a=6, b=-4$, and $c=-3$. (Lesson 1-6)
42. 9 C
43. $-8 a$
44. $2 b c$
45. $5 b^{2}$

## Problem Solving in Science

Math Genes It's time to complete your project. Use the information and data you have gathered about genetics and pet traits to prepare a poster. Be sure to include a chart displaying your data with your project.

## IN Math Online

## FOLDABLES

Study Organizer
Be sure the following Big Ideas are noted in your Foldable.

BIC ICeas


Probability
(Lesson 9-1)

- The probability of a simple event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

Fundamental Counting Principle (Lesson 9-3)

- If event $M$ has $m$ possible outcomes and is followed by event $N$ that has $n$ possible outcomes, then the event $M$ followed by $N$ has $m \times n$ possible outcomes.


## Theoretical and Experimental <br> Probability (Lesson 9-7)

- Theoretical probability is based on what should happen when conducting a probability experiment.
- Experimental probability is based on what actually occurred during a probability experiment.

Independent Events (Lesson 9-8)

- The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

Dependent Events (Lesson 9-8)

- If two events, $A$ and $B$, are dependent, then the probability of both events occurring is the product of the probability of $A$ and the probability of $B$ after $A$ occurs.


## Disjoint Events (Lesson 9-8)

- If two events are disjoint, then the probability that either event will occur is the sum of their individual probabilities.
- Key Vocabulary
combination (p. 480)
complementary events (p. 462)
compound event (p. 492)
dependent events (p. 493)
disjoint events (p. 494)
experimental probability
(p. 486)

Fundamental Counting
Principle (p. 471)
independent events (p. 492) outcome (p. 460)
permutation (p. 475)
probability (p. 460)
random (p. 461)
sample space (p. 465)
simple event (p. 460)
theoretical probability
(p. 486)
tree diagram (p. 466)

## Vocabulary Check

State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. Compound events consist of two or more simple events.
2. A random outcome is an outcome that occurs by chance.
3. $P(\operatorname{not} A)$ is read the permutation of the complement of $A$.
4. The Fundamental Counting Principle counts the number of possible outcomes using the operation of addition.
5. Events in which the outcome of the first event does not affect the outcome of the other event(s) are simple events.
6. The sample space of an event is the set of outcomes not included in the event.
7. Events that cannot occur at the same time are called dependent events.

## Lesson-by-Lesson Review

## 9-1 Simple Events (pp. 460-464)


7.4.5,
8.4.7

A bag of animal crackers contains 5 monkeys, 4 giraffes, 6 elephants, and 3 tigers. Suppose you draw a cracker at random. Find the probability of each event. Write as a fraction in simplest form.
8. $P$ (monkey)
9. $P$ (tiger)
10. $P$ (giraffe or elephant)
11. P(not monkey)
12. $P$ (monkey, giraffe, or elephant)
13. ARRIVALS The probability that a plane will arrive at the airport on time is $\frac{23}{25}$. Find the probability that the plane will not arrive on time. Write as a percent.

Example 1 What is the probability of rolling a number less than 3 on a number cube?
$P(1$ or 2$)=\frac{\text { numbers less than } 3}{\text { total number of possible outcomes }}$ $=\frac{2}{6}$ Two numbers are less than 3 .
$=\frac{1}{3}$ Simplify.
Therefore, $P(1$ or 2$)=\frac{1}{3}$.

## 9-2

7.4.5, 8.4.7

Sample Spaces (pp. 465-470)
For each situation, find the sample space using a table or tree diagram.
14. rolling a number cube and tossing a coin
15. choosing from pepperoni, mushroom, or cheese pizza and water, juice, or milk
16. GAMES Eliza and Zeke are playing a game in which Zeke spins the spinner shown and rolls a number cube. If the sum of the numbers is less than six, Eliza wins. Otherwise Zeke wins. Find the sample space. Then find the probability that Zeke wins.


Example 2 Ginger and Micah are playing a game in which a coin is tossed twice. If heads comes up exactly once, Ginger wins. Otherwise, Micah wins. Find the sample space. Then find the probability that Ginger wins.
Make a tree diagram.

| First | Second | Sample |
| :--- | :---: | :---: |
| Toss | Toss | Space |



There are four equally likely outcomes with 2 favoring each player. The probability that Ginger wins is $\frac{2}{4}$ or $\frac{1}{2}$.

