

# Unit 5

## Geometry and Measurement

### FOCUS

Use formulas to determine surface areas and volumes of three-dimensional shapes.

### CHAPTER 10 Geometry: Polygons

**BIG Idea** Identify and describe properties of two-dimensional figures.

### CHAPTER 11 Measurement: Two- and Three-Dimensional Figures

**BIG Idea** Use formulas to determine area and volume of two- and three-dimensional figures.

**BIG Idea** Derive and determine the area of a circle.

### CHAPTER 12 Looking Ahead to Next Year: Geometry and Measurement

**BIG Idea** Find length using the Pythagorean Theorem.

**BIG Idea** Find surface areas of rectangular prisms and cylinders.





## Problem Solving in Life Skills



### Real-World Unit Project

**Design That House** Are you ready to become an architect? You've been selected to design and decorate a dream house. Along the way, you'll determine actual angle measures and wall lengths of the blueprint you'll be creating of your dream house. Your journey will begin soon, so pack your geometry tool kit.

**IN Math Online**

Log on to [glencoe.com](http://glencoe.com) to begin.



# CHAPTER

# 10

# Geometry: Polygons

## Indiana Academic Standards

**7.3.2** Identify, describe, and use transformations (translations, rotations, and reflections and simple compositions of these transformations) to solve problems.

**7.3.5** Identify, describe and construct similarity relationships and solve problems involving similarity (including similar triangles) and scale drawings by using proportional reasoning.

## Key Vocabulary

**line of symmetry** (p. 558)

**similar figures** (p. 540)

## Real-World Link

**Tulips** Holland, Michigan, hosts a Tulip Time Festival each May. Geometry is used to explain how a tulip shows rotational symmetry.

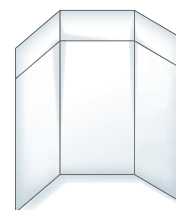
## FOLDABLES<sup>®</sup> Study Organizer

**Geometry: Polygons** Make this Foldable to help you organize your notes. Begin with a piece of 11" by 17" paper.

- 1** Fold a 2" tab along the long side of the paper.



- 2** Unfold the paper and fold in thirds widthwise.



- 3** Open and draw lines along the folds. Label the head of each column as shown. Label the front of the folded table with the chapter title.

What I Know About Polygons	What I Need to Know	What I've Learned

# GET READY for Chapter 10

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Option 2

**IN Math Online**

Take the Online Readiness Quiz at [glencoe.com](http://glencoe.com).

## Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

### QUICK Quiz

Multiply or divide. Round to the nearest hundredth if necessary.

(Prior Grade)

- $360 \times 0.85$
- $48 \div 191$
- $24 \div 156$
- $0.37 \times 360$
- $33 \div 307$
- $0.69 \times 360$

Solve each equation. (Lesson 3-2)

- $122 + x + 14 = 180$
- $45 + 139 + k + 17 = 360$
- SCHOOL** There are 180 school days at Lee Middle School. If school has been in session for 62 days and there are 13 days until winter break, how many school days are after the break? (Lesson 3-2)

Solve each proportion. (Lesson 6-5)

- $\frac{4}{a} = \frac{3}{9}$
- $\frac{5}{8} = \frac{15}{y}$
- $\frac{7}{16} = \frac{h}{32}$
- $\frac{t}{42} = \frac{6}{7}$
- READING** Sandra can read 28 pages of a novel in 45 minutes. At this rate, how many pages can she read in 135 minutes? (Lesson 6-6)

### QUICK Review

#### Example 1

Find  $0.92 \times 360$ .

$$\begin{array}{r} 360 \\ \times 0.92 \leftarrow \text{two decimal places} \\ \hline 720 \\ + 32400 \\ \hline 331.20 \leftarrow \text{two decimal places} \end{array}$$

So,  $0.92 \times 360 = 331.2$ .

#### Example 2

Solve the equation.

$$46 + 90 + p = 180.$$

$$\begin{array}{r} 46 + 90 + p = 180 \quad \text{Write the equation.} \\ 136 + p = 180 \quad \text{Add 46 and 90.} \\ \underline{-136} \quad \underline{-136} \quad \text{Subtract 136 from} \\ p = 44 \quad \text{each side.} \end{array}$$

The solution to the equation  $46 + 90 + p = 180$  is  $p = 44$ .

#### Example 3

Solve the proportion  $\frac{3}{8} = \frac{g}{48}$ .

$$\frac{3}{8} = \frac{g}{48} \quad \text{Write a proportion.}$$
$$\frac{3}{8} = \frac{18}{48} \quad \text{Since } 8 \times 6 = 48, \text{ multiply 3 by 6 to find } g.$$

So,  $g = 18$ .

# 10-1

# Angle Relationships

## MAIN IDEA

Classify angles and identify vertical and adjacent angles.

## IN Academic Standards

**Reinforcement of 6.3.1** Identify, draw and use the properties of vertical, adjacent, complementary, and supplementary angles, and properties of triangles and quadrilaterals to solve problems involving a missing angle. Also addresses P.4.1.

## New Vocabulary

angle  
degrees  
vertex  
congruent angles  
right angle  
acute angle  
obtuse angle  
straight angle  
vertical angles  
adjacent angles

## IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Reading in the Content Area

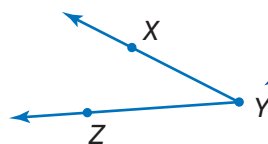
## ▶ GET READY for the Lesson

**ROLLER COASTERS** The angles of descent of a roller coaster are shown.

1. The roller coaster at the right shows two angles of descent. Draw an angle between  $44^\circ$  and  $70^\circ$ .
2. Some roller coasters have an angle of descent that is  $90^\circ$ , known as a vertical angle of descent. Draw a vertical angle of descent.



An **angle** has two sides that share a common endpoint and is measured in units called **degrees**. If a circle were divided into 360 equal-sized parts, each part would have an angle measure of 1 degree ( $1^\circ$ ).



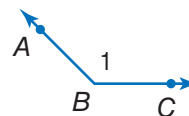
A **vertex** is the point where the sides meet.

An angle can be named in several ways. The symbol for angle is  $\angle$ .

## EXAMPLE Naming Angles

**1** Name the angle at the right.

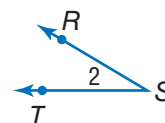
- Use the vertex as the middle letter and a point from each side.  
 $\angle ABC$  or  $\angle CBA$
- Use the vertex only.  
 $\angle B$
- Use a number.  
 $\angle 1$



The angle can be named in four ways:  $\angle ABC$ ,  $\angle CBA$ ,  $\angle B$ , or  $\angle 1$ .

## ✓ CHECK Your Progress

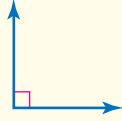
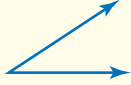


- a. Name the angle shown in four ways.





Angles are classified according to their measure. Two angles that have the same measure are said to be **congruent**.

**Study Tip**  
**Right Angles** The square corner indicates a right angle.

Types of Angles				Key Concept
<b>right angle</b>	<b>acute angle</b>	<b>obtuse angle</b>	<b>straight angle</b>	
				
exactly $90^\circ$	less than $90^\circ$	between $90^\circ$ and $180^\circ$	exactly $180^\circ$	

**EXAMPLES** Classify Angles

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.



The angle is less than  $90^\circ$ , so it is an acute angle.

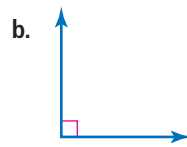


The angle is between  $90^\circ$  and  $180^\circ$ , so it is an obtuse angle.

**Study Tip**  
**Classifying Angles** You do not need to use a protractor to measure an angle in order to classify it as acute, obtuse, right, or straight.

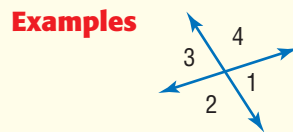
**CHECK Your Progress**

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.



**Vertical Angles** Key Concept

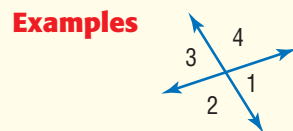
**Words** Two angles are **vertical** if they are opposite angles formed by the intersection of two lines.



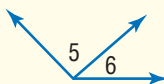
$\angle 1$  and  $\angle 3$  are vertical angles.  
 $\angle 2$  and  $\angle 4$  are vertical angles.

**Adjacent Angles**

**Words** Two angles are **adjacent** if they share a common vertex, a common side, and do not overlap.



Adjacent angle pairs are  $\angle 1$  and  $\angle 2$ ,  $\angle 2$  and  $\angle 3$ ,  $\angle 3$  and  $\angle 4$ , and  $\angle 4$  and  $\angle 1$ .



$\angle 5$  and  $\angle 6$  are adjacent angles.



## Real-World EXAMPLE

**4 INTERSECTIONS** Identify a pair of vertical angles in the diagram at the right. Justify your response.

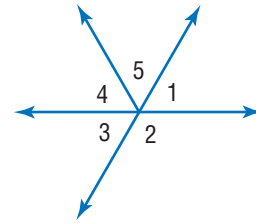
Since  $\angle 2$  and  $\angle 4$  are opposite angles formed by the intersection of two lines, they are vertical angles. Similarly,  $\angle 1$  and  $\angle 3$  are also vertical angles.



### ✓ CHECK Your Progress

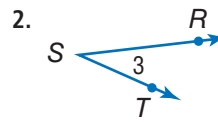
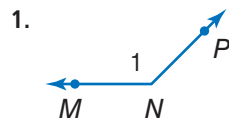
Refer to the diagram at the right. Identify each of the following. Justify your response.

- e. a pair of vertical angles
- f. a pair of adjacent angles



## ✓ CHECK Your Understanding

**Examples 1–3** Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*. (pp. 510–511)



**Example 4** 3. **RAILROADS** Identify a pair of vertical angles on the railroad crossing sign. Justify your response. (p. 512)

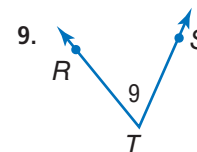
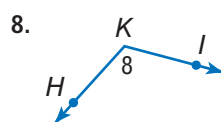
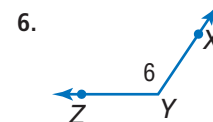
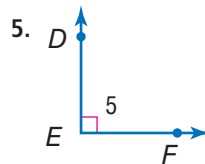
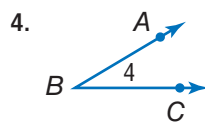


## ▶ Practice and Problem Solving

### HOMework HELP

For Exercises	See Examples
4–9	1–3
10–17	4

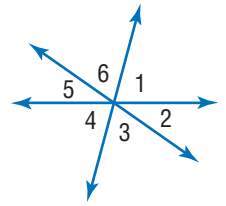
Name each angle in four ways. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.





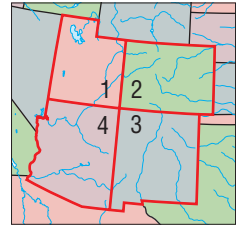
For Exercises 10–15, refer to the diagram at the right. Identify each angle pair as *adjacent*, *vertical*, or *neither*.

10.  $\angle 2$  and  $\angle 5$     11.  $\angle 4$  and  $\angle 6$     12.  $\angle 3$  and  $\angle 4$   
 13.  $\angle 5$  and  $\angle 6$     14.  $\angle 1$  and  $\angle 3$     15.  $\angle 1$  and  $\angle 4$



**GEOGRAPHY** For Exercises 16 and 17, use the diagram at the right and the following information.

The corner where the states of Utah, Arizona, New Mexico, and Colorado meet is called the Four Corners.



Academic Standards • ISTEP+

Extra Practice, pp. 693, 713

16. Identify a pair of vertical angles. Justify your response.  
 17. Identify a pair of adjacent angles. Justify your response.

**H.O.T. Problems**

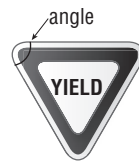
**CHALLENGE** For Exercises 18 and 19, determine whether each statement is *true* or *false*. If the statement is true, provide a diagram to support it. If the statement is false, explain why.

18. A pair of obtuse angles can also be vertical angles.  
 19. A pair of straight angles can also be adjacent angles.  
 20. **WRITING IN MATH** Describe the differences between vertical and adjacent angles.

**ISTEP+ PRACTICE**

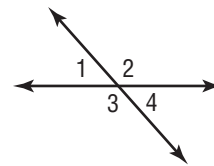
Reinforcement of 6.3.1

21. Which word best describes the angle marked in the figure?



- A acute  
 B obtuse  
 C right  
 D straight

22. Which statement is true?



- F  $\angle 1$  and  $\angle 4$  are adjacent angles.  
 G  $\angle 2$  and  $\angle 3$  are vertical angles.  
 H  $\angle 3$  and  $\angle 4$  are vertical angles.  
 J  $\angle 2$  and  $\angle 3$  are adjacent angles.

**Spiral Review**

A coin is tossed twice and a number cube is rolled. Find each probability. (Lesson 9-8)

23.  $P(2 \text{ heads and } 6)$     24.  $P(1 \text{ head, } 1 \text{ tail, and a } 3)$     25.  $P(2 \text{ tails and not } 4)$   
 26. **PROBABILITY** A spinner is spun 20 times, and it lands on the color red 5 times. What is the experimental probability of *not* landing on red? (Lesson 9-7)

**GET READY for the Next Lesson**

**ALGEBRA** Solve each equation. Check your solution. (Lesson 3-2)

27.  $44 + x = 90$     28.  $117 + x = 180$     29.  $90 = 36 + x$     30.  $180 = 75 + x$



# 10-2

## Complementary and Supplementary Angles

### MAIN IDEA

Identify complementary and supplementary angles and find missing angle measures.

### IN Academic Standards

**Reinforcement of 6.3.1** Identify, draw and use the properties of vertical, adjacent, complementary, and supplementary angles, and properties of triangles and quadrilaterals to solve problems involving a missing angle. Also addresses P.4.1.

### New Vocabulary

**complementary angles**  
**supplementary angles**

### IN Math Online

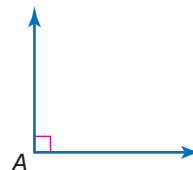
[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

### MINI Lab

**GEOMETRY** Refer to  $\angle A$  shown at the right.

1. Classify it as *acute*, *right*, *obtuse*, or *straight*.
2. Copy the angle onto a piece of paper. Then draw a ray that separates the angle into two congruent angles. Label these angles  $\angle 1$  and  $\angle 2$ .
3. What is  $m\angle 1$  and  $m\angle 2$ ?
4. What is the sum of  $m\angle 1$  and  $m\angle 2$ ?
5. Copy the original angle onto a piece of paper. Then draw a ray that separates the angle into two non-congruent angles. Label these angles  $\angle 3$  and  $\angle 4$ .
6. What is true about the sum of  $m\angle 3$  and  $m\angle 4$ ?
7. Complete Exercises 1–6 for  $\angle B$  shown at the right.



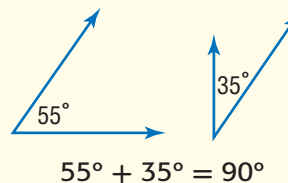
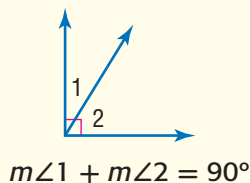
A special relationship exists between two angles whose sum is  $90^\circ$ .  
A special relationship also exists between two angles whose sum is  $180^\circ$ .

### Complementary Angles

#### Key Concept

**Words** Two angles are **complementary** if the sum of their measures is  $90^\circ$ .

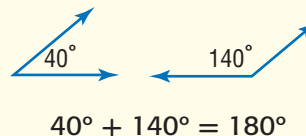
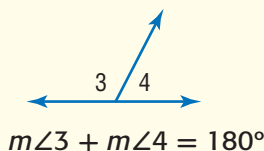
#### Examples



### Supplementary Angles

**Words** Two angles are **supplementary** if the sum of their measures is  $180^\circ$ .

#### Examples



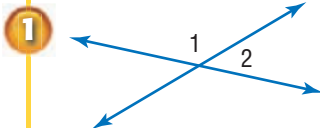
### Reading Math

**Angle Measure** The notation  $m\angle 1$  is read *the measure of angle 1*.

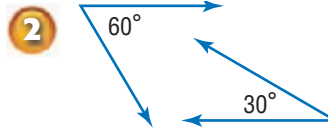
You can use these relationships to identify complementary and supplementary angles.

## EXAMPLES Identify Angles

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.



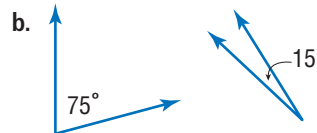
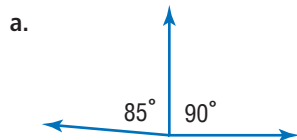
$\angle 1$  and  $\angle 2$  form a straight angle. So, the angles are supplementary.



$60^\circ + 30^\circ = 90^\circ$   
The angles are complementary.

### CHECK Your Progress

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.



You can use angle relationships to find missing measures.

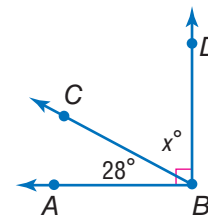
### Reading Math

**Perpendicular** Lines or sides that meet to form right angles are perpendicular.

## EXAMPLE Find a Missing Angle Measure

3 **ALGEBRA** Find the value of  $x$ .

Since the two angles form a right angle, they are complementary.



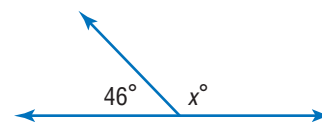
<b>Words</b>	The sum of the measures of $\angle ABC$ and $\angle CBD$	is	$90^\circ$ .
<b>Variable</b>	Let $x$ represent the measure of $\angle CBD$ .		
<b>Equation</b>	$28 + x$	=	90

$$\begin{array}{r} 28 + x = 90 \quad \text{Write the equation.} \\ - 28 \quad - 28 \quad \text{Subtract 28 from each side.} \\ \hline x = 62 \end{array}$$

So, the value of  $x$  is 62.

### CHECK Your Progress

- c. **ALGEBRA** Find the value of  $x$ .
- d. **ALGEBRA** If  $\angle J$  and  $\angle K$  are complementary and the measure of  $\angle K$  is  $65^\circ$ , what is the measure of  $\angle J$ ?

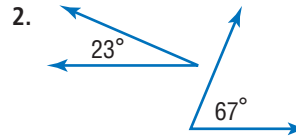
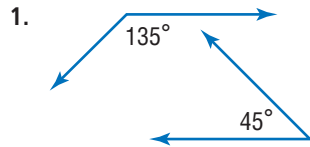




# CHECK Your Understanding

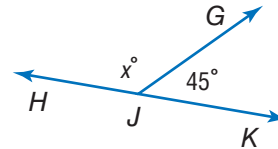
**Examples 1, 2**  
(p. 515)

Identify each pair of angles as *complementary*, *supplementary*, or *neither*.



**Example 3**  
(p. 515)

3. **ALGEBRA** Find the value of  $x$ .

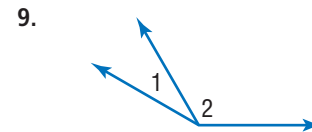
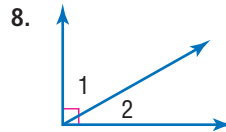
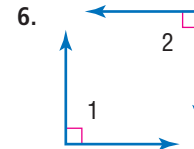
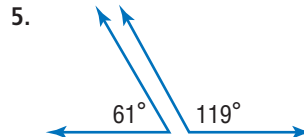
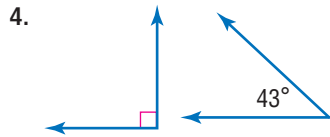


## Practice and Problem Solving

### HOMEWORK HELP

For Exercises	See Examples
4–9	1, 2
10–11	3

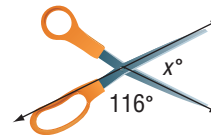
Identify each pair of angles as *complementary*, *supplementary*, or *neither*.



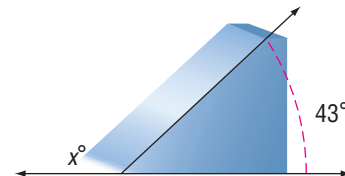
10. **ALGEBRA** If  $\angle A$  and  $\angle B$  are complementary and the measure of  $\angle B$  is  $67^\circ$ , what is the measure of  $\angle A$ ?

11. **ALGEBRA** What is the measure of  $\angle J$  if  $\angle J$  and  $\angle K$  are supplementary and the measure of  $\angle K$  is  $115^\circ$ ?

12. **SCHOOL SUPPLIES** What is the measure of the angle given by the opening of the scissors,  $x$ ?

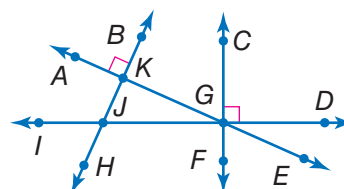


13. **SKATEBOARDING** A skateboard ramp forms a  $43^\circ$  angle as shown. Find the unknown angle.



Use the figure at the right to name the following.

- a pair of supplementary angles
- a pair of complementary angles
- a pair of vertical angles



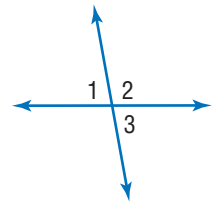
Academic Standards • ISTEP+

Extra Practice, pp. 693, 713



**GEOMETRY** For Exercises 17–20, use the figure at the right.

17. Are  $\angle 1$  and  $\angle 2$  vertical angles, adjacent angles, or neither?  $\angle 2$  and  $\angle 3$ ?  $\angle 1$  and  $\angle 3$ ?
18. Write an equation representing the sum of  $m\angle 1$  and  $m\angle 2$ . Then write an equation representing the sum of  $m\angle 2$  and  $m\angle 3$ .
19. Solve the equations you wrote in Exercise 18 for  $m\angle 1$  and  $m\angle 3$ , respectively. What do you notice?
20. **MAKE A CONJECTURE** Use your answer to Exercise 19 to make a conjecture as to the relationship between vertical angles.

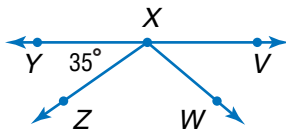


**H.O.T. Problems**

21. **CHALLENGE** Angles  $E$  and  $F$  are complementary. If  $m\angle E = x - 10$  and  $m\angle F = x + 2$ , find the measure of each angle.
22. **WRITING IN MATH** Describe a strategy for determining whether two angles are *complementary*, *supplementary*, or *neither* without knowing or measuring each angle using a protractor.

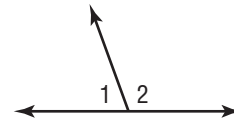
**ISTEP+ PRACTICE** Reinforcement of 6.3.1

23. In the figure below,  $m\angle YXZ = 35^\circ$  and  $m\angle WXV = 40^\circ$ . What is  $m\angle ZXW$ ?



- A  $180^\circ$       C  $75^\circ$   
 B  $105^\circ$       D  $15^\circ$

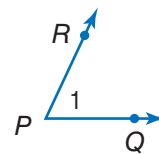
24. Which is a true statement about angles 1 and 2 shown below?



- F  $\angle 1$  is complementary to  $\angle 2$ .  
 G  $\angle 1$  and  $\angle 2$  are vertical angles.  
 H  $\angle 1$  is supplementary to  $\angle 2$ .  
 J Both angles are obtuse.

**Spiral Review**

25. Name the angle at the right in four ways. Then classify it as *acute*, *right*, *obtuse*, or *straight*. (Lesson 10-1)
26. **MEASUREMENT** A house for sale has a rectangular lot with a length of 250 feet and a width of 120 feet. What is the area of the lot? (Lesson 3-6)



**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Multiply or divide. Round to the nearest hundredth if necessary. (pp. 674 and 676)

27.  $0.62 \cdot 360$       28.  $360 \cdot 0.25$       29.  $17 \div 146$       30.  $63 \div 199$

# 10-3

## Statistics: Display Data in a Circle Graph

### MAIN IDEA

Construct and interpret circle graphs.

### IN Academic Standards

**7.4.1 Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs. Justify the choice of data display.**

### New Vocabulary

**circle graph**

### IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

### ▶ GET READY for the Lesson

**VEGETABLES** The students at Pine Ridge Middle School were asked to identify their favorite vegetable. The table shows the results of the survey.

Favorite Vegetable	
Vegetable	Percent
Carrots	45%
Green Beans	23%
Peas	17%
Other	15%

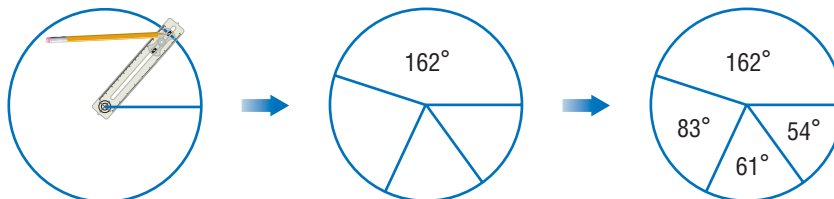
1. Explain how you know that each student only selected one favorite vegetable.
2. If 400 students participated in the survey, how many students preferred carrots?

A graph that shows data as parts of a whole is called a **circle graph**. In a circle graph, the percents add up to 100.

### EXAMPLE Display Data in a Circle Graph

**1 VEGETABLES** Display the data above in a circle graph.

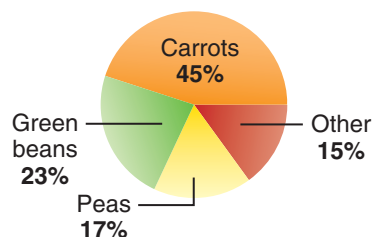
- There are  $360^\circ$  in a circle. Find the degrees for each part.
  - $45\%$  of  $360^\circ = 0.45 \cdot 360^\circ$  or  $162^\circ$
  - $23\%$  of  $360^\circ = 0.23 \cdot 360^\circ$  or  $83^\circ$  Round to the nearest whole degree.
  - $17\%$  of  $360^\circ = 0.17 \cdot 360^\circ$  or about  $61^\circ$
  - $15\%$  of  $360^\circ = 0.15 \cdot 360^\circ$  or about  $54^\circ$
- Draw a circle with a radius as shown. Then use a protractor to draw the first angle, in this case  $162^\circ$ . Repeat this step for each section or *sector*.



- Label each section of the graph with the category and percent it represents. Give the graph a title.

**Check** The sum of the angle measures should equal to  $360^\circ$ .  
 $162^\circ + 83^\circ + 61^\circ + 54^\circ = 360^\circ$

**Favorite Vegetable**





### ✓ CHECK Your Progress

- a. **SCIENCE** The table shows the present composition of Earth's atmosphere. Display the data in a circle graph.

Composition of Earth's Atmosphere	
Element	Percent
Nitrogen	78%
Oxygen	21%
Other gases	1%

When constructing a circle graph, you may need to first convert the data to ratios and decimals and then to degrees and percents.

### EXAMPLE Construct a Circle Graph

- 2 **ANIMALS** The table shows endangered species in the United States. Make a circle graph of the data.

- Find the total number of species:  
 $68 + 77 + 14 + 11 = 170$ .
- Find the ratio that compares each number with the total. Write the ratio as a decimal rounded to the nearest hundredth.  
mammals:  $\frac{68}{170} = 0.40$       birds:  $\frac{77}{170} \approx 0.45$   
reptiles:  $\frac{14}{170} \approx 0.08$       amphibians:  $\frac{11}{170} \approx 0.06$

- Find the number of degrees for each section of the graph.

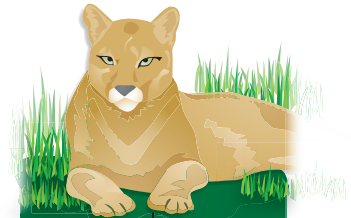
$$\begin{aligned} \text{mammals: } & 0.40 \cdot 360^\circ = 144^\circ \\ \text{birds: } & 0.45 \cdot 360^\circ \approx 162^\circ \\ \text{reptiles: } & 0.08 \cdot 360^\circ \approx 29^\circ \\ \text{amphibians: } & 0.06 \cdot 360^\circ \approx 22^\circ \end{aligned}$$

Because of rounding, the sum of the degrees is  $357^\circ$ .

- Draw the circle graph.

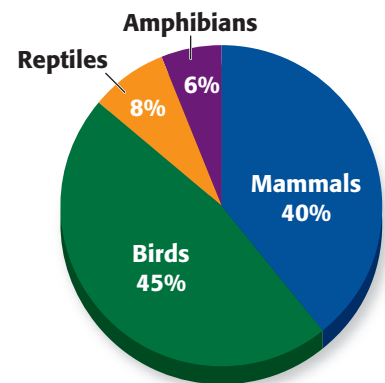
$$\begin{aligned} 0.40 &= 40\%, 0.45 = 45\%, \\ 0.08 &= 8\%, 0.06 = 6\% \end{aligned}$$

**Check** After drawing the first two sections, you can measure the last section of a circle graph to verify that the angles have the correct measures.



Species	Number of Species
Mammals	68
Birds	77
Reptiles	14
Amphibians	11

Source: U.S. Fish & Wildlife Service



#### Real-World Link . . . .

The Carolina Northern and Virginian Northern Flying Squirrel are both endangered. The northern flying squirrel is a small nocturnal gliding mammal that is about 10 to 12 inches in total length and weighs about 3–5 ounces.

Source: U.S. Fish and Wildlife Service

### ✓ CHECK Your Progress

- b. **OLYMPICS** The number of Winter Olympic medals won by the U.S. from 1924 to 2006 is shown in the table. Display the data in a circle graph.

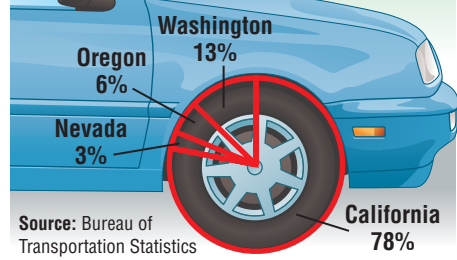
U.S. Winter Olympic Medals	
Type	Number
Gold	78
Silver	81
Bronze	59



## EXAMPLES Analyze a Circle Graph

**AUTOMOBILES** The graph shows the percent of automobiles registered in the western United States in a recent year.

### U.S. Registered Automobiles in West



- 3 Which state had the most registered automobiles?

The largest section of the circle is the one representing California. So, California has the most registered automobiles.

- 4 If 24.0 million automobiles were registered in these states, how many more automobiles were registered in California than Oregon?

California: 78% of 24.0 million  $\rightarrow 0.78 \times 24.0$ , or 18.72 million

Oregon: 6% of 24.0 million  $\rightarrow 0.06 \times 24.0$ , or 1.44 million

There were 18.72 million  $-$  1.44 million, or 17.28 million more registered automobiles in California than in Oregon.

### Study Tip

**Check for Reasonableness**  
To check Example 4, you can estimate and solve the problem another way.

$78\% - 6\% \approx 70\%$   
70% of 24 is about 17.

Since 17.28 is about 17, the answer is reasonable.

### CHECK Your Progress

- c. Which state had the least number of registered automobiles? Explain.  
d. What was the total number of registered automobiles in Washington and Oregon?

## CHECK Your Understanding

Examples 1, 2  
(pp. 518–519)

Display each set of data in a circle graph.

1.

Blood Types in the U.S.	
Blood Type	Percent
O	44%
A	42%
B	10%
AB	4%

Source: Stanford School of Medicine

2.

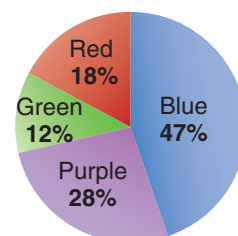
Favorite Musical Instrument	
Type	Number of Students
Piano	54
Guitar	27
Drum	15
Flute	24

Examples 3, 4  
(p. 520)

**COLORS** For Exercises 3 and 4, use the graph that shows the results of a survey.

3. What color is most favored?  
4. If 400 people were surveyed, how many more people favored purple than red?

### Favorite Color





# Practice and Problem Solving

## HOMEWORK HELP

For Exercises	See Examples
5–6	1
7–8	2
9–14	3,4

Display each set of data in a circle graph.

5. **U.S. Steel Roller Coasters**

Type	Percent
Sit down	86%
Inverted	8%
Other	6%

6. **U.S. Orange Production**

State	Orange Production
California	18%
Florida	81%
Texas	1%

Source: National Agriculture Statistics Service

7. **Animals in Pet Store**

Animal	Number of Pets
Birds	13
Cats	11
Dogs	9
Fish	56
Other	22

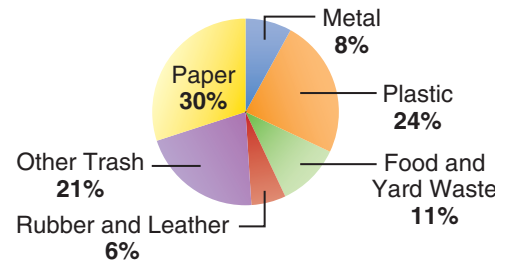
8. **Favorite Games**

Type of Game	Number of Students
Card	7
Board	9
Video	39
Sports	17
Drama	8

**LANDFILLS** For Exercises 9–11, use the circle graph that shows what is in United States landfills.

- What takes up the most space in landfills?
- About how many times more paper is there than food and yard waste?
- If a landfill contains 200 million tons of trash, how much of it is plastic?

What is in U.S. Landfills?



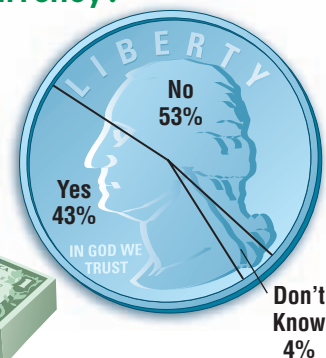
Source: The World Almanac for Kids

**MONEY** For Exercises 12–14, use the graph that shows the results of a survey about a common currency for North America.

Do Americans Favor Common North American Currency?



Source: Coinstar



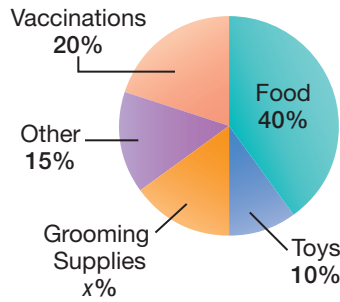
- What percent of Americans favor a common North American currency?
- Based on these results, about how many of the approximately 298 million Americans would say “Don’t Know” in response to this survey?
- About how many more Americans oppose a common currency than favor it?



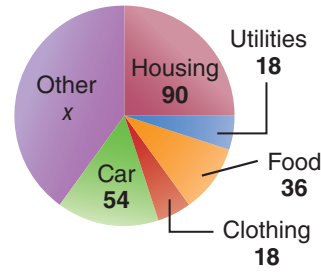


**DATA SENSE** For each graph, find the missing values.

15. **Dog Expenses**



16. **Family Budget**



Select an appropriate type of graph to display each set of data: line graph, bar graph, or circle graph. Then display the data using the graph.

17. **Top 5 Presidential Birth States**

Place	Presidents
Virginia	8
Ohio	7
Massachusetts	4
New York	4
Texas	3

Source: *The World Book of Facts*

18. **Tanya's Day**

Activity	Percent
School	25%
Sleep	33%
Homework	12%
Sports	8%
Other	22%

**GEOGRAPHY** For Exercises 19–21, use the table.

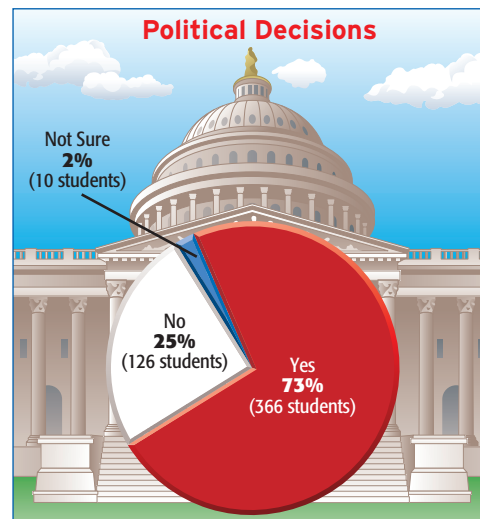
- Display the data in a circle graph.
- Use your graph to find which two lakes equal the size of Lake Superior.
- Compare the size of Lake Ontario to the size of Lake Michigan.

Lake	Size (sq mi)
Erie	9,930
Huron	23,010
Michigan	22,400
Ontario	7,520
Superior	31,820

**POLITICS** For Exercises 22 and 23, use the graph and information below.

A group of students were asked whether people their age could make a difference in the political decisions of elected officials.

- How many students participated in the survey?
- Write a convincing argument explaining whether or not it is reasonable to say that 50% more students said they could make a difference than those who said they could not make a difference.



Source: *Mom's Life and Mothering Magazine*

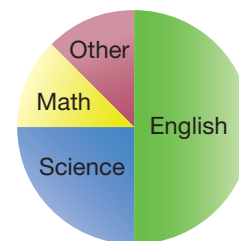
- FIND THE DATA** Refer to the Data File on pages 16–19. Choose some data that can be displayed in a circle graph. Then display the data in a circle graph and write one statement analyzing the data.

**Academic Standards • ISTEP+**  
Extra Practice, pp. 694, 713

**H.O.T. Problems**

25. **CHALLENGE** The graph shows the results of a survey about students' favorite school subject. About what percent of those surveyed said that math was their favorite subject? Explain your reasoning.
26. **COLLECT THE DATA** Collect some data from your classmates that can be represented in a circle graph. Then create the circle graph and write one statement analyzing the data.
27. **WRITING IN MATH** The table shows the percent of people that like each type of fruit juice. Can the data be represented in a circle graph? Justify your answer.

**Favorite Subject**

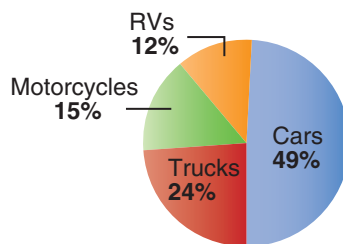


Fruit Juice	Percent
Apple	54%
Grape	48%
Orange	37%
Cranberry	15%

**ISTEP+ PRACTICE 7.4.1**

28. The graph shows the type of vehicles that used Highway 82 during one month.

**Types of Vehicles**

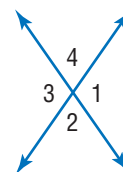


Which statement is true according to the circle graph shown?

- A More cars used the highway than RVs and trucks combined.
- B More than half the vehicles that used the highway were cars.
- C More RVs used the highway than trucks.
- D More trucks used the highway than cars.

**Spiral Review**

29. **GEOMETRY** Refer to the diagram at the right. Identify a pair of vertical angles. (Lesson 10-1)
30. **ALGEBRA**  $\angle A$  and  $\angle B$  are complementary. If  $m\angle A = 15^\circ$ , find  $m\angle B$ . (Lesson 10-2)



**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (Lesson 3-2)

31.  $x + 112 = 180$       32.  $50 + t = 180$       33.  $180 = 79 + y$       34.  $180 = h + 125$

# 10-4

# Triangles

## MAIN IDEA

Identify and classify triangles.