## 0 <br> Study Guide and Review

## $9-3$

The Fundamental Counting Principle (pp. 471-474)

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.
17. rolling two number cubes
18. creating an outfit from 6 different shirts and 4 different pants
19. TUXEDOS A tuxedo shop offers a tuxedo in three colors, black, gray, and white. The tie can be a bow tie or a regular tie. The tuxedo can come with tails or no tails. If a tuxedo is selected at random, what is the probability that it will be black, with a bow tie, and no tails?

Example 3 Use the Fundamental Counting Principle to find the total number of outcomes for a coin that is tossed four times.
There are 2 possible outcomes, heads or tails, each time a coin is tossed. For a coin that is tossed four times, there are $2 \cdot 2 \cdot 2 \cdot 2$, or 16 outcomes.

Example 4 Find the probability that, in a family of four children, all four children are girls.
There are 16 outcomes. There is one possible outcome resulting in four girls. So, the probability that all four children are girls is $\frac{1}{16}$.

Permutations (pp. 475-478)
20. PORTRAITS In how many ways can a family of five arrange themselves in a line for a family portrait?
21. LETTERS How many permutations are there of the letters in the word computer?
22. RUNNING Jacinda and Raul are entered in a race with 5 other runners. If each runner is equally likely to win, what is the probability that Jacinda will finish first and Raul will finish second?

Example 5 Nathaniel needs to choose two of the chores shown to do after school. If he is equally likely to choose the chores, what is the probability that he will walk the dog first and rake the leaves second?
There are $4 \cdot 3$, or 12 , arrangements in which Nathaniel can complete the chores. There is one way in which he will walk the dog first and rake the leaves second. So, the probability that he will walk the dog first and rake the leaves second is $\frac{1}{12}$.

9-5

Combinations (pp. 480-483)
23. DVD In how many ways can Toni select 2 DVDs from the 15 in her collection?
24. SPORTS How many ways can a coach select 3 players from a roster of 9 ?
25. GAMES Marcus has enough money on his gift card to purchase 3 new games from the 14 displayed in the New Release section. In how many ways can Marcus select 3 different games?
26. QUIZ Frances must answer 3 of the 5 questions on a quiz, numbered 1-5. What is the probability that Frances will answer questions 2, 3, and 4?

Example 6 Caitlin and Román are playing a game in which Román chooses four different numbers from 1-15. What is the probability that Caitlin will guess all four numbers correctly?
There are $15 \cdot 14 \cdot 13 \cdot 12$ permutations of four numbers chosen from 15 numbers. There are $4 \cdot 3 \cdot 2 \cdot 1$ ways to arrange the 4 numbers.
$\frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1}=\frac{32,760}{24}$ or 1,365
There are 1,365 ways to choose four numbers from 15 numbers. There is one way to guess all four numbers correctly, so the probability that Caitlin will guess all four numbers correctly is $\frac{1}{1,365}$.

## 9-6 <br> PSI: Act It Out (pp. 484-485)

P.1.3, P.5. 1

Solve each problem. Use the act it out strategy.
27. QUIZ Determine whether tossing a coin is a good way to answer a 6-question true-false quiz. Justify your answer.
28. FAMILY PORTRAIT In how many ways can the Maxwell family pose for a portrait if Mr. and Mrs. Maxwell are sitting in the middle and their three children are standing behind them?
29. AMUSEMENT PARK In how many ways can 4 friends be seated in 2 rows of 2 seats each on a roller coaster if Judy and Harold must ride together?

Example 7 In how many ways can three females and two males sit in a row of five seats at a concert if the females must sit in the first three seats?

Place five desks or chairs in a row. Have three females and two males sit in any of the seats as long as the females sit in the first three seats. Continue rearranging until you find all the possibilities. Record the results.
$\mathrm{F}_{1} \mathrm{~F}_{2} \quad \mathrm{~F}_{3} \quad \mathrm{M}_{1} \quad \mathrm{M}_{2} \quad \mathrm{~F}_{2} \quad \mathrm{~F}_{3} \mathrm{~F}_{1} \mathrm{M}_{1} \mathrm{M}_{2}$
$F_{1} \quad F_{2} \quad F_{3} \quad M_{2} \quad M_{1} \quad F_{2} \quad F_{3} \quad F_{1} \quad M_{2} \quad M_{1}$
$\mathrm{F}_{1} \mathrm{~F}_{3} \quad \mathrm{~F}_{2} \mathrm{M}_{1} \quad \mathrm{M}_{2} \quad \mathrm{~F}_{3} \quad \mathrm{~F}_{2} \mathrm{~F}_{1} \mathrm{M}_{1} \quad \mathrm{M}_{2}$ $\mathrm{F}_{1} \mathrm{~F}_{3} \mathrm{~F}_{2} \mathrm{M}_{2} \quad \mathrm{M}_{1} \quad \mathrm{~F}_{3} \quad \mathrm{~F}_{2} \mathrm{~F}_{1} \mathrm{M}_{2} \quad \mathrm{M}_{1}$



There are 12 possible arrangements.