## IN Academic Standards

**Reinforcement of 6.3.2** Recognize that the sum of the interior angles of any triangle is  $180^\circ$  and that the sum of the interior angles of any quadrilateral is  $360^\circ$ . Use this information to solve problems. *Also addresses P.3.4.*

## New Vocabulary

triangle  
congruent segments  
acute triangle  
right triangle  
obtuse triangle  
scalene triangle  
isosceles triangle  
equilateral triangle

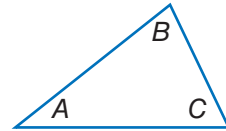
## IN Math Online

[glencoe.com](http://glencoe.com)

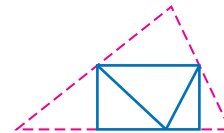
- Extra Examples
- Personal Tutor
- Self-Check Quiz

## MINI Lab

**STEP 1** Use a straightedge to draw a triangle with three acute angles. Label the angles  $A$ ,  $B$ , and  $C$ . Cut out the triangle.



**STEP 2** Fold  $\angle A$ ,  $\angle B$ , and  $\angle C$  so the vertices meet on the line between angles  $A$  and  $C$ .



1. What kind of angle is formed where the three vertices meet?
2. Repeat the activity with another triangle. Make a conjecture about the sum of the measures of the angles of any triangle.

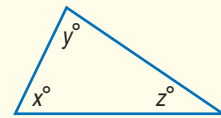
A **triangle** is a figure with three sides and three angles. The symbol for triangle is  $\triangle$ . There is a relationship among the three angles in a triangle.

## Angles of a Triangle

### Key Concept

**Words** The sum of the measures of the angles of a triangle is  $180^\circ$ .

**Model**



**Algebra**  $x + y + z = 180$

## EXAMPLE Find a Missing Measure

**1 ALGEBRA** Find  $m\angle Z$  in the triangle.

Since the sum of the angle measures in a triangle is  $180^\circ$ ,  $m\angle Z + 43^\circ + 119^\circ = 180^\circ$ .

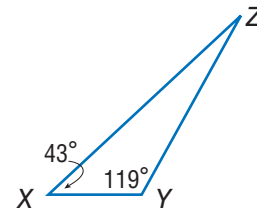
$$m\angle Z + 43^\circ + 119^\circ = 180^\circ$$

$$m\angle Z + 162^\circ = 180^\circ$$

$$\underline{\quad - 162^\circ = - 162^\circ}$$

$$m\angle Z = 18^\circ$$

So,  $m\angle Z$  is  $18^\circ$ .



Write the equation.

Simplify.

Subtract  $162^\circ$  from each side.

## CHECK Your Progress

- a. **ALGEBRA** In  $\triangle ABC$ , if  $m\angle A = 25^\circ$  and  $m\angle B = 108^\circ$ , what is  $m\angle C$ ?



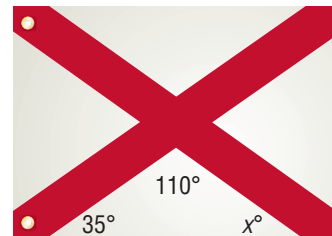
### ISTEP+ EXAMPLE

Reinforcement of 6.3.2

2

The Alabama state flag is constructed with the triangles shown. What is the missing measure in the design?

- A  $145^\circ$
- B  $75^\circ$
- C  $35^\circ$
- D  $25^\circ$



#### Test-Taking Tip

##### Check the Results

To check the results, add the three angle measures to see if their sum is 180.

$$35 + 110 + 35 = 180 \quad \checkmark$$

The answer is correct.



#### Read the Item

To find the missing measure, write and solve an equation.

#### Solve the Item

$$x + 110 + 35 = 180 \quad \text{The sum of the measures is 180.}$$

$$x + 145 = 180 \quad \text{Simplify.}$$

$$\underline{-145 = -145} \quad \text{Subtract 145 from each side.}$$

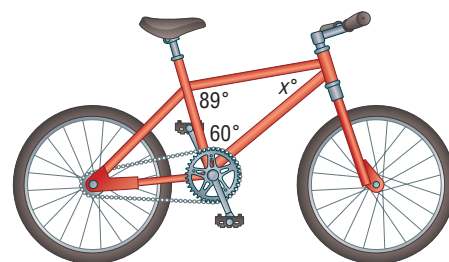
$$x = 35$$

The answer is C.

#### CHECK Your Progress

b. The frame of a bicycle shows a triangle. What is the missing measure?

- F  $31^\circ$
- G  $40^\circ$
- H  $45^\circ$
- J  $50^\circ$



Every triangle has at least two acute angles. One way you can classify a triangle is by using the third angle. Another way to classify triangles is by their sides. Sides with the same length are **congruent segments**.

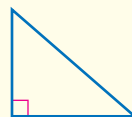
### Classify Triangles Using Angles

Key Concept



all acute angles

**acute triangle**



1 right angle

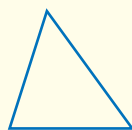
**right triangle**



1 obtuse angle

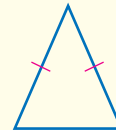
**obtuse triangle**

### Classify Triangles Using Sides



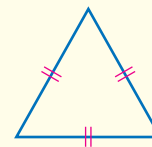
no congruent sides

**scalene triangle**



at least 2 congruent sides

**isosceles triangle**



3 congruent sides

**equilateral triangle**

#### Study Tip

**Congruent Segments**  
The marks on the sides of the triangle indicate that those sides are congruent.



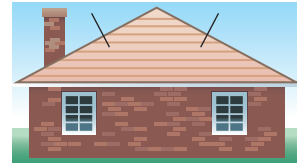
### Real-World Link . . . .

There are two main types of roofs—flat and pitched. Most houses have pitched, or sloped, roofs. A pitched roof generally lasts 15 to 20 years.

Source: National Association of Certified Home Inspectors

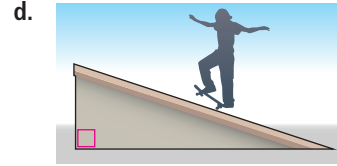
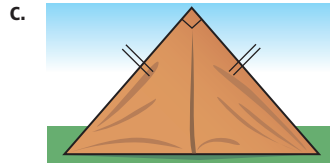
## Real-World EXAMPLE

- 3 Classify the marked triangle at the right by its angles and by its sides.



The triangle on the side of a house has one obtuse angle and two congruent sides. So, it is an obtuse, isosceles triangle.

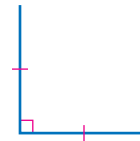
## CHECK Your Progress



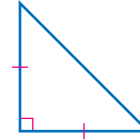
## EXAMPLES Draw Triangles

- 4 Draw a triangle with one right angle and two congruent sides. Then classify the triangle.

Draw a right angle. The two segments should be congruent.



Connect the two segments to form a triangle.



The triangle is a right isosceles triangle.

- 5 Draw a triangle with one obtuse angle and no congruent sides. Then classify the triangle.

Draw an obtuse angle. The two segments of the angle should have different lengths.



Connect the two segments to form a triangle.



The triangle is an obtuse scalene triangle.

## CHECK Your Progress

Draw a triangle that satisfies each set of conditions below. Then classify each triangle.

- e. a triangle with three acute angles and three congruent sides
- f. a triangle with one right angle and no congruent sides

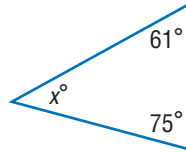


# CHECK Your Understanding

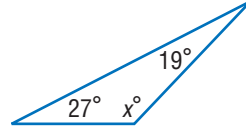
**Example 1**  
(p. 524)

Find the value of  $x$ .

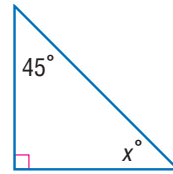
1.



2.



3.

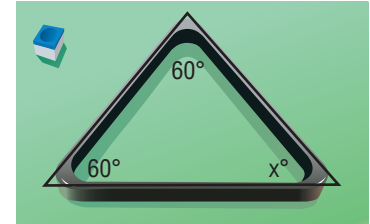


4. **ALGEBRA** Find  $m\angle T$  in  $\triangle RST$  if  $m\angle R = 37^\circ$  and  $m\angle S = 55^\circ$ .

**Example 2**  
(p. 525)

5. **MULTIPLE CHOICE** A triangle is used in the game of pool to rack the pool balls. Find the missing measure of the triangle.

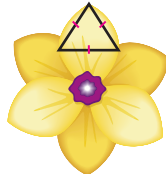
- A  $30^\circ$                       C  $60^\circ$   
B  $40^\circ$                       D  $75^\circ$



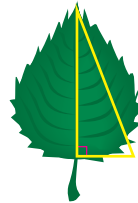
**Example 3**  
(p. 526)

**NATURE** Classify the marked triangle in each object by its angles and by its sides.

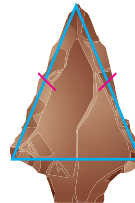
6.



7.



8.



**Examples 4, 5**  
(p. 526)

**DRAWING TRIANGLES** For Exercises 9 and 10, draw a triangle that satisfies each set of conditions. Then classify each triangle.

9. a triangle with three acute angles and two congruent sides  
10. a triangle with one obtuse angle and two congruent sides

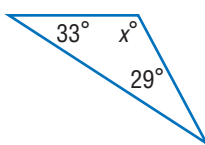
# Practice and Problem Solving

## HOMEWORK HELP

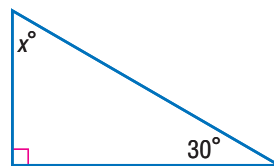
For Exercises	See Examples
11–18, 47, 48	1–2
19–26	3
27–30	4, 5

Find the value of  $x$ .

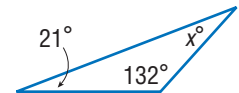
11.



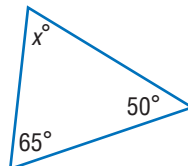
12.



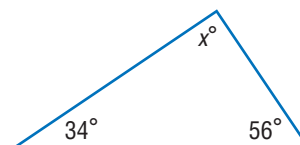
13.



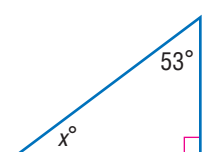
14.



15.



16.



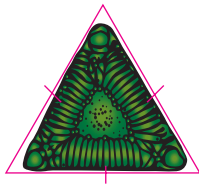
17. **ALGEBRA** Find  $m\angle Q$  in  $\triangle QRS$  if  $m\angle R = 25^\circ$  and  $m\angle S = 102^\circ$ .

18. **ALGEBRA** In  $\triangle EFG$ ,  $m\angle F = 46^\circ$  and  $m\angle G = 34^\circ$ . What is  $m\angle E$ ?

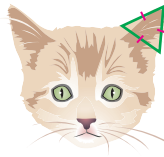


Classify the marked triangle in each object by its angles and by its sides.

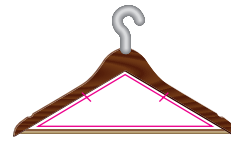
19.



20.



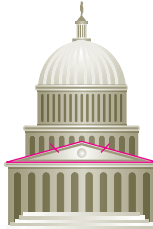
21.



22.



23.



24.



25. **ART** The sculpture at the right is entitled *Texas Triangles*. It is located in Lincoln, Massachusetts. What type of triangle is shown: *acute*, *right*, or *obtuse*?



Source: DeCordova Museum and Sculpture Park

26. **ARCHITECTURE** Use the photo at the left to classify the side view of the Transamerica building by its angles and by its sides.



27. **DRAWING TRIANGLES** For Exercises 27–30, draw a triangle that satisfies each set of conditions. Then classify each triangle.

27. a triangle with three acute angles and no congruent sides  
 28. a triangle with one obtuse angle and two congruent sides  
 29. a triangle with three acute angles and three congruent sides  
 30. a triangle with one right angle and no congruent sides

Find the missing measure in each triangle with the given angle measures.

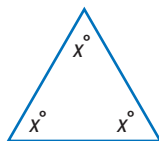
31.  $80^\circ, 20.5^\circ, x^\circ$       32.  $75^\circ, x^\circ, 50.2^\circ$       33.  $x^\circ, 10.8^\circ, 90^\circ$   
 34.  $45.5^\circ, x^\circ, 105.6^\circ$       35.  $x^\circ, 140.1^\circ, 18.6^\circ$       36.  $110.2^\circ, x^\circ, 35.6^\circ$

37. **ALGEBRA** Find the third angle measure of a right triangle if one of the angles measures is  $10^\circ$ .

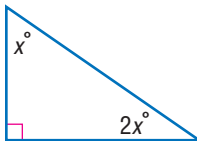
38. **ALGEBRA** What is the third angle measure of a right triangle if one of the angle measures is  $45.8^\circ$ ?

**ALGEBRA** Find the value of  $x$  in each triangle.

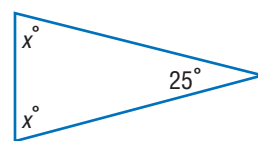
39.



40.



41.



**Real-World Link** . . . . .

The Transamerica building in San Francisco is 853 feet tall.

Source: Transamerica Corporation

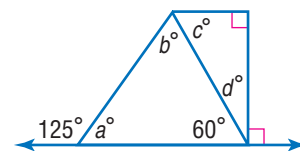


**Academic Standards • ISTEP+**  
 Extra Practice, pp. 694, 713



### H.O.T. Problems

42. **CHALLENGE** Apply what you know about triangles to find the missing angle measures in the figure.
43. **OPEN ENDED** Draw an acute scalene triangle. Describe the angles and sides of the triangle.



**REASONING** Determine whether each statement is *sometimes*, *always*, or *never* true. Justify your answer.

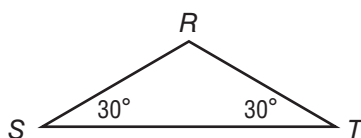
44. It is possible for a triangle to have two right angles.
45. It is possible for a triangle to have two obtuse angles.
46. **WRITING IN MATH** An equilateral triangle not only has three congruent sides, but also has three congruent angles. Based on this, explain why it is impossible to draw an equilateral triangle that is either right or obtuse.



### ISTEP+ PRACTICE

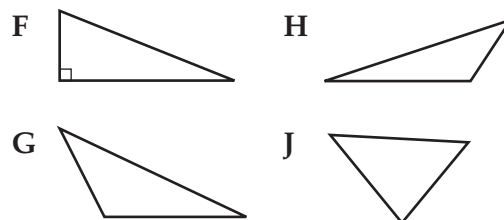
#### Reinforcement of 6.3.2

47. How would you find  $m\angle R$ ?



- A Add  $30^\circ$  to  $180^\circ$ .
- B Subtract  $60^\circ$  from  $180^\circ$ .
- C Subtract  $30^\circ$  from  $90^\circ$ .
- D Subtract  $180^\circ$  from  $60^\circ$ .

48. Which of the following is an acute triangle?

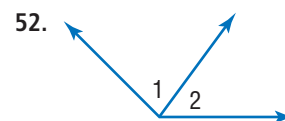
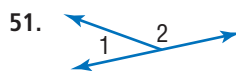
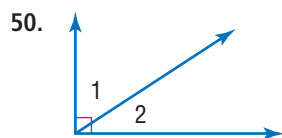


### Spiral Review

49. **SURVEYS** A circle graph shows that 28% of people chose tea as their favorite drink. What is the measure of the angle of the tea section of the graph? (Lesson 10-3)

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

(Lesson 10-2)



### GET READY for the Next Lesson

53. **PREREQUISITE SKILL** If Jade buys 5 notebooks at \$1.75 each, what will be the total cost of the notebooks; about \$6, \$7, or \$9? (Lesson 7-5)



# 10-5

## Problem-Solving Investigation

**MAIN IDEA:** Solve problems by using logical reasoning.

Academic Standards

**P.2.1** Recognize reasoning and proof as fundamental aspects of mathematics. **P.2.2** Make and investigate mathematical conjectures. Also addresses P.2.3, P.2.4.

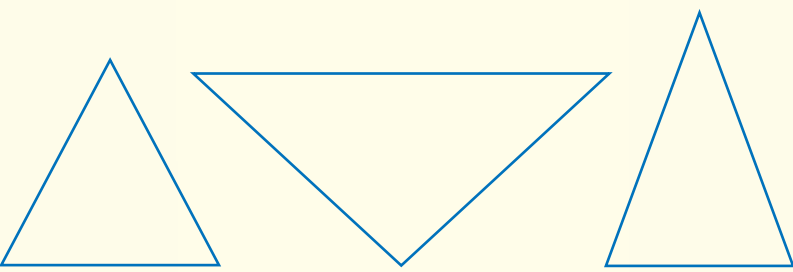
### P.S.I. TEAM +

#### e-Mail: USE LOGICAL REASONING

SANTOS: I know that at least two sides of an isosceles triangle are congruent. It also looks like two of the angles in an isosceles triangle are congruent.

YOUR MISSION: Use **logical reasoning** to find if the angles in an isosceles triangles are congruent.



<b>Understand</b>	Isosceles triangles have at least two congruent sides. We need to find if there is a relationship between the angles.
<b>Plan</b>	Draw several isosceles triangles and measure their angles.
<b>Solve</b>	 <p>In each triangle, two angles are congruent. So, it seems like an isosceles triangle has two congruent angles.</p>
<b>Check</b>	Try drawing several more isosceles triangles and measuring their angles. Although this is not a proof, it is likely your conclusion is valid.

### Analyze The Strategy

1. When you use *inductive reasoning*, you make a rule after seeing several examples. When you use *deductive reasoning*, you use a rule to make a decision. What type of reasoning did Santos use to solve the problem? Explain your reasoning.
2. Explain how the *look for a pattern* strategy is similar to inductive reasoning.

## Mixed Problem Solving

For Exercises 3–5, use logical reasoning to solve the problem. Justify your response.

- GEOMETRY** Draw several scalene triangles and measure their angles. What do you notice about the measures of the angles of a scalene triangle?
- HOUSE NUMBERS** Rico's house number contains four digits. The digits are 5, 8, 3, and 2. If his house number is odd, divisible by 3, and the middle two numbers are a perfect square, what is his house number?
- FRUIT** Julio, Rashanda, and Perry each brought a fruit with their lunch. The fruits brought were a mango, a banana, and an orange. If Perry did not bring a banana and Julio brought a mango, what type of fruit did each student bring?



Use any strategy to solve Exercises 6–14. Some strategies are shown below.

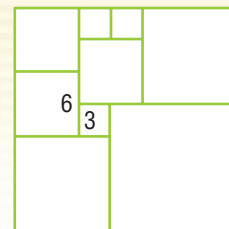
### PROBLEM-SOLVING STRATEGIES

- Look for a pattern.
- Use a graph.
- Use logical reasoning.

- GEOMETRY** Draw several rectangles and measure their diagonals. Find a relationship between the diagonals of a rectangle.
- MONEY** Corelia has twice as many quarters in her purse as dimes and half as many nickels as dimes. If she has 12 quarters in her purse, what is the total amount of money she has?
- ALGEBRA** Find the next three numbers in the pattern below.

71, 64, 57, 50, ■, ■, ■

- MEASUREMENT** The large square has been divided into 9 squares. The lengths of the squares are given. Find the area of the entire square.



- READING** Sonia read 10 pages of a 150-page book on Monday. She plans to read twice as many pages each day than she did the previous day. On what day will she finish the book?
- SUPPLIES** Bianca has \$55 to buy school supplies. She bought a backpack for \$23.50, a combination lock for \$6.25, and 4 binders that are \$3.99 each. If the costs include tax and mechanical pencils are \$2.50 per pack, how many packs can she buy?
- BOWLING** Colin and three of his friends are going bowling, and they have a total of \$70 to spend. Suppose they buy a large pizza, four beverages, and each rent bowling shoes. How many games can they bowl if they all bowl the same number of games?

Bowling Costs	
Item	Price
large pizza	\$15.75
beverage	\$1.50
shoe rental	\$3.50
game	\$4.00

- STATISTICS** David has earned scores of 73, 85, 91, and 82 on the first four out of five math tests for the grading period. He would like to finish the grading period with a test average of at least 82. What is the minimum score David needs to earn on the fifth test in order to achieve his goal?
- ALLOWANCE** Samantha earns \$520 in allowance yearly. Her parents promise to give her a \$60 raise each year. At this rate, what will her yearly allowance be in 4 years?

## Explore 10-6

# Geometry Lab Investigating Quadrilaterals

### MAIN IDEA

Investigate the properties of special quadrilaterals.

### IN Academic Standards

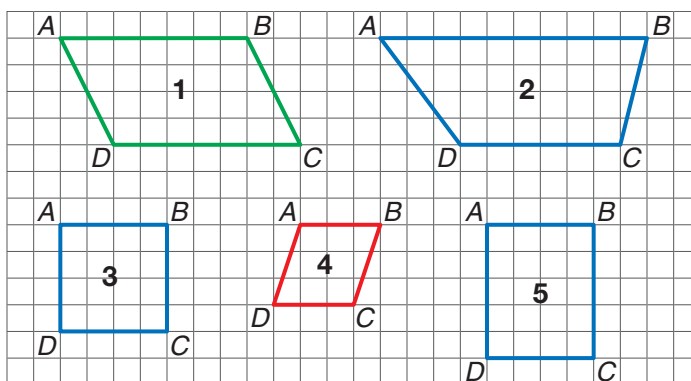
#### Reinforcement of 6.3.2

Recognize that the sum of the interior angles of any triangle is  $180^\circ$  and that the sum of the interior angles of any quadrilateral is  $360^\circ$ . Use this information to solve problems. Also addresses P.1.1, P.1.4, P.2.2, P.2.3, P.3.1, P.3.2, P.4.1.

Four-sided figures are called *quadrilaterals*. In this lab, you will explore the properties of different types of quadrilaterals.

### ACTIVITY

**STEP 1** Draw the quadrilaterals shown on grid paper.



**STEP 2** Use a ruler and a protractor to measure the sides and angles of each quadrilateral. Record your results in a table.

Quadrilateral	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle D$	AB	BC	CD	DA
1								
2								

### ANALYZE THE RESULTS

- Describe any similarities or patterns in the angle measurements.
- Describe any similarities or patterns in the side measurements.
- MAKE A VENN DIAGRAM** Cut out the quadrilaterals you drew in the activity. Then sort them into categories according to their similarities and differences. Arrange and record your categories in a two-circled Venn diagram. Be sure to label each circle with its category.
- Create two other Venn diagrams illustrating two different ways of categorizing these quadrilaterals.
- WRITING IN MATH** Did you find shapes that did not fit a category? Where did you place these shapes? Did any shapes have properties allowing them to belong to more than one category? Could you arrange these quadrilaterals into a three-circled Venn diagram? If so, how?

# 10-6

# Quadrilaterals

## MAIN IDEA

Identify and classify quadrilaterals.

## IN Academic Standards

### Reinforcement of 6.3.2

Recognize that the sum of the interior angles of any triangle is  $180^\circ$  and that the sum of the interior angles of any quadrilateral is  $360^\circ$ . Use this information to solve problems. *Also addresses P.3.4.*

## New Vocabulary

quadrilateral  
parallelogram  
trapezoid  
rhombus

## IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

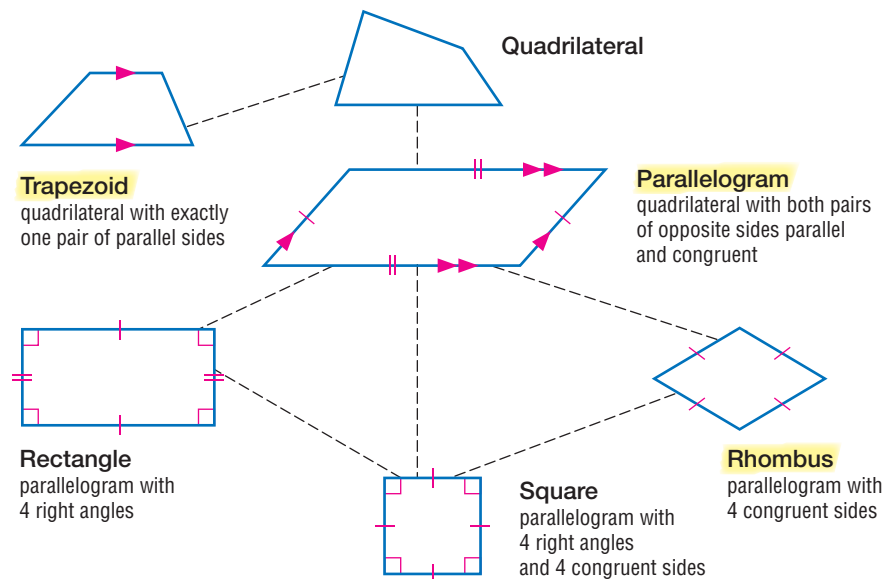
## ▶ GET READY for the Lesson

**VIDEO GAMES** The general shape of a video game controller is shown.

1. Describe the angles inside the four-sided figure.
2. Which sides of the figure appear to be parallel?
3. Which sides appear to be congruent?



A **quadrilateral** is a closed figure with four sides and four angles. Quadrilaterals are named based on their sides and angles. The diagram shows how quadrilaterals are related. Notice how it goes from the most general to the most specific.



The name that *best* describes a quadrilateral is the one that is most specific.

- If a quadrilateral has all the properties of a parallelogram and a rhombus, then the *best* description of the quadrilateral is a rhombus.
- If a quadrilateral has all the properties of a parallelogram, rhombus, rectangle, and square, then the *best* description of the quadrilateral is a square.

## Study Tip

**Parallel Lines** The sides with matching arrows are parallel.



## EXAMPLES

## Draw and Classify Quadrilaterals

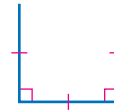
Draw a quadrilateral that satisfies each set of conditions. Then classify each quadrilateral with the name that best describes it.

- 1** a parallelogram with four right angles and four congruent sides

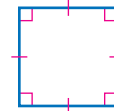
Draw one right angle. The two segments should be congruent.



Draw a second right angle that shares one of the congruent segments. The third segment drawn should be congruent to the first two segments drawn.



Connect the fourth side of the quadrilateral. All four angles should be right angles, and all four sides should be congruent.



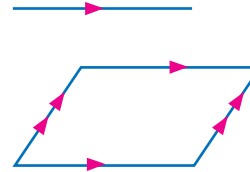
The figure is a square.

- 2** a quadrilateral with opposite sides parallel

Draw two parallel sides of equal length.



Connect the endpoints of these two sides so that two new parallel sides are drawn.



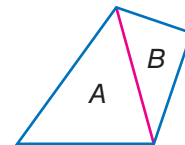
The figure is a parallelogram.

### CHECK Your Progress

Draw a quadrilateral that satisfies each set of conditions. Then classify each quadrilateral with the name that best describes it.

- a quadrilateral with exactly one pair of parallel sides
- a parallelogram with four congruent sides

A quadrilateral can be separated into two triangles,  $A$  and  $B$ . Since the sum of the angle measures of each triangle is  $180^\circ$ , the sum of the angle measures of the quadrilateral is  $2 \cdot 180$ , or  $360^\circ$ .



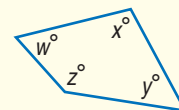
## Angles of a Quadrilateral

### Key Concept

**Words** The sum of the measures of the angles of a quadrilateral is  $360^\circ$ .

**Algebra**  $w + x + y + z = 360$

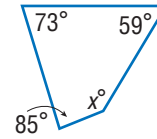
**Model**





## EXAMPLE Find a Missing Measure

- 3 **ALGEBRA** Find the value of  $x$  in the quadrilateral shown.



Write and solve an equation.

**Words**

The sum of the measures is  $360^\circ$ .

**Variable**

Let  $x$  represent the missing measure.

**Equation**

$$85 + 73 + 59 + x = 360$$

### Study Tip

**Check for Reasonableness**  
Since  $\angle x$  is an obtuse angle,  $m\angle x$  should be between  $90^\circ$  and  $180^\circ$ .  
Since  $90^\circ < 143^\circ < 180^\circ$ , the answer is reasonable.

$$85 + 73 + 59 + x = 360$$

Write the equation.

$$217 + x = 360$$

Simplify.

$$\underline{-217} \quad \underline{= -217}$$

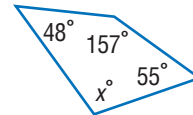
Subtract 217 from each side.

$$x = 143$$

So, the missing angle measure is  $143^\circ$ .

### CHECK Your Progress

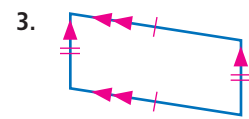
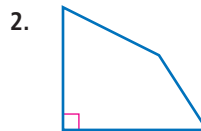
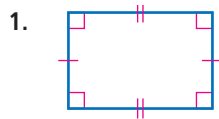
- c. **ALGEBRA** Find the value of  $x$  in the quadrilateral shown.



## CHECK Your Understanding

Examples 1, 2  
(p. 534)

Classify each quadrilateral with the name that best describes it.



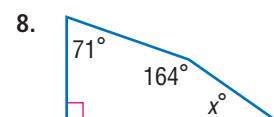
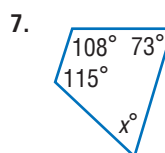
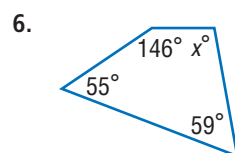
4. **BOATS** The photo shows a sailboat called a schooner. What type of quadrilateral does the indicated sail best represent?



Example 3  
(p. 535)

5. **ALGEBRA** In quadrilateral  $DEFG$ ,  $m\angle D = 57^\circ$ ,  $m\angle E = 78^\circ$ ,  $m\angle G = 105^\circ$ . What is  $m\angle F$ ?

**ALGEBRA** Find the missing angle measure in each quadrilateral.



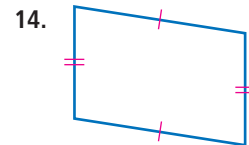
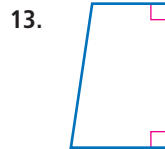
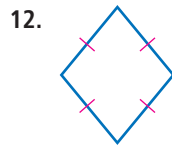
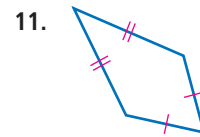
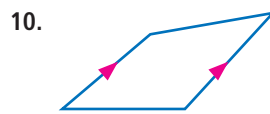
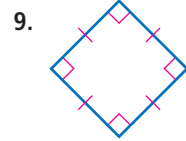


# Practice and Problem Solving

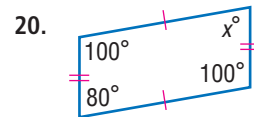
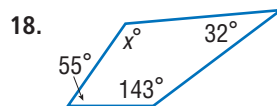
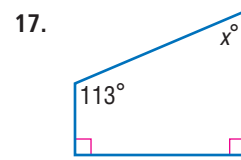
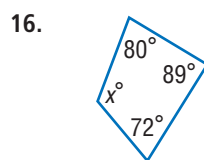
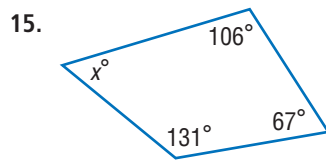
## HOMEWORK HELP

For Exercises	See Examples
9–14, 23–24	1, 2
15–22	3

Classify each quadrilateral with the name that best describes it.



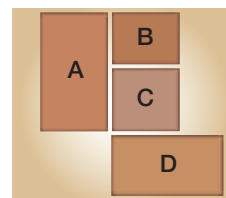
**ALGEBRA** Find the missing angle measure in each quadrilateral.



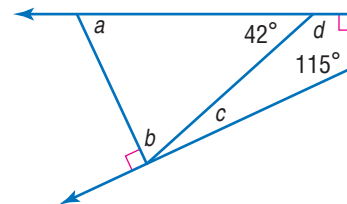
21. **ALGEBRA** Find  $m\angle B$  in quadrilateral  $ABCD$  if  $m\angle A = 87^\circ$ ,  $m\angle C = 135^\circ$ , and  $m\angle D = 22^\circ$ .

22. **ALGEBRA** What is  $m\angle X$  in quadrilateral  $WXYZ$  if  $m\angle W = 45^\circ$ ,  $m\angle Y = 128^\circ$ , and  $\angle Z$  is a right angle?

23. **LANDSCAPE** Identify the shapes of the bricks used in the design at the right. Use the name that *best* describes each brick.



24. **MEASUREMENT** Find each of the missing angle measures  $a$ ,  $b$ ,  $c$ , and  $d$  in the figure at the right. Justify your answers.



Find the missing measure in each quadrilateral with the given angle measures.

25.  $37.5^\circ, 78^\circ, 115.4^\circ, x^\circ$

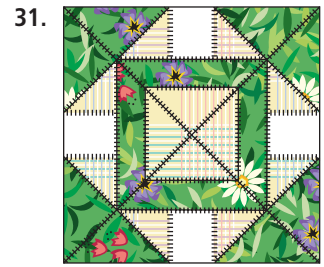
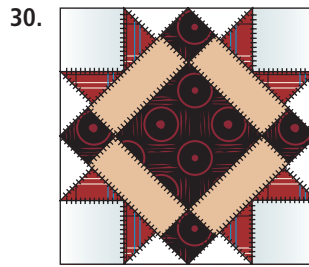
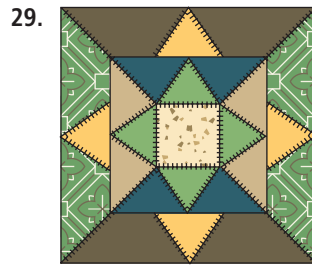
26.  $x^\circ, 108.3^\circ, 49.8^\circ, 100^\circ$

27.  $25.5^\circ, x^\circ, 165.9^\circ, 36.8^\circ$

28.  $79.1^\circ, 120.8^\circ, x^\circ, 65.7^\circ$



**ART** For Exercises 29–31, identify the types of triangles and quadrilaterals used in each quilt block pattern. Use the names that *best* describe the figures.

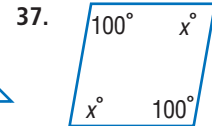
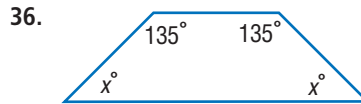
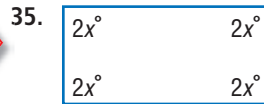


**DRAWING QUADRILATERALS** Determine whether each figure described below can be drawn. If the figure can be drawn, draw it. If not, explain why not.

- 32. a quadrilateral that is both a rhombus and a rectangle
- 33. a trapezoid with three right angles
- 34. a trapezoid with two congruent sides

**ALGEBRA** Find the value of  $x$  in each quadrilateral.

**Academic Standards • ISTEP+**  
Extra Practice, pp. 694, 713



**H.O.T. Problems**

**CHALLENGE** For Exercises 38 and 39, refer to the table that gives the properties of several parallelograms. Property A states that both pairs of opposite sides are parallel and congruent.

Parallelogram	Properties
1	A, C
2	A, B, C
3	A, B

- 38. If property C states that all four sides are congruent, classify parallelograms 1–3. Justify your response.
- 39. If parallelogram 3 is a rectangle, describe Property B. Justify your response.

**REASONING** Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

- 40. A quadrilateral is a trapezoid.
- 41. A trapezoid is a parallelogram.
- 42. A square is a rectangle.
- 43. A rhombus is a square.

44. **FIND THE ERROR** Isabelle and John are describing a rectangle. Who is more accurate? Explain.



a quadrilateral with 2 pairs of parallel sides

Isabelle



a parallelogram with 4 right angles

John

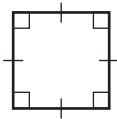
45. **WRITING IN MATH** The diagonals of a rectangle are congruent, and the diagonals of a rhombus are perpendicular. Based on this information, what can you conclude about the diagonals of a square? of a parallelogram? Explain your reasoning.



## ISTEP+ PRACTICE

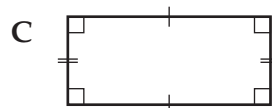
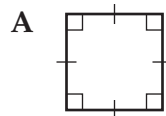
### Reinforcement of 6.3.2

46. Identify the name that does *not* describe the quadrilateral shown.



- A square
  - B rectangle
  - C rhombus
  - D trapezoid
47. Which statement is always true about a rhombus?
- F It has 4 right angles.
  - G The sum of the measures of the angles is  $180^\circ$ .
  - H It has exactly one pair of parallel sides.
  - J It has 4 congruent sides.

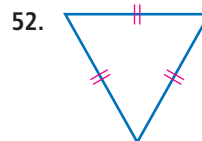
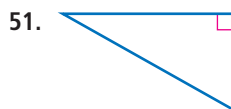
48. Which of the following is a correct drawing of a quadrilateral with all sides congruent and with four right angles?



## Spiral Review

49. **REASONING** Neva, Sophie, and Seth have a turtle, a dog, and a hamster for a pet, but not in that order. Sophie's pet lives in a glass aquarium and does not have fur. Neva never has to give her pet a bath. Who has what pet? Use the *logical reasoning* strategy. (Lesson 10-5)

Classify each triangle by its angles and by its sides. (Lesson 10-4)



53. **LETTERS** How many permutations are possible of the letters in the word *Fresno*? (Lesson 9-4)

Find the sales tax or discount to the nearest cent. (Lesson 7-7)

54. \$54 jacket; 7% sales tax

55. \$23 hat; 15% discount

## GET READY for the Next Lesson

**PREREQUISITE SKILL** Solve each proportion. (Lesson 6-6)

56.  $\frac{3}{5} = \frac{x}{75}$

57.  $\frac{a}{7} = \frac{18}{42}$

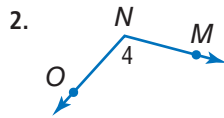
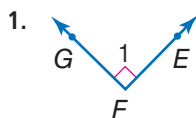
58.  $\frac{7}{9} = \frac{28}{m}$

59.  $\frac{3.5}{t} = \frac{16}{32}$

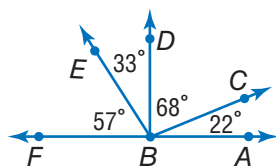
60.  $\frac{3}{6} = \frac{c}{5}$

Name each angle in four ways. Then classify each angle as *acute*, *obtuse*, *right*, or *straight*.

(Lesson 10-1)



3. **MULTIPLE CHOICE** Which angle is complementary to  $\angle CBD$ ? (Lesson 10-2)

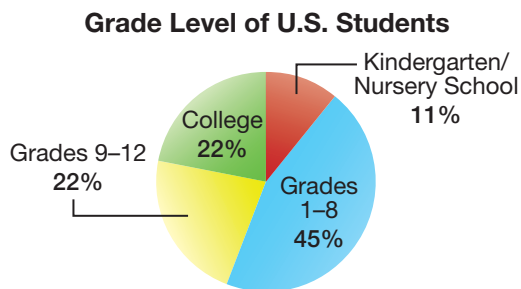


- A  $\angle ABC$                       C  $\angle DBE$   
B  $\angle FBC$                       D  $\angle EBF$

4. **SOCCER** Display the data in a circle graph. (Lesson 10-3)

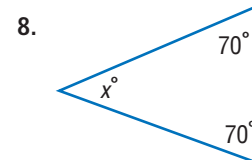
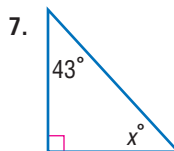
Injuries of High School Girls' Soccer Players	
Position	Percent
Halfbacks	37%
Fullbacks	23%
Forward Line	28%
Goalkeepers	12%

5. **EDUCATION** For Exercises 5 and 6, use the circle graph that shows the percent of students by grade level in U.S. schools. (Lesson 10-3)



5. In which grades are most students?  
6. How many times as many students are there in grades 1–8 than in grades 9–12?

**ALGEBRA** Find the value of  $x$ . (Lesson 10-4)



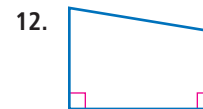
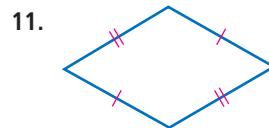
9. **MULTIPLE CHOICE** In triangle  $ABC$ ,  $m\angle A = 62^\circ$  and  $m\angle C = 44^\circ$ . What is  $m\angle B$ ? (Lesson 10-4)

- F  $90^\circ$                       H  $64^\circ$   
G  $74^\circ$                       J  $42^\circ$

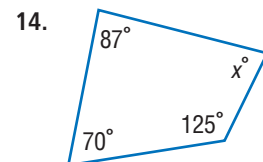
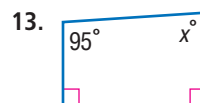
10. **RACES** Norberto, Isabel, Fiona, Brock, and Elizabeth were the first five finishers of a race. From the given clues, find the order in which they finished. Use the *logical reasoning* strategy. (Lesson 10-5)

- Norberto passed Fiona just before the finish line.
- Elizabeth finished 5 seconds ahead of Norberto.
- Isabel crossed the finish line after Fiona.
- Brock was fifth at the finish line.