## 0 Study Guide and Review

9-7 Theoretical and Experimental Probability (pp. 486-490)

The student council surveyed their classmates to find what they want to eat for their end-of-year celebration.
The results are shown

| Meal | Number of <br> Students |
| :--- | :---: |
| pizza | 26 |
| chicken | 20 |
| hamburger | 13 |
| hot dog | 5 | in the table. Find the experimental probability of each event.

30. $P($ hot dog $)$
31. P(chicken)
32. $P$ (hamburger)
33. P(hot dog or hamburger)
34. $P$ (not pizza) 35. $P$ (pizza or chicken)
35. PROBABILITY If a spinner has four equal sections labeled $1-4$, what is the theoretical probability of landing on 2 ?

Example 8 A coin is tossed 65 times, and it lands on tails 40 times. What is the experimental probability of the coin landing on heads?
The coin landed on heads 25 times.

$$
\begin{aligned}
P(\text { heads }) & =\frac{\text { number of times heads occurs }}{\text { total number of possible outcomes }} \\
& =\frac{25}{65} \text { or } \frac{5}{13}
\end{aligned}
$$

So, the experimental probability of the coin landing on heads is $\frac{5}{13}$ or about $38 \%$.

Compound Events (pp. 492-497)
A bag contains 6 green, 8 white, and 2 blue counters. Two are randomly drawn. Find each probability if the first counter is replaced before the second counter is drawn. Then find each probability if the first counter is not replaced.
37. $P$ (green, blue)
38. $P$ (2 white)
39. $P$ (blue, not white)
40. $P$ (white, not green)
41. PROBABILITY A coin is tossed and a number cube is rolled. Find the probability that tails and a number less than 5 comes up.
42. COMPUTERS A computer randomly generates a digit from 0-9. Find the probability that an odd number or the number 8 is generated.

Example 9 A number cube is rolled and a spinner with 8 equal sections labeled $A$ through H is spun. Find the probability of rolling an even number and spinning a vowel.
$P($ even $)=\frac{3}{6}$
$P($ vowel $)=\frac{2}{8}$
$\frac{3}{6} \cdot \frac{2}{8}=\frac{6}{48}$ or $\frac{1}{8}$
So, the probability of rolling an even number and spinning a vowel is $\frac{1}{8}$, or about 12.5\%.

The spinner shown has an equal chance of landing on each number. Find each probability.

1. $P$ (odd number)
2. $P(1$ or 7$)$
3. $P$ (not a prime number)
4. $P$ (number greater than 1 )

For each situation, use a table or a tree diagram to find the sample space.
5. A coin is tossed two times.

11. DOGS There are 40 dogs entered in a dog show. How many ways can a first place and a second place ribbon be awarded?
12. MULTIPLE CHOICE There is a choice of 8 toppings for a pizza. Which of the following gives the number of ways you can order a pizza with 3 different toppings?
F 336
H 56
G 58
J 24
13. CAMPING Four campers are chosen from nine to pitch the tents. If Sandy, Jarrod, Dyami, and Clara are among the nine campers, find the probability that they are chosen.
14. SCHOOL Determine whether spinning a spinner with five equal sections would be a good way to answer a 5-question multiplechoice quiz. Justify your answer.
15. PROBABILITY A spinner is spun 60 times. The results are shown in the table. What is the experimental probability that the spinner lands on section C? Write as a percent.

| Section | Number <br> of Times |
| :---: | :---: |
| A | 12 |
| B | 17 |
| C | 12 |
| D | 4 |
| E | 15 |

juice boxes, 12 orange juice boxes, and 4 apple juice boxes. If a juice box is selected at random, what is the probability that it will be grape?
A $50 \%$
C $0.1 \overline{6}$
B $\frac{1}{3}$
D $\frac{5}{6}$
 $A$ and $B$. If a number less than 4 and a consonant comes up, Lucy wins. Otherwise Randall wins. Find the sample space. Then find the probability that Lucy wins.