Classify the quadrilateral with the name that best describes it. (Lesson 10-6)



**ALGEBRA** Find the value of  $x$  in each quadrilateral. (Lesson 10-6)



15. **ALGEBRA** What is  $m\angle A$  in quadrilateral  $ABCD$  if  $m\angle B = 36^\circ$ ,  $m\angle C = 74^\circ$ , and  $\angle D$  is a right angle? (Lesson 10-6)

# 10-7

# Similar Figures

## MAIN IDEA

Determine whether figures are similar and find a missing length in a pair of similar figures.

## IN Academic Standards

**7.3.5** Identify, describe and construct similarity relationships and solve problems involving similarity (including similar triangles) and scale drawings by using proportional reasoning.

## New Vocabulary

similar figures  
corresponding sides  
corresponding angles  
indirect measurement

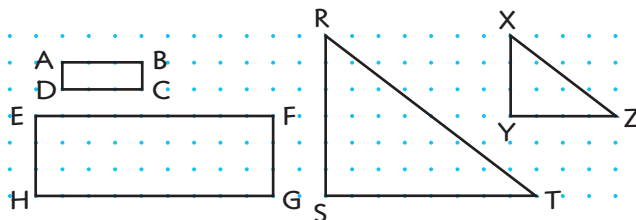
## IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

## MINI Lab

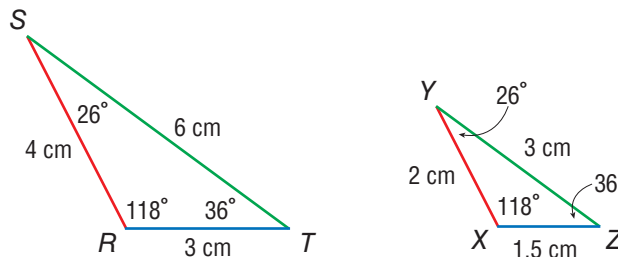
The figures in each pair below have the same shape but different sizes. Copy each pair onto dot paper. Then find the measure of each angle using a protractor and the measure of each side using a centimeter ruler.



- $\overline{AB}$  on the smaller rectangle matches  $\overline{EF}$  on the larger rectangle. Name all pairs of matching sides in each pair of figures. The notation  $\overline{AB}$  means the segment with endpoints at  $A$  and  $B$ .
- Write each ratio in simplest form. The notation  $AB$  means the *measure* of segment  $AB$ .
  - $\frac{AB}{EF}, \frac{BC}{FG}, \frac{DC}{HG}, \frac{AD}{EH}$
  - $\frac{RS}{XY}, \frac{ST}{YZ}, \frac{RT}{XZ}$
- What do you notice about the ratios of matching sides?
- Name all pairs of matching angles in the figures above. What do you notice about the measure of these angles?
- MAKE A CONJECTURE** about figures that have the same shape but not necessarily the same size.



Figures that have the same shape but not necessarily the same size are **similar figures**. In the figures below, triangle  $RST$  is similar to triangle  $XYZ$ . We write this as  $\triangle RST \sim \triangle XYZ$ .



The matching sides are  $\overline{ST}$  and  $\overline{YZ}$ ,  $\overline{SR}$  and  $\overline{YX}$ , and  $\overline{RT}$  and  $\overline{XZ}$ . The sides of similar figures that “match” are called **corresponding sides**. The matching angles are  $\angle S$  and  $\angle Y$ ,  $\angle R$  and  $\angle X$ , and  $\angle T$  and  $\angle Z$ . The angles of similar figures that “match” are called **corresponding angles**.



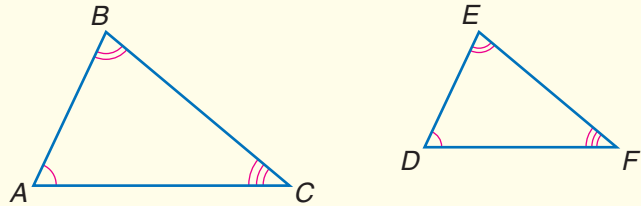
The Mini Lab illustrates the following statements.

## Similar Figures

Key Concept

- Words** If two figures are similar, then
- the corresponding sides are proportional, and
  - the corresponding angles are congruent.

**Models**



- Symbols**  $\triangle ABC \sim \triangle DEF$  The symbol  $\sim$  means *is similar to*.
- corresponding sides:  $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
- corresponding angles:  $\angle A \cong \angle D; \angle B \cong \angle E; \angle C \cong \angle F$

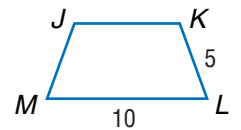
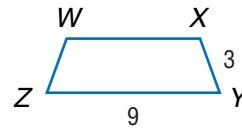
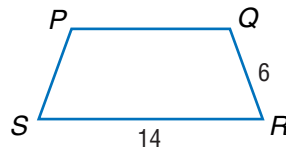
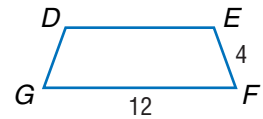
### Reading Math

#### Geometry Symbols

- $\sim$  is similar to
- $\cong$  is congruent to

### EXAMPLE Identify Similar Figures

- 1** Which trapezoid below is similar to trapezoid  $DEFG$ ?



Find the ratios of the corresponding sides to see if they form a constant ratio.

**Trapezoid PQRS**

$$\frac{EF}{QR} = \frac{4}{6} \text{ or } \frac{2}{3}$$

$$\frac{FG}{RS} = \frac{12}{14} \text{ or } \frac{6}{7}$$

Not similar

**Trapezoid WXYZ**

$$\frac{EF}{XY} = \frac{4}{3}$$

$$\frac{FG}{YZ} = \frac{12}{9} \text{ or } \frac{4}{3}$$

Similar

**Trapezoid JKLM**

$$\frac{EF}{KL} = \frac{4}{5}$$

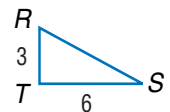
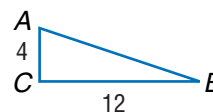
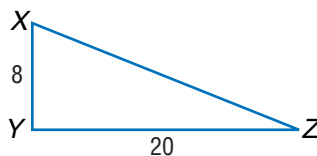
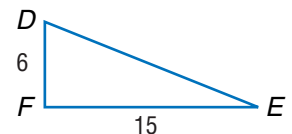
$$\frac{FG}{LM} = \frac{12}{10} \text{ or } \frac{6}{5}$$

Not similar

So, trapezoid  $WXYZ$  is similar to trapezoid  $DEFG$ .

### CHECK Your Progress

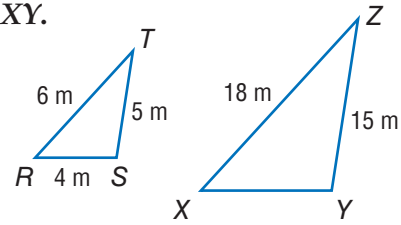
- a. Which triangle below is similar to triangle  $DEF$ ?



**EXAMPLE****Find Side Measures of Similar Triangles**

- 2 If  $\triangle RST \sim \triangle XYZ$ , find the length of  $\overline{XY}$ .

Since the two triangles are similar, the ratios of their corresponding sides are equal. Write and solve a proportion to find  $XY$ .



$$\frac{RT}{XZ} = \frac{RS}{XY}$$

Write a proportion.

$$\frac{6}{18} = \frac{4}{n}$$

Let  $n$  represent the length of  $\overline{XY}$ . Then substitute.

$$6n = 18(4)$$

Find the cross products.

$$6n = 72$$

Simplify.

$$n = 12$$

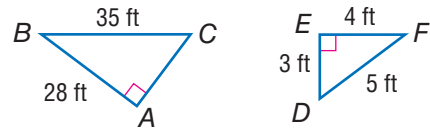
Divide each side by 6. The length of  $\overline{XY}$  is 12 meters.

**Reading Math****Geometry Symbols**

Just as the measure of angle  $A$  can be written as  $m\angle A$ , there is a special way to indicate the measure of a segment. The measure of  $\overline{AB}$  is written as  $AB$ , without the bar over it.

**CHECK Your Progress**

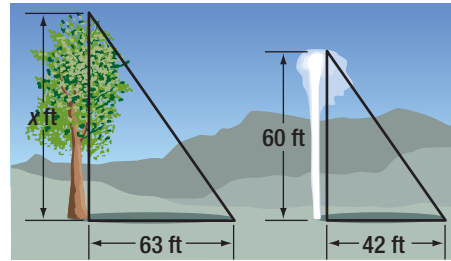
- b. If  $\triangle ABC \sim \triangle EFD$ , find the length of  $\overline{AC}$ .



**Indirect measurement** uses similar figures to find the length, width, or height of objects that are too difficult to measure directly.

**Real-World EXAMPLE**

- 3 **GEYSERS** Old Faithful in Yellowstone National Park shoots water 60 feet into the air that casts a shadow of 42 feet. What is the height of a nearby tree that casts a shadow 63 feet long? Assume the triangles are similar.



**Tree** Old Faithful

$$\frac{x}{63} = \frac{60}{42} \quad \begin{array}{l} \leftarrow \text{height} \\ \leftarrow \text{shadow} \end{array}$$

$$42x = 60(63) \quad \text{Find the cross products.}$$

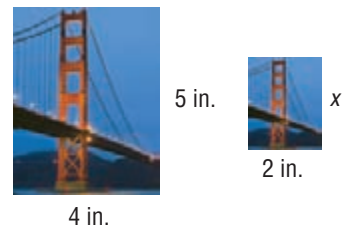
$$42x = 3,780 \quad \text{Simplify.}$$

$$x = 90 \quad \text{Divide each side by 42.}$$

The tree is 90 feet tall.

**CHECK Your Progress**

- c. **PHOTOGRAPHY** Destiny wants to resize a 4-inch wide by 5-inch long photograph for the school newspaper. It is to fit in a space that is 2 inches wide. What is the length of the resized photograph?

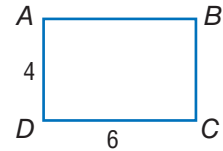
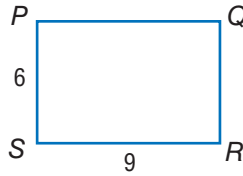
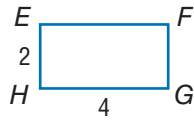




# CHECK Your Understanding

**Example 1**  
(p. 541)

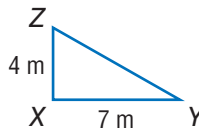
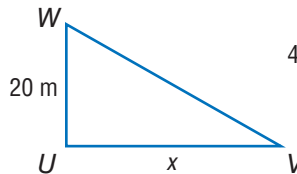
1. Which rectangle below is similar to rectangle  $ABCD$ ?



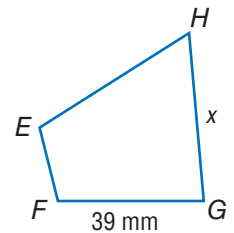
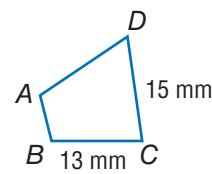
**Example 2**  
(p. 542)

**ALGEBRA** Find the value of  $x$  in each pair of similar figures.

2.

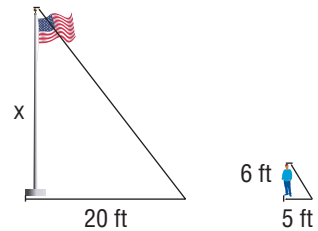


3.



**Example 3**  
(p. 542)

4. **SHADOWS** A flagpole casts a 20-foot shadow. At the same time, Humberto, who is 6 feet tall, casts a 5-foot shadow. What is the height of the flagpole? Assume the triangles are similar.

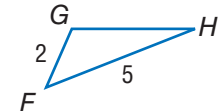
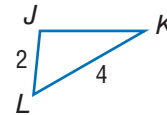
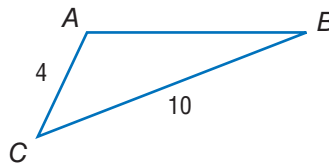
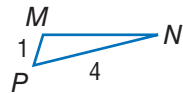


## Practice and Problem Solving

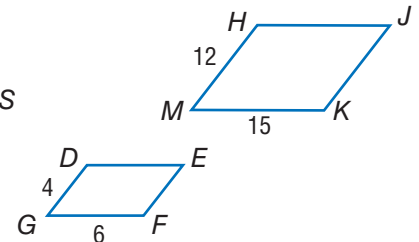
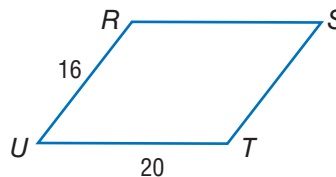
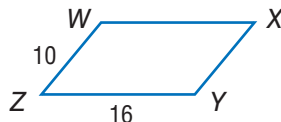
### HOMEWORK HELP

For Exercises	See Examples
5–6	1
7–10	2
11–12	3

5. Which triangle below is similar to triangle  $FGH$ ?

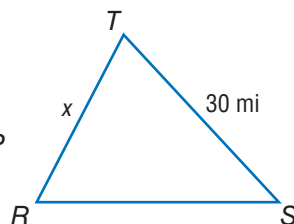
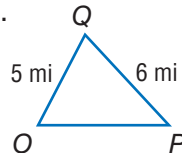


6. Which parallelogram below is similar to parallelogram  $HJKM$ ?

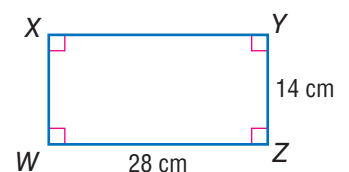
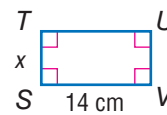


**ALGEBRA** Find the value of  $x$  in each pair of similar figures.

7.

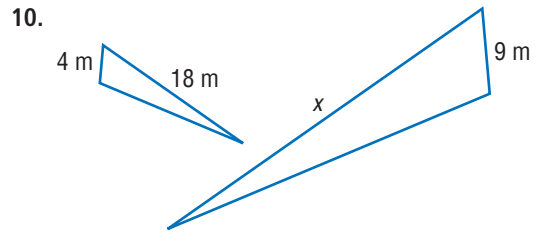
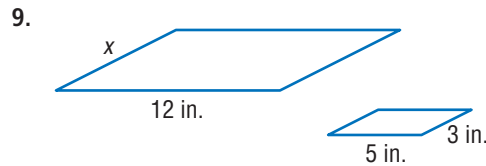


8.

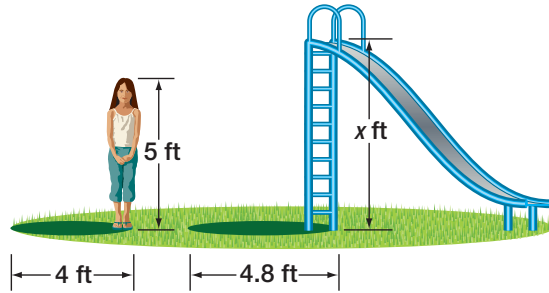




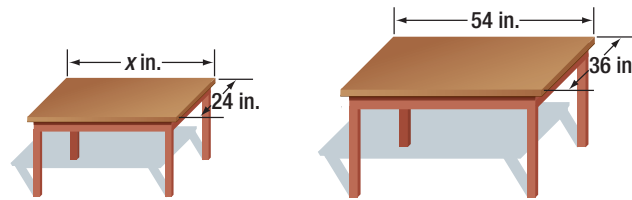
**ALGEBRA** Find the value of  $x$  in each pair of similar figures.



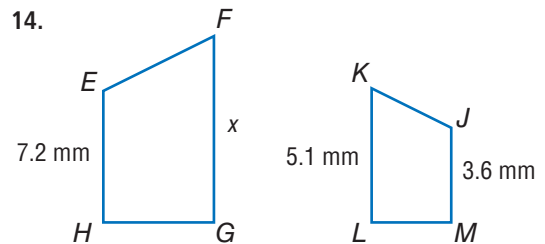
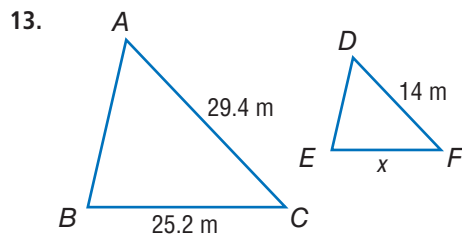
11. **PARKS** Ruth is at the park standing next to a slide. Ruth is 5 feet tall, and her shadow is 4 feet long. If the shadow of the slide is 4.8 feet long, what is the height of the slide? Assume the triangles are similar.



12. **FURNITURE** A child's desk is made so that it is a replica of a full-size adult desk. Suppose the top of the full-size desk measures 54 inches long by 36 inches wide. If the top of a child's desk is 24 inches wide and is similar to the full-size desk, what is the length?



**ALGEBRA** Find the value of  $x$  in each pair of similar figures.



**SKYSCRAPERS** For Exercises 15 and 16, use the information below and at the left.

Tricia has a miniature replica of the Empire State Building. The replica is 10 inches tall, and the height of the observation deck is 8.3 inches.

15. About how tall is the actual observation deck?
16. Tricia's sister has a larger replica in which the height of the observation deck is 12 inches. How tall is the larger replica?

17. **MEASUREMENT** The ratio of the length of square  $A$  to the length of square  $B$  is 3:5. If the length of square  $A$  is 18 meters, what is the perimeter of square  $B$ ?



**Real-World Link** . . . . .

The height to the top of the lightning rod on the Empire State Building is 1,454 feet.

Source: Wired New York



**Academic Standards • ISTEP+**  
Extra Practice, pp. 695, 713

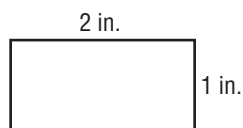
**H.O.T. Problems****CHALLENGE** For Exercises 18 and 19, use the following information.

Two rectangles are similar. The ratio of their corresponding sides is 1 : 4.

18. Find the ratio of their perimeters.
19. What is the ratio of their areas?
20. **WRITING IN MATH** Write a problem about a real-world situation that could be solved using proportions and the concept of similarity. Then use what you have learned in this lesson to solve the problem.

**ISTEP+ PRACTICE** 7.3.5

21. Which rectangle is similar to the rectangle shown?

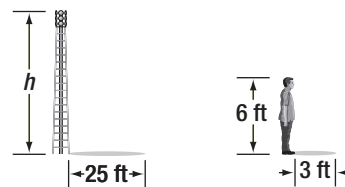


- A 3 ft  
 B 16 yd  
 C 28 yd  
 D 12 m

22. Which of the following equations is a correct use of cross-multiplication in solving the proportion  $\frac{12}{15} = \frac{m}{6}$ ?

- F  $12 \cdot m = 15 \cdot 6$   
 G  $12 \cdot 6 = m \cdot 15$   
 H  $12 \cdot 15 = m \cdot 6$   
 J  $12 \div 6 = m \div 15$

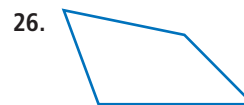
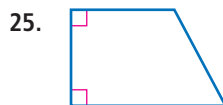
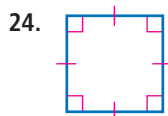
23. Horatio is 6 feet tall and casts a shadow 3 feet long. What is the height of a nearby tower if it casts a shadow 25 feet long at the same time?



- A 25 feet      C 50 feet  
 B 45 feet      D 75 feet

**Spiral Review**

**GEOMETRY** Classify the quadrilateral using the name that *best* describes it. (Lesson 10-6)



27. **MEASUREMENT** A triangular-shaped sail has angle measures of  $44^\circ$  and  $67^\circ$ . Find the measure of the third angle. (Lesson 10-4)

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation. (Lesson 3-3)

28.  $5a = 120$       29.  $360 = 4x$       30.  $940 = 8n$       31.  $6t = 720$



### MAIN IDEA

Classify polygons and determine which polygons can form a tessellation.

### IN Academic Standards

**7.3.4** Recognize, describe, or extend geometric patterns using tables, graphs, words, or symbols. Also addresses P.3.1, P.4.1.

### New Vocabulary

polygon  
 pentagon  
 hexagon  
 heptagon  
 octagon  
 nonagon  
 decagon  
 regular polygon  
 tessellation

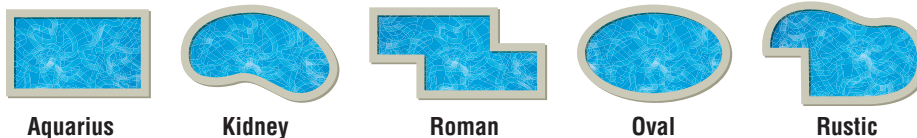
### IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

## ▶ GET READY for the Lesson

**POOLS** Prairie Pools designs and builds swimming pools in various shapes and sizes. The shapes of five swimming pool styles are shown in their catalog.



1. In the pool catalog, the Aquarius and the Roman styles are listed under Group A. The remaining three pools are listed under Group B. Describe one difference between the shapes of the pools in the two groups.
2. Create your own drawing of the shape of a pool that would fit into Group A. Group B.



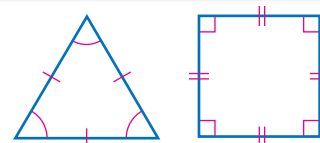
A **polygon** is a simple, closed figure formed by three or more straight line segments. A *simple figure* does not have lines that cross each other. You have drawn a *closed figure* when your pencil ends up where it started.

Polygons	Not Polygons
<ul style="list-style-type: none"> <li>• Line segments are called sides.</li> <li>• Sides meet only at their endpoints.</li> <li>• Points of intersection are called vertices.</li> </ul>	<ul style="list-style-type: none"> <li>• Figures with sides that cross each other.</li> <li>• Figures that are open.</li> <li>• Figures that have curved sides.</li> </ul>

A polygon can be classified by the number of sides it has.

Words	pentagon	hexagon	heptagon	octagon	nonagon	decagon
Number of Sides	5	6	7	8	9	10
Models						

A **regular polygon** has all sides congruent and all angles congruent. Equilateral triangles and squares are examples of regular polygons.





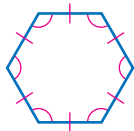
### Reading Math

**Regular Polygons** Since regular polygons have equal-sized angles, they are also called *equiangular*.

## EXAMPLES

### Classify Polygons

1



The figure has 6 congruent sides and 6 congruent angles. It is a regular hexagon.

2



The figure is not a polygon since it has a curved side.



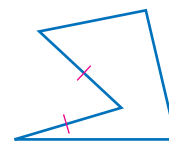
### CHECK Your Progress

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

a.



b.



The sum of the measures of the angles of a triangle is  $180^\circ$ . You can use this relationship to find the measures of the angles of regular polygons.

### Study Tip

**Angle Measures**  
The number of triangles formed is 2 less than the number of sides in the polygon. The equation  $(n - 2) \times 180 = s$  gives the sum  $s$  of angle measures in a polygon with  $n$  sides.

## EXAMPLE

### Angle Measures of a Polygon

3

**ALGEBRA** Find the measure of each angle of a regular pentagon.

- Draw all of the diagonals from one vertex as shown and count the number of triangles formed.
- Find the sum of the angle measures in the polygon.



$$\text{number of triangles formed} \times 180^\circ = \text{sum of angle measures in polygon}$$

$$3 \times 180^\circ = 540^\circ$$

- Find the measure of each angle of the polygon. Let  $n$  represent the measure of one angle in the pentagon.

$$5n = 540 \quad \text{There are five congruent angles.}$$

$$n = 108 \quad \text{Divide each side by 5.}$$

The measure of each angle in a regular pentagon is  $108^\circ$ .



### CHECK Your Progress

Find the measure of an angle in each polygon.

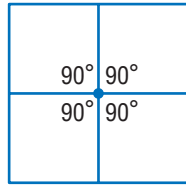
c. regular octagon

d. equilateral triangle

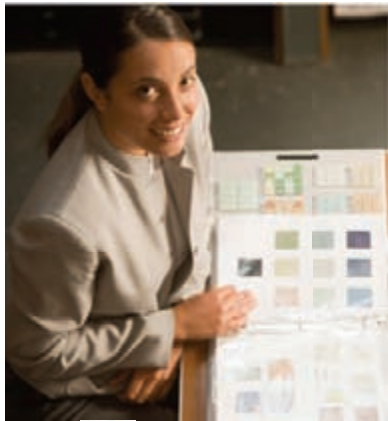


A repetitive pattern of polygons that fit together with no overlaps or holes is called a **tessellation**. The surface of these bricks is an example of a tessellation of squares.

The sum of the measures of the angles where the vertices meet in a tessellation is  $360^\circ$ .



$$4 \times 90^\circ = 360^\circ$$



### Real-World EXAMPLE

**4 DESIGN** Ms. Evans wants to design a floor tile using pentagons. Can Ms. Evans make a tessellation using pentagons?

The measure of each angle in a regular pentagon is  $108^\circ$ .

The sum of the measures of the angles where the vertices meet must be  $360^\circ$ . So, solve  $108^\circ n = 360$ .



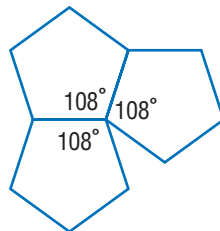
$$108n = 360 \quad \text{Write the equation.}$$

$$\frac{108n}{108} = \frac{360}{108} \quad \text{Divide each side by 108.}$$

$$n \approx 3.3 \quad \text{Use a calculator.}$$

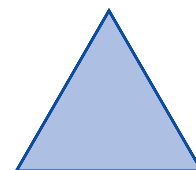
Since  $108^\circ$  does not divide evenly into  $360^\circ$ , the sum of the measures of the angles where the vertices meet is not  $360^\circ$ . So, Ms. Evans cannot make a tessellation using the pentagons.

**Check**



### CHECK Your Progress

e. **DESIGN** Can Ms. Evans use tiles that are equilateral triangles to cover the floor? Explain.



### Real-World Career . . .

**How Does a Textile Designer Use Math?** A textile designer uses math when creating tile patterns for flooring and walls.

### IN Math Online

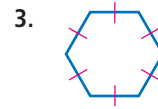
For more information, go to [glencoe.com](http://glencoe.com).



## CHECK Your Understanding

**Examples 1, 2**  
(p. 547)

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



**Example 3**  
(p. 547)

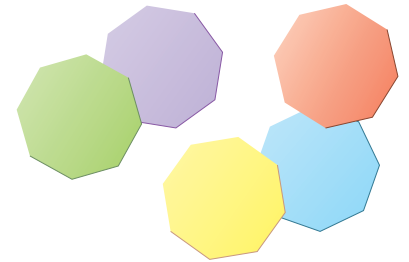
Find the measure of an angle in each polygon if the polygon is regular. Round to the nearest tenth of a degree if necessary.

4. hexagon

5. heptagon

**Example 4**  
(p. 548)

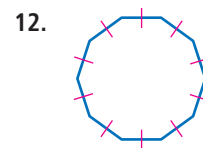
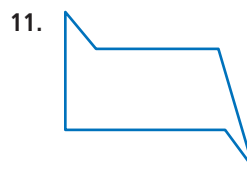
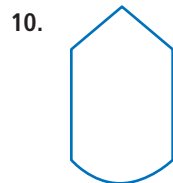
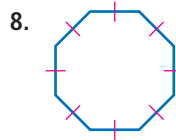
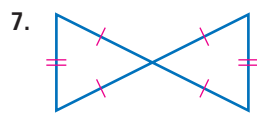
6. **ART** In art class, Trisha traced and then cut several regular octagons out of tissue paper. Can she use the figures to create a tessellation? Explain.



## Practice and Problem Solving

HOMEWORK HELP	
For Exercises	See Examples
7–12	1, 2
13–16	3
17–18	4

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



Find the measure of an angle in each polygon if the polygon is regular. Round to the nearest tenth of a degree if necessary.

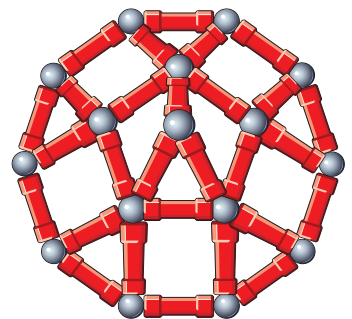
13. decagon

14. nonagon

15. quadrilateral

16. 11-gon

17. **TOYS** Marty used his magnetic building set to build the regular decagon at the right. Assume he has enough building parts to create several of these shapes. Can the figures be arranged in a tessellation? Explain.

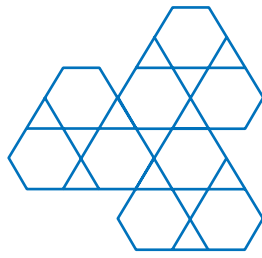


18. **COASTERS** Paper coasters are placed under a beverage glass to protect the table surface. The coasters are shaped like regular heptagons. Can the coasters be arranged in a tessellation? Explain your reasoning.

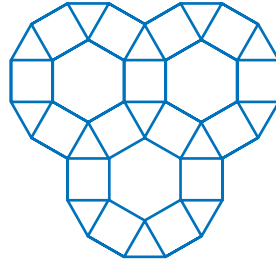


Classify the polygons that are used to create each tessellation.

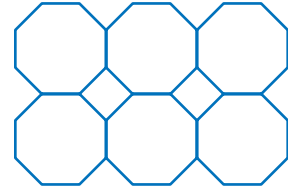
19.



20.



21.



22. What is the perimeter of a regular nonagon with sides 4.8 centimeters?

23. Find the perimeter of a regular pentagon having sides  $7\frac{1}{4}$  yards long.

24. **ART** The mosaic shown at the right is from a marble floor in Venice, Italy. Name the polygons used in the floor.



25. **SIGNS** Refer to the photo at the left. Stop signs are made from large sheets of steel. Suppose one sheet of steel is large enough to cut nine signs. Can all nine signs be arranged on the sheet so that none of the steel goes to waste? Explain.

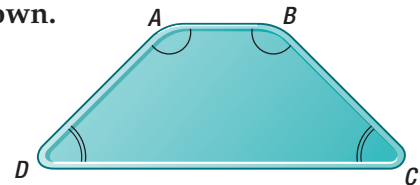


26. **RESEARCH** Use the Internet or another source to find the shape of other road signs. Name the type of sign, its shape, and state whether or not it is regular.



**SCHOOL** For Exercises 27–29, use the information below and the graphic of the cafeteria tray shown.

A company designs cafeteria trays so that four students can place their trays around a square table without bumping corners. The top and bottom sides of the tray are parallel.



27. Classify the shape of the tray.

28. If  $\angle A \cong \angle B$ ,  $\angle C \cong \angle D$ , and  $m\angle A = 135^\circ$ , find  $m\angle B$ ,  $m\angle C$ , and  $m\angle D$ .

29. Name the polygon formed by the outside of four trays when they are placed around the table with their sides touching. Justify your answer.

30. **REASONING** True or False? Only a regular polygon can tessellate a plane.

31. **OPEN ENDED** Draw examples of a pentagon and a hexagon that represent real-world objects.

32. **CHALLENGE** You can make a tessellation with equilateral triangles. Can you make a tessellation with any isosceles or scalene triangles? If so, explain your reasoning and make a drawing.

**Real-World Link** . . . . .



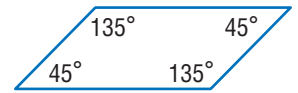
All stop signs have the same shape and meaning. In Spanish-speaking countries, the signs read "ALTO" (halt) or "PARE" (stop).

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Extra Practice, pp. 695, 713

**H.O.T. Problems**

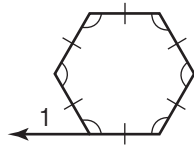


33. **WRITING IN MATH** Analyze the parallelogram at the right and then explain how you know the parallelogram can be used by itself to make a tessellation.



**ISTEP+ PRACTICE** 7.3.4

34. **SHORT RESPONSE** What is the measure of  $\angle 1$ ?



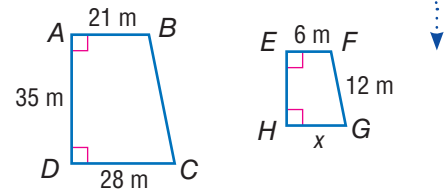
35. Which statement is *not* true about polygons?

- A A polygon is classified by the number of sides it has.
- B The sides of a polygon overlap.
- C A polygon is formed by 3 or more line segments.
- D The sides of a polygon meet only at its endpoints.

**Spiral Review**

For Exercises 36 and 37, use the figures at the right.

36. **ALGEBRA** The quadrilaterals are similar. Find the value of  $x$ . (Lesson 10-7)
37. **GEOMETRY** Classify figure  $ABCD$ . (Lesson 10-6)
38. **PROBABILITY** Two students will be randomly selected from a group of seven to present their reports. If Carla and Pedro are in the group of 7, what is the probability that Carla will be selected first and Pedro selected second? (Lesson 9-4)
39. **RUNNING** Use the information in the table to find the rate of change. (Lesson 6-3)



Distance (m)	15	30	45	60
Time (s)	10	20	30	40

Add or subtract. Write each sum or difference in simplest form. (Lesson 5-3)

40.  $3\frac{2}{9} + 5\frac{4}{9}$       41.  $5\frac{1}{3} - 2\frac{1}{6}$       42.  $1\frac{3}{7} + 6\frac{1}{4}$       43.  $9\frac{4}{5} - 4\frac{7}{8}$

**GET READY for the Next Lesson**

**PREREQUISITE SKILL** Graph and label each point on the same coordinate plane. (Lesson 2-3)

44.  $A(-2, 3)$       45.  $B(4, 3)$       46.  $C(2, -1)$       47.  $D(-4, -1)$

# Geometry Lab Tessellations

In this lab, you will create tessellations.

## MAIN IDEA

Create tessellations.

### IN Academic Standards

#### Preparation for

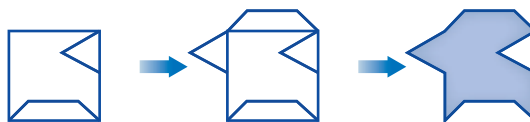
**7.3.2** Identify, describe, and use transformations (translations, rotations and reflections and simple compositions of these transformations) to solve problems.

## ACTIVITY

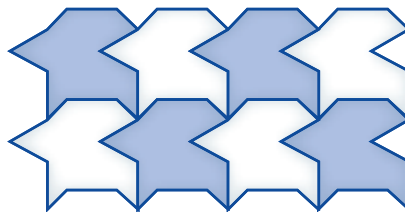
**STEP 1** Draw a square on the back of an index card. Then draw a triangle on the inside of the square and a trapezoid on the bottom of the square as shown.

**STEP 2** Cut out the square. Then cut out the triangle and slide it from the right side of the square to the left side of the square. Cut out the trapezoid and slide it from the bottom to the top of the square.

**STEP 3** Tape the figures together to form a pattern.

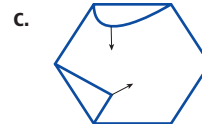
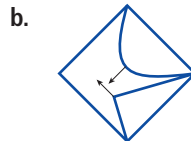
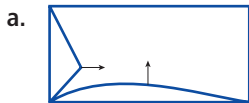


**STEP 4** Trace this pattern onto a sheet of paper as shown to create a tessellation.



## ✓ CHECK Your Progress

Create a tessellation using each pattern.



## ANALYZE THE RESULTS

- Design and describe your own tessellation pattern.
- MAKE A CONJECTURE** *Congruent figures* have corresponding sides of equal length and corresponding angles of equal measure. Explain how congruent figures are used in your tessellation.

# 10-9

# Translations

## MAIN IDEA

Graph translations of polygons on a coordinate plane.

## IN Academic Standards

**7.3.2** Identify, describe and use transformations (translations, rotations, and reflections and simple compositions of the transformations) to solve problems. Also addresses P.3.4.

## New Vocabulary

**transformation**  
**translation**  
**congruent figures**

## IN Math Online

[glencoe.com](http://glencoe.com)

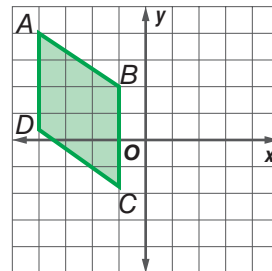
- Concepts In Motion
- Extra Examples
- Personal Tutor
- Self-Check Quiz

## MINI Lab

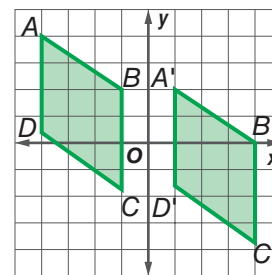
**STEP 1** Trace a parallelogram-shaped pattern block onto a coordinate grid. Label the vertices  $ABCD$ .

**STEP 2** Slide the pattern block over 5 units to the right and 2 units down.

**STEP 3** Trace the figure in its new position. Label the vertices  $A'$ ,  $B'$ ,  $C'$ , and  $D'$ .



1. Trace the horizontal and vertical path between corresponding vertices. What do you notice?
2. Add 5 to each  $x$ -coordinate of the vertices of the original figure. Then subtract 2 from each  $y$ -coordinate of the vertices of the original figure. What do you notice?



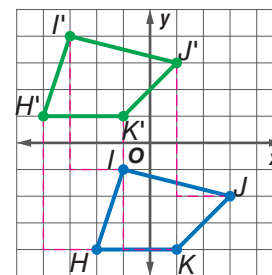
A **transformation** maps one figure onto another. When you move the figure without turning it, the motion is called a **translation**.

When translating a figure, every point of the original figure is moved the same distance and in the same direction.

## EXAMPLE Graph a Translation

- 1 Translate quadrilateral  $HIJK$  2 units left and 5 units up. Graph quadrilateral  $H'I'J'K'$ .

- Move each vertex of the figure 2 units left and 5 units up. Label the new vertices  $H'$ ,  $I'$ ,  $J'$ , and  $K'$ .
- Connect the vertices to draw the trapezoid. The coordinates of the vertices of the new figure are  $H'(-4, 1)$ ,  $I'(-3, 4)$ ,  $J'(1, 3)$ , and  $K'(-1, 1)$ .



## CHECK Your Progress

- a. Translate quadrilateral  $HIJK$  4 units up and 2 units right. Graph quadrilateral  $H'I'J'K'$ .