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.
8. A 4-digit security code is chosen.
9. A number cube is rolled five times.

10. MULTIPLE CHOICE A cooler contains 8 grape
16. PRIZES Josh is choosing from two prize bags that each contain 5 packs of baseball cards, 11 packages of putty, and 9 hats. What is the probability that Josh randomly picks a hat from the first bag and a pack of baseball cards from the second bag?

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## PART 1 Multiple Choice

Read each question. Then fill in the correct answer on the answer sheet provided by your teacher or on a sheet of paper.

1. Jessica played a game where she spun each of the spinners shown below once. If she spins an even number on Spinner 1, red or yellow on Spinner 2, and a B on Spinner 3, how many possible unique combinations are there?


Spinner 1


Spinner 2


Spinner 3
A 4
C 10
B 8
D 16
2. Mr. Campos bought 40 pencils priced at 8 for $\$ 0.99$ and 3 dozen notebooks priced at 4 for $\$ 2.49$. Find the total amount, not including tax, Mr. Campos spent on pencils and notebooks.
F \$28.86
H $\$ 17.88$
G $\$ 27.36$
J \$15.96
3. In a movie theater there are 168 seats. If $75 \%$ of the theater is filled, how many people are sitting in the movie theater?
A 156
C 134
B 148
D 126
4. Mr. Blackwell gave his math students a pop quiz. The students' scores are listed below. What is the median quiz score?

$$
\begin{array}{|lllllll}
23 & 19 & 18 & 12 & 21 & 24 & 25
\end{array}
$$

F 12
H 24
G 21
J 25
5. The results of an election for student body president showed that Trey received 250 votes, Marta 100 votes, and Ed 50 votes. Which of the following correctly displays the election results?

A
Election Results


B
Election Results

C

D
Election Results

6. What is $4 \div \frac{1}{3}$ ?

$$
\text { F } \frac{1}{12}
$$

$$
\text { H } 7
$$

$$
\text { G } \frac{4}{3}
$$12

7. Hanako spins each spinner shown below once. Find the total possible letter/number combinations that could have resulted from Hanako's spins.


First Spinner
A 3
B 6


Second Spinner
C 9
D 12
8. Juan rolled a number cube four times. Each time, the number 3 appeared. If Juan rolls the number cube one more time, what is the probability that the number 3 will appear?
F $\frac{2}{3}$
H $\frac{1}{6}$
G $\frac{1}{2}$
J $\frac{5}{6}$

## TEST-TAKING TIP

Question 8 You may want to find your own answer before looking at the answer choices. This keeps you from choosing an answer that looks correct, but is still wrong.
9. The owner of a fruit stand has $x$ pounds of apples on display. She sells 30 pounds and then adds $4 y$ pounds of apples to the display. Which of the following expressions represents the weight in pounds of the apples that are now on the display?
A $x+30+4 y$
B $x-30+4 y$
C $x+30-4 y$
D $x-30-4 y$

## PART 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.
10. A building is 182 meters tall. About how tall is the building in feet and inches? (1 meter $\approx 39$ inches)
11. Wilson has 7 different pieces of fruit in his refrigerator. If he randomly selects 3 pieces of fruit, how many possible unique combinations are there?
12. In how many ways can you select 3 flowers out of 9 ?

## PART 3 Extended Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper. Show your work.
13. Audrey is going on vacation. She packs 4 shirts and 3 pairs of pants. The shirts are red, green, yellow, and pink. The pants are black, white, and brown.
a. Make a tree diagram that shows all of the outfits that Audrey can make.
b. If Audrey chooses a shirt and pair of pants at random, what is the probability that she will wear a red shirt with white pants on the first day of her vacation?
c. Audrey does not repeat outfits. What is the probability that she will wear a yellow shirt with black pants on the first day and a green shirt with brown pants on the second day?

## NEED EXTRA HELP?

| If You Missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Go to Lesson... | $9-2$ | $1-1$ | $7-1$ | $8-2$ | $8-4$ | $5-7$ | $9-2$ | $9-8$ | $3-1$ | $6-8$ | $9-2$ | $9-5$ | $9-8$ |
| IN Academic Standards | 7.4 .5 | 7.2 .1 | 7.1 .8 | 7.4 .3 | 7.4 .1 | 7.1 .7 | 7.4 .5 | 7.4 .5 | 7.2 .1 | 7.3 .5 | 7.4 .5 | 7.4 .5 | 7.4 .5 |