## Reading Math

**Prime Symbols** Use prime symbols for vertices in a transformed image.

$$A \rightarrow A'$$

$$B \rightarrow B'$$

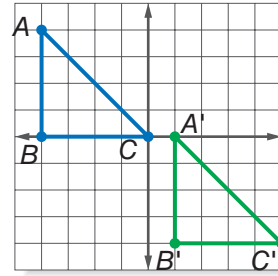
$$C \rightarrow C'$$

$A'$  is read *A prime*.

When a figure has been translated, the original figure and the translated figure, or *image*, are congruent.

**Congruent figures** have the same size and same shape, and the corresponding sides and angles have equal measures.

You can increase or decrease the coordinates of the vertices of a figure by a fixed amount to find the coordinates of the translated vertices.



$$\triangle ABC \cong \triangle A'B'C'$$

A *positive* integer describes a translation right or up on a coordinate plane. A *negative* integer describes a translation left or down.

## EXAMPLE Find Coordinates of a Translation

- 2 Triangle  $LMN$  has vertices  $L(-1, -2)$ ,  $M(6, -3)$ , and  $N(2, -5)$ . Find the vertices of  $\triangle L'M'N'$  after a translation of 6 units left and 4 units up. Then graph the figure and its translated image.

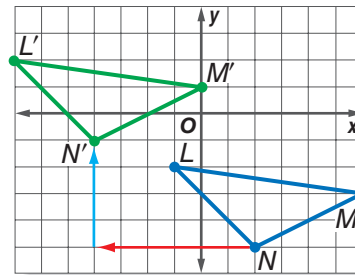
The vertices can be found by adding  $-6$  to the  $x$ -coordinates and 4 to the  $y$ -coordinates.

Add  $-6$  to each  $x$ -coordinate.

Add 4 to each  $y$ -coordinate.

Vertices of $\triangle LMN$	$(x + (-6), y + 4)$	Vertices of $\triangle L'M'N'$
$L(-1, -2)$	$(-1 + (-6), -2 + 4)$	$L'(-7, 2)$
$M(6, -3)$	$(6 + (-6), -3 + 4)$	$M'(0, 1)$
$N(2, -5)$	$(2 + (-6), -5 + 4)$	$N'(-4, -1)$

Use the vertices of  $\triangle LMN$  and of  $\triangle L'M'N'$  to graph each triangle.



## CHECK Your Progress

- b. Triangle  $TUV$  has vertices  $T(6, -3)$ ,  $U(-2, 0)$ , and  $V(-1, 2)$ . Find the vertices of  $\triangle T'U'V'$  after a translation of 3 units right and 4 units down. Then graph the figure and its translated image.

In Example 2,  $\triangle LMN$  was translated 6 units left and 4 units up. This translation can be described using the ordered pair  $(-6, 4)$ . In Check Your Progress b.,  $\triangle TUV$  was translated 3 units right and 4 units down. This translation can be described using the ordered pair  $(3, -4)$ .

## Study Tip

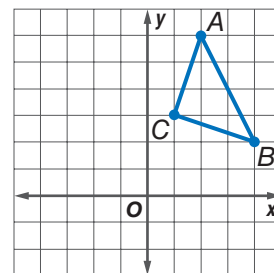
### Transformations

A transformation of the form  $(x', y') = (x + a, y + b)$  is a translation which moves the point  $(x, y)$   $a$  units horizontally and  $b$  units vertically.

# CHECK Your Understanding

**Example 1**  
(p. 553)

1. Translate  $\triangle ABC$  3 units left and 3 units down. Graph  $\triangle A'B'C'$ .

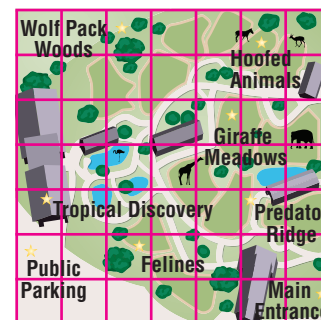


**Example 2**  
(p. 554)

Quadrilateral  $DEFG$  has vertices  $D(1, 0)$ ,  $E(-2, -2)$ ,  $F(2, 4)$ , and  $G(6, -3)$ . Find the vertices of  $D'E'F'G'$  after each translation. Then graph the figure and its translated image.

2. 4 units right, 5 units down
3. 6 units right

4. **MAPS** Nakos explores part of the Denver Zoo in Colorado as shown. He starts at the felines and then visits the hoofed animals. Describe this translation in words and as an ordered pair.

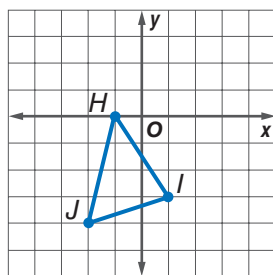


## Practice and Problem Solving

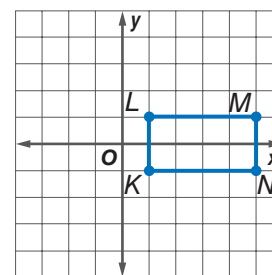
### HOMEWORK HELP

For Exercises	See Examples
5-6	1
7-12	2

5. Translate  $\triangle HIJ$  2 units right and 6 units down. Graph  $\triangle H'I'J'$ .



6. Translate rectangle  $KLMN$  1 unit left and 3 units up. Graph rectangle  $K'L'M'N'$ .

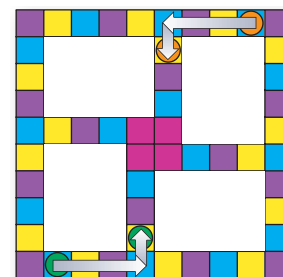


Triangle  $PQR$  has vertices  $P(0, 0)$ ,  $Q(5, -2)$ , and  $R(-3, 6)$ . Find the vertices of  $P'Q'R'$  after each translation. Then graph the figure and its translated image.

7. 6 units right, 5 units up
8. 8 units left, 1 unit down
9. 3 units left
10. 9 units down

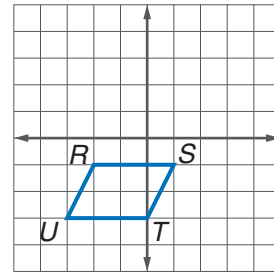
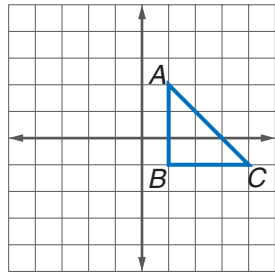
**GAMES** When playing the game shown at the right, the player can move horizontally or vertically across the board. Describe each of the following as a translation in words and as an ordered pair.

11. Green player
12. Orange player





13. Triangle  $ABC$  is translated 2 units left and 3 units down. Then the translated figure is translated 3 units right. Graph the resulting triangle.
14. Parallelogram  $RSTU$  is translated 3 units right and 5 units up. Then the translated figure is translated 2 units left. Graph the resulting parallelogram.



15. **ART** Marjorie Rice creates art using tessellations. At the right is her artwork of fish. Explain how translations and tessellations were used in the figure.
16. **RESEARCH** Use the Internet or another source to find other pieces of art that contain tessellations of translations. Describe how the artists incorporated both ideas into their work.
17. Triangle  $FGH$  has vertices  $F(7, 6)$ ,  $G(3, 4)$ ,  $H(1, 5)$ . Find the coordinates of  $\triangle F'G'H'$  after a translation  $1\frac{1}{2}$  units right and  $3\frac{1}{2}$  units down. Then graph the figure and its translated image.



**REASONING** The coordinates of a point and its image after a translation are given. Describe the translation in words and as an ordered pair.

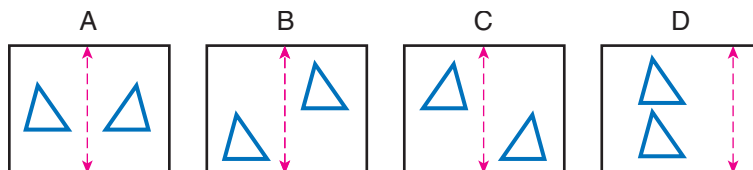
Academic Standards • ISTEP+

Extra Practice, pp. 696, 713

18.  $N(0, -3) \rightarrow N'(2, 2)$       19.  $M(2, 4) \rightarrow M'(-3, 1)$   
 20.  $P(-2, -1) \rightarrow P'(3, -2)$       21.  $Q(-4, 0) \rightarrow Q'(1, 4)$

**H.O.T. Problems**

22. **CHALLENGE** Is it possible to make a tessellation with translations of an equilateral triangle? Explain your reasoning.
23. **Which One Doesn't Belong?** Identify the transformation that is not the same as the other three. Explain your reasoning.



24. **WRITING IN MATH** Triangle  $ABC$  is translated 4 units right and 2 units down. Then the translated image is translated again 7 units left and 5 units up. Describe the final translated image in words.

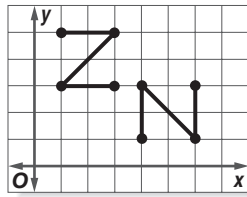


### ISTEP+ PRACTICE

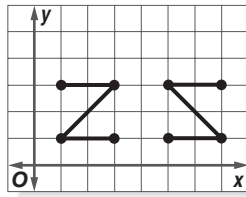
7.3.2

25. Which graph shows a translation of the letter Z?

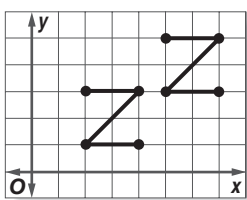
A



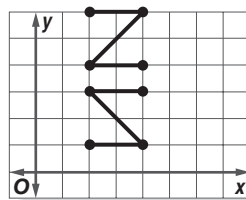
C



B



D



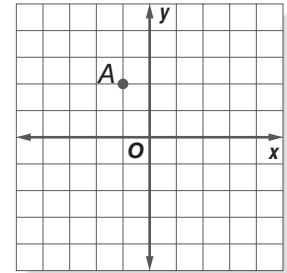
26. If point A is translated 4 units left and 3 units up, what will be the coordinates of point A in its new position?

F (4, 4)

G (-5, 5)

H (-5, -1)

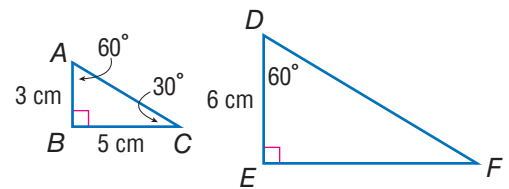
J (-4, 3)



## Spiral Review

27. **GEOMETRY** What is the name of a polygon with eight sides? (Lesson 10-8)

28. **GEOMETRY** The triangles at the right are similar. What is the measure of  $\angle F$ ? (Lesson 10-7)



29. **FOOD** For dinner, you can choose one of two appetizers, one of four entrées, and one of three desserts. How many possible unique dinners can you choose? (Lesson 9-3)

For each set of data, describe how the range would change if the value 15 was added to the data set. (Lesson 8-2)

30. {8, 17, 32}

31. {22, 38, 41, 77}

32. {10, 10, 19}

33. {7, 11, 13}

Write each percent as a decimal. (Lesson 4-7)

34. 83.8%

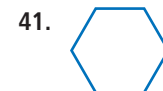
35. 56.7%

36. 3.8%

37. 102.6%

### GET READY for the Next Lesson

**PREREQUISITE SKILL** Determine whether each figure can be folded in half so that one side matches the other. Write *yes* or *no*.



### MAIN IDEA

Identify figures with line symmetry and graph reflections on a coordinate plane.

### IN Academic Standards

**7.3.2 Identify, describe, and use transformations** (translations, rotations, and **reflections** and simple compositions of these transformations) **to solve problems.** Also addresses P.3.4.

### New Vocabulary

**line symmetry**  
**line of symmetry**  
**reflection**  
**line of reflection**

### IN Math Online

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

### MINI Lab

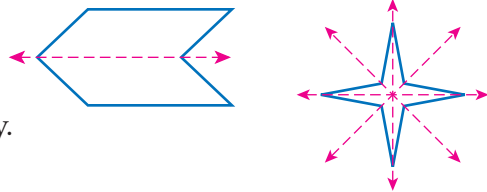
**VISION** Scientists have determined that the human eye uses symmetry to see. It is possible to understand what you are looking at, even if you do not see all of it.

1. The top half of the words at the right are missing. Identify the words.
2. List all the capital letters of the alphabet that, when folded across a horizontal line, look exactly the same.
3. On a piece of paper, write the bottom half of other words that, when reflected across a horizontal line, look exactly the same.

DUX

GUDE

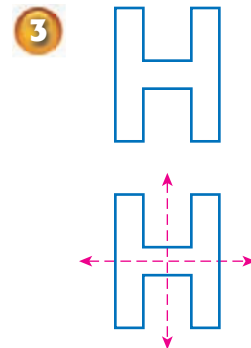
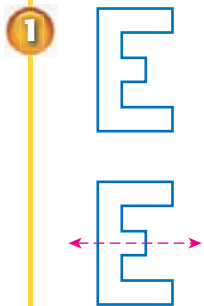
Figures that match exactly when folded in half have **line symmetry**. The figures at the right have line symmetry.



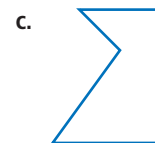
Each fold line is called a **line of symmetry**.

### Real-World EXAMPLES

**GRAPHIC DESIGN** Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.



### CHECK Your Progress



## Study Tip

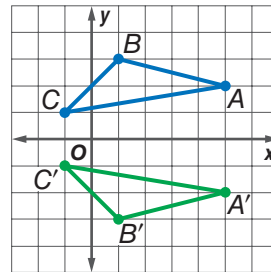
**Congruent Figures** As with translations, the original figure and the reflected image are congruent.

A **reflection** is a mirror image of the original figure. It is the result of a transformation of a figure over a line called a **line of reflection**.

### EXAMPLE Reflect a Figure Over the $x$ -axis

- 4 Triangle  $ABC$  has vertices  $A(5, 2)$ ,  $B(1, 3)$ , and  $C(-1, 1)$ . Graph the figure and its reflected image over the  $x$ -axis. Then find the coordinates of the vertices of the reflected image.

The  $x$ -axis is the line of reflection. So, plot each vertex of  $A'B'C'$  the same distance from the  $x$ -axis as its corresponding vertex on  $ABC$ .



Point  $A$  is 2 units above the  $x$ -axis, ...

... so point  $A'$  is plotted 2 units below the  $x$ -axis.

The coordinates are  $A'(5, -2)$ ,  $B'(1, -3)$ , and  $C'(-1, -1)$ .

### CHECK Your Progress

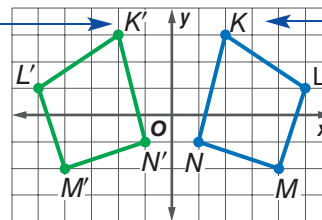
- d. Rectangle  $GHIJ$  has vertices  $G(3, -4)$ ,  $H(3, -1)$ ,  $I(-2, -1)$ , and  $J(-2, -4)$ . Graph the figure and its image after a reflection over the  $x$ -axis. Then find the coordinates of the reflected image.

### EXAMPLE Reflect a Figure Over the $y$ -axis

- 5 Quadrilateral  $KLMN$  has vertices  $K(2, 3)$ ,  $L(5, 1)$ ,  $M(4, -2)$ , and  $N(1, -1)$ . Graph the figure and its reflected image over the  $y$ -axis. Then find the coordinates of the vertices of the reflected image.

The  $y$ -axis is the line of reflection. So, plot each vertex of  $K'L'M'N'$  the same distance from the  $y$ -axis as its corresponding vertex on  $KLMN$ .

Point  $K'$  is 2 units to the left of the  $y$ -axis.



Point  $K$  is 2 units to the right of the  $y$ -axis.

The coordinates are  $K'(-2, 3)$ ,  $L'(-5, 1)$ ,  $M'(-4, -2)$ , and  $N'(-1, -1)$ .

### CHECK Your Progress

- e. Triangle  $PQR$  has vertices  $P(1, 5)$ ,  $Q(3, 7)$ , and  $R(5, -1)$ . Graph the figure and its reflection over the  $y$ -axis. Then find the coordinates of the reflected image.



## CHECK Your Understanding

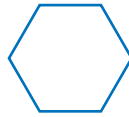
**Examples 1–3**  
(p. 558)

Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

1.



2.



3.



4. **INSECTS** Identify the number of lines of symmetry in the photo of the butterfly at the right.



**Example 4**  
(p. 559)

Graph each figure and its reflection over the  $x$ -axis. Then find the coordinates of the reflected image.

5.  $\triangle ABC$  with vertices  $A(5, 8)$ ,  $B(1, 2)$ , and  $C(6, 4)$

6. quadrilateral  $DEFG$  with vertices  $D(-4, 6)$ ,  $E(-2, -3)$ ,  $F(2, 2)$ , and  $G(4, 9)$

**Example 5**  
(p. 559)

Graph each figure and its reflection over the  $y$ -axis. Then find the coordinates of the reflected image.

7.  $\triangle QRS$  with vertices  $Q(2, -5)$ ,  $R(5, -5)$ , and  $S(2, 3)$

8. parallelogram  $WXYZ$  with vertices  $W(-4, -2)$ ,  $X(-4, 3)$ ,  $Y(-2, 4)$ , and  $Z(-2, -1)$

## Practice and Problem Solving

### HOMEWORK HELP

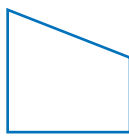
For Exercises	See Examples
9–14 23–24	1, 3
15–18	4
19–22	5

Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

9.



10.



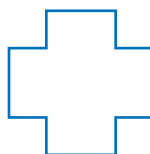
11.



12.



13.



14.



Graph each figure and its reflection over the  $x$ -axis. Then find the coordinates of the reflected image.

15.  $TUV$  with vertices  $T(-6, -1)$ ,  $U(-2, -3)$ , and  $V(5, -4)$

16.  $MNP$  with vertices  $M(2, 1)$ ,  $N(-3, 1)$ , and  $P(-1, 4)$

17. square  $ABCD$  with vertices  $A(2, 4)$ ,  $B(-2, 4)$ ,  $C(-2, 8)$ , and  $D(2, 8)$

18.  $WXYZ$  with vertices  $W(-1, -1)$ ,  $X(4, 1)$ ,  $Y(4, 5)$ , and  $Z(1, 7)$



Graph each figure and its reflection over the  $y$ -axis. Then find the coordinates of the reflected image.

19.  $\triangle RST$  with vertices  $R(-5, 3)$ ,  $S(-4, -2)$ , and  $T(-2, 3)$
20.  $\triangle GHJ$  with vertices  $G(4, 2)$ ,  $H(3, -4)$ , and  $J(1, 1)$
21. parallelogram  $HJK$  with vertices  $H(-1, 3)$ ,  $I(-1, -1)$ ,  $J(2, -2)$ , and  $K(2, 2)$
22. quadrilateral  $DEFG$  with vertices  $D(1, 0)$ ,  $E(1, -5)$ ,  $F(4, -1)$ , and  $G(3, 2)$

23. **GATES** Describe the location of the line(s) of symmetry in the photograph of Brandenburg Gate in Berlin, Germany.



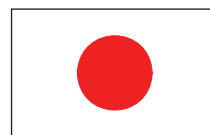
24. **FLAGS** Flags of some countries have line symmetry. Of the flags shown below, which flags have line symmetry? Copy and draw all lines of symmetry.



Nigeria



Ghana



Japan

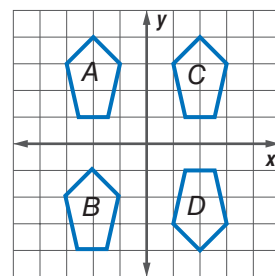


Mexico

25. **MUSIC** Use the photo at the left to determine how many lines of symmetry the body of a violin has.

For Exercises 26–29, use the graph shown at the right.

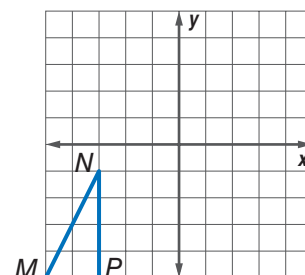
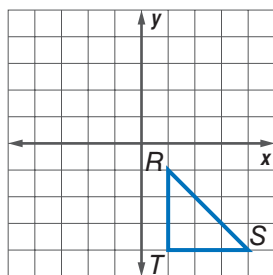
26. Identify the pair(s) of figures for which the  $x$ -axis is the line of reflection.
27. For which pair(s) of figures is the line of reflection the  $y$ -axis?
28. What type of transformation do figures  $B$  and  $C$  represent?
29. Describe the possible transformation(s) required to move figure  $A$  onto figure  $D$ .
30.  $\triangle RST$  is reflected over the  $x$ -axis and then translated 3 units to the left and 2 units down. Graph the resulting triangle.
31.  $\triangle MNP$  is translated 2 units right and 3 units up. Then the translated figure is reflected over the  $y$ -axis. Graph the resulting triangle.



**Real-World Link** . . . . .

A violin is usually around 14 inches long.

**Academic Standards • ISTEP+**  
Extra Practice, pp. 696, 713





The coordinates of a point and its image after a reflection are given. Describe the reflection as over the  $x$ -axis or  $y$ -axis.

32.  $A(-3, 5) \rightarrow A'(3, 5)$

33.  $M(3, 3) \rightarrow M'(3, -3)$

34.  $X(-1, -4) \rightarrow X'(-1, 4)$

35.  $W(-4, 0) \rightarrow W'(4, 0)$

**H.O.T. Problems**

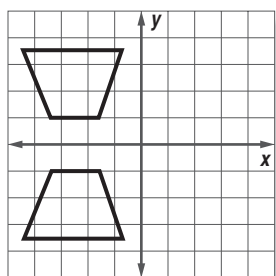
36. **OPEN ENDED** Make a tessellation using a combination of translations and reflections of polygons. Explain your method.

37. **CHALLENGE** Triangle  $JKL$  has vertices  $J(-7, 4)$ ,  $K(7, 1)$ , and  $L(2, -2)$ . Without graphing, find the new coordinates of the vertices of the triangle after a reflection first over the  $x$ -axis and then over the  $y$ -axis.

38. **WRITING IN MATH** Draw a figure on a coordinate plane and its reflection over the  $y$ -axis. Explain how the  $x$ - and  $y$ -coordinates of the reflected figure relate to the  $x$ - and  $y$ -coordinates of the original figure. Then repeat, this time reflecting the figure over the  $x$ -axis.

**ISTEP+ PRACTICE 7.3.2**

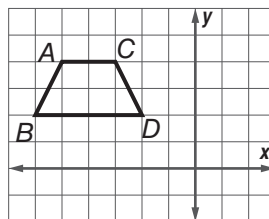
39. The figure shown was transformed from quadrant II to quadrant III.



This transformation best represents which of the following?

- A translation      C reflection  
B tessellation    D rotation

40. If  $ABCD$  is reflected over the  $x$ -axis and translated 5 units to the right, which is the resulting image of point  $B$ ?



- F  $(-1, -2)$       H  $(-1, 2)$   
G  $(-11, 2)$      J  $(11, 2)$

**Spiral Review**

41. **GEOMETRY** Triangle  $FGH$  has vertices  $F(-3, 7)$ ,  $G(-1, 5)$ , and  $H(-2, 2)$ . Graph the figure and its image after a translation 4 units right and 1 unit down. Write the ordered pairs for the vertices of the image. (Lesson 10-9)

42. **GEOMETRY** Melissa wishes to construct a tessellation for a wall hanging made only from regular decagons. Is this possible? Explain. (Lesson 10-8)

Estimate. (Lesson 5-1)

43.  $\frac{4}{9} + 8\frac{1}{9}$

44.  $\frac{1}{9} \times \frac{2}{5}$

45.  $12\frac{1}{4} \div 5\frac{6}{7}$

**FOLDABLES**  
 Study Organizer

**GET READY to Study**

Be sure the following Big Ideas are noted in your Foldable.

What I Know About Polygons	What I Need to Know	What I've Learned

**BIG Ideas**
**Angles** (Lessons 10-1 and 10-2)

- Two angles are adjacent if they have the same vertex, share a common side, and do not overlap.
- Two angles are vertical if they are opposite angles formed by the intersection of two lines.
- Two angles are complementary if the sum of their measures is  $90^\circ$ .
- Two angles are supplementary if the sum of their measures is  $180^\circ$ .

**Triangles** (Lesson 10-4)

- The sum of the measures of the angles of a triangle is  $180^\circ$ .

**Quadrilaterals** (Lesson 10-6)

- The sum of the measures of the angles of a quadrilateral is  $360^\circ$ .

**Similar Figures** (Lesson 10-7)

- If two figures are similar then the corresponding sides are proportional and the corresponding angles are congruent.

**Transformations** (Lessons 10-9 and 10-10)

- When translating a figure, every point in the original figure is moved the same distance in the same direction.
- When reflecting a figure, every point in the original figure is the same distance from the line of reflection as its corresponding point on the original figure.

**Key Vocabulary**

<b>acute angle</b> (p. 511)	<b>pentagon</b> (p. 546)
<b>angle</b> (p. 510)	<b>polygon</b> (p. 546)
<b>circle graph</b> (p. 518)	<b>quadrilateral</b> (p. 533)
<b>complementary angles</b> (p. 514)	<b>reflection</b> (p. 559)
<b>congruent angles</b> (p. 511)	<b>regular polygon</b> (p. 546)
<b>congruent figures</b> (p. 554)	<b>rhombus</b> (p. 533)
<b>degrees</b> (p. 510)	<b>right angle</b> (p. 511)
<b>hexagon</b> (p. 546)	<b>similar figures</b> (p. 540)
<b>indirect measurement</b> (p. 542)	<b>straight angle</b> (p. 511)
<b>line of symmetry</b> (p. 558)	<b>supplementary angles</b> (p. 514)
<b>line symmetry</b> (p. 558)	<b>tessellation</b> (p. 548)
<b>obtuse angle</b> (p. 511)	<b>transformation</b> (p. 553)
<b>octagon</b> (p. 546)	<b>translation</b> (p. 553)
<b>parallelogram</b> (p. 533)	<b>trapezoid</b> (p. 533)
	<b>triangle</b> (p. 524)
	<b>vertex</b> (p. 510)

**Vocabulary Check**

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

- Two angles with measures adding to  $180^\circ$  are called complementary angles.
- A hexagon is a polygon with 6 sides.
- An angle with a measure of less than  $90^\circ$  is called a right angle.
- The vertex is where the sides of an angle meet.
- The point  $(3, -2)$  when translated up 3 units and to the left 5 units becomes  $(6, -7)$ .
- A trapezoid has both pairs of opposite sides parallel.

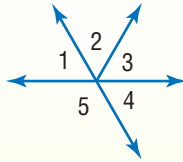
## Lesson-by-Lesson Review

## 10-1 Angle Relationships (pp. 510–513)



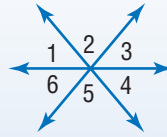
6.3.1

For Exercises 7 and 8, refer to the figure at the right to identify each pair of angles. Justify your response.



7. a pair of vertical angles
8. a pair of adjacent angles

**Example 1** Refer to the figure below. Identify a pair of vertical angles.



$\angle 1$  and  $\angle 4$  are opposite angles formed by the intersection of two lines.

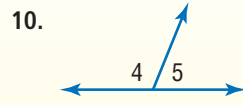
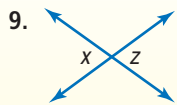
$\angle 1$  and  $\angle 4$  are vertical angles.

## 10-2 Complementary and Supplementary Angles (pp. 514–517)



6.3.1

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.

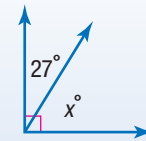


**Example 2** Find the value of  $x$ .

$$x + 27 = 90$$

$$\underline{-27 = -27}$$

$$x = 63$$



## 10-3 Statistics: Display Data in a Circle Graph (pp. 518–523)



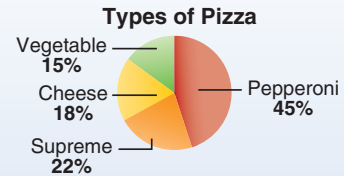
7.4.1

11. **COLORS** The table shows favorite shades of blue. Display the set of data in a circle graph.

Shade	Percent
Navy	35%
Sky/Light Blue	30%
Aquamarine	17%
Other	18%

**Example 3** Which pizza was chosen by about twice as many people as supreme?

Pepperoni was chosen by about twice as many people as supreme.

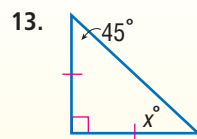
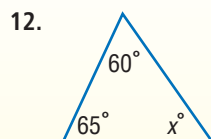


## 10-4 Triangles (pp. 524–529)



6.3.2

**ALGEBRA** Find the value of  $x$ .



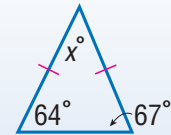
**Example 4** Find the value of  $x$ .

$$x + 64 + 67 = 180$$

$$x + 131 = 180$$

$$\underline{-131 = -131}$$

$$x = 49$$



**10-5** **PSI: Logical Reasoning** (pp. 530–531)



14. **SPORTS** Donnie, Jenna, Milo, and Barbara play volleyball, field hockey, golf, and soccer but not in that order. Use the clues given below to find the sport each person plays.
- Donnie does not like golf, volleyball, or soccer.
  - Neither Milo nor Jenna likes golf.
  - Milo does not like soccer.
15. **FOOD** Angelo's Pizza Parlor makes square pizzas. After baking, the pizzas are cut along one diagonal into two triangles. Classify the triangles made.

**Example 5** Todd, Virginia, Elaine, and Peter are siblings. Todd was born after Peter, but before Virginia. Elaine is the oldest. Who is the youngest in the family?

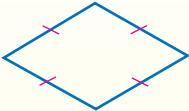

Use logical reasoning to determine the youngest of the family.

You know that Elaine is the oldest, so she is first on the list. Todd was born after Peter, but before Virginia. So, Peter was second and then Todd was born. Virginia is the youngest of the family.

**10-6** **Quadrilaterals** (pp. 533–538)

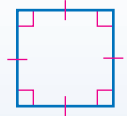


Classify the quadrilateral with the name that best describes it.

16. 
17. 

18. **GEOMETRY** What quadrilateral does not have opposite sides congruent?

**Example 6** Classify the quadrilateral using the name that *best* describes it.

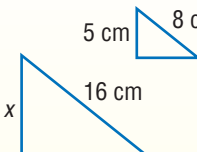
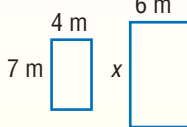


The quadrilateral is a parallelogram with 4 right angles and 4 congruent sides. It is a square.

**10-7** **Similar Figures** (pp. 540–545)



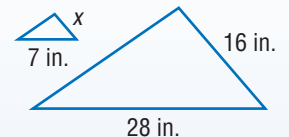
Find the value of  $x$  in each pair of similar figures.

19. 
20. 

21. **FLAGPOLES** Hiroshi is 1.6 meters tall and casts a shadow 0.53 meter in length. How tall is a flagpole if it casts a shadow 2.65 meters in length?

**Example 7**

Find the value of  $x$  in the pair of similar figures.



$$\frac{7}{28} = \frac{x}{16}$$

$$28 \cdot x = 7 \cdot 16$$

$$28x = 112$$

$$\frac{28x}{28} = \frac{112}{28}$$

$$x = 4$$

So, the value of  $x$  is 4.

Write a proportion.

Find the cross products.

Simplify.

Divide each side by 28.

Simplify.

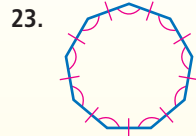
## 10-8

## Polygons and Tessellations (pp. 546–551)



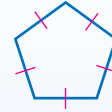
7.3.4

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



24. **ALGEBRA** Find the measure of each angle of a regular 12-gon.

**Example 8** Determine whether the figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



Since the polygon has 5 congruent sides and 5 congruent angles, it is a regular pentagon.

## 10-9

## Translations (pp. 553–557)



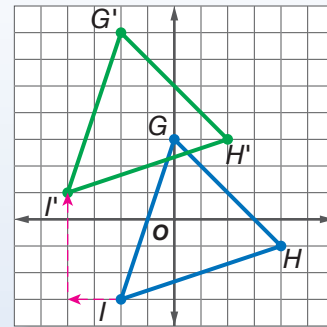
7.3.2

Triangle  $PQR$  has coordinates  $P(4, -2)$ ,  $Q(-2, -3)$ , and  $R(-1, 6)$ . Find the coordinates of  $P'Q'R'$  after each translation. Then graph each translation.

25. 6 units left, 3 units up
26. 4 units right, 1 unit down
27. 3 units left
28. 7 units down

**Example 9** Find the coordinates of  $\triangle G'H'I'$  after a translation of 2 units left and 4 units up.

The vertices of  $\triangle GHI$  are  $G(-2, 7)$ ,  $H(2, 3)$ , and  $I(-4, 1)$ .



## 10-10

## Reflections (pp. 558–562)



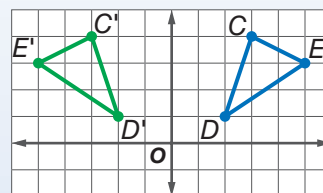
7.3.2

Find the coordinates of each figure after a reflection over the given axis. Then graph the figure and its reflected image.

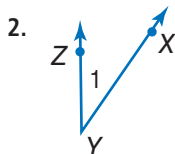
29.  $\triangle RST$  with coordinates  $R(-1, 3)$ ,  $S(2, 6)$ , and  $T(6, 1)$ ;  $x$ -axis
30. parallelogram  $ABCD$  with coordinates  $A(1, 3)$ ,  $B(2, -1)$ ,  $C(5, -1)$ , and  $D(4, 3)$ ;  $y$ -axis
31. rectangle  $EFGH$  with coordinates  $E(4, 2)$ ,  $F(-2, 2)$ ,  $G(-2, 5)$ , and  $H(4, 5)$ ;  $x$ -axis

**Example 10** Find the coordinates of  $\triangle C'D'E'$  after a reflection over the  $y$ -axis. Then graph its reflected image.

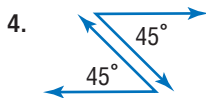
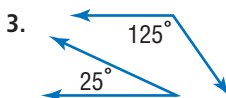
The vertices of  $\triangle CDE$  are  $C(-3, 4)$ ,  $D(-2, 1)$ , and  $E(-5, 3)$ .



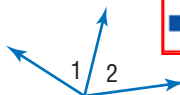
Name each angle in four ways. Then classify each angle as *acute*, *obtuse*, *right*, or *straight*.



Classify each pair of angles as *complementary*, *supplementary*, or *neither*.



5. **GEOMETRY** Classify the angle pair at the right as *vertical*, *adjacent*, or *neither*.



6. **MULTIPLE CHOICE** The table shows the results of a survey. The results are to be displayed in a circle graph. Which statement about the graph is *not* true?

Favorite Type of Bagels	
Type	Students
blueberry	8
cinnamon raisin	9
everything	18
plain	32

- A About 12% of students chose blueberry as their favorite bagel.
- B The blueberry section on the graph will have an angle measure of about  $43^\circ$ .
- C The everything and plain sections on the circle graph form supplementary angles.
- D Plain bagels were preferred more than any other type of bagel.

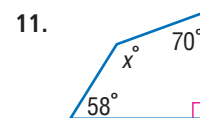
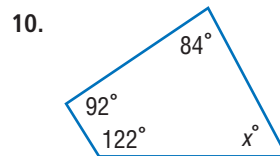
**ALGEBRA** Find the missing measure in each triangle with the given angle measures.

- 7.  $75^\circ$ ,  $25.5^\circ$ ,  $x^\circ$
- 8.  $23.5^\circ$ ,  $x^\circ$ ,  $109.5^\circ$



9. **ALGEBRA** Numbers ending in zero or five are divisible by five. Are the numbers 25, 893, and 690 divisible by 5? Use the *logical reasoning* strategy.

**ALGEBRA** Find the value of  $x$  in each quadrilateral.



12. **ART** A drawing is enlarged so that it is 14 inches long and 11 inches wide. If the original length of the drawing is 8 inches, what is its width?



13. **GEOMETRY** Can a regular heptagon, with angle measures that total  $900^\circ$ , be used by itself to make a tessellation? Explain.

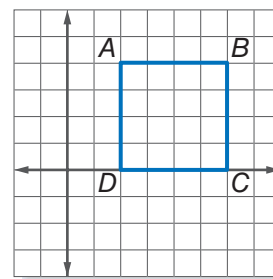


14. **MULTIPLE CHOICE** Which quadrilateral does *not* have opposite sides congruent?

- F parallelogram
- G square
- H trapezoid
- J rectangle



15. **ALGEBRA** Square  $ABCD$  is shown. What are the vertices of  $A'B'C'D'$  after a translation 2 units right and 2 units down? Graph the translated image.

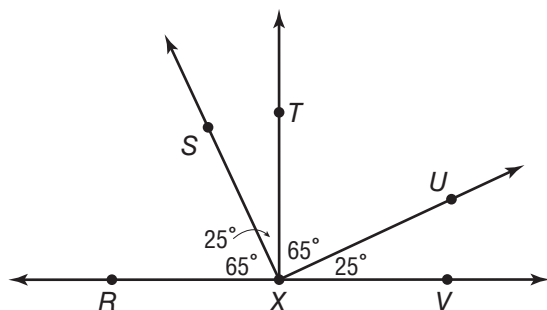


16. **GEOMETRY** Draw a figure with one line of symmetry. Then draw a figure with no lines of symmetry.

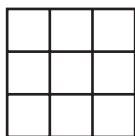
**PART 1 Multiple Choice**

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following two angles are complementary?



- A  $\angle RXS$  and  $\angle TXU$   
 B  $\angle SXT$  and  $\angle TXU$   
 C  $\angle RXS$  and  $\angle SXV$   
 D  $\angle SXR$  and  $\angle SXV$
2. A square is divided into 9 congruent squares. Which of the following methods can be used to find the area of the larger square, given the area of one of the smaller squares?



- F Multiply the area of the larger square by 9.  
 G Add 9 to the area of one of the smaller squares.  
 H Multiply the area of one of the smaller squares by 9.  
 J Add the area of the larger square to the sum of the areas of each of the 9 smaller squares.

3. Which of the following groups does *not* contain equivalent fractions, decimals, and percents?

- A  $\frac{9}{20}$ , 0.45, 45%  
 B  $\frac{3}{10}$ , 0.3, 30%  
 C  $\frac{7}{8}$ , 0.875, 87.5%  
 D  $\frac{1}{100}$ , 0.1, 1%

4. The table below shows all the possible outcomes when tossing two fair coins at the same time.

1st Coin	2nd Coin
H	H
H	T
T	H
T	T

Which of the following must be true?

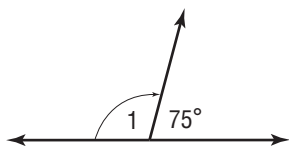
- F The probability that both coins have the same outcome is  $\frac{1}{4}$ .  
 G The probability of getting at least one tail is higher than the probability of getting two heads.  
 H The probability that exactly one coin will turn up heads is  $\frac{3}{4}$ .  
 J The probability of getting at least one tail is lower than the probability of getting two tails.
5. Seth has \$858.60 in his savings account. He plans to spend 15% of his savings on a bicycle. Which of the following represents the amount Seth plans to spend on the bicycle?
- A \$182.79                      C \$128.79  
 B \$171.72                      D \$122.79



6. A manager took an employee to lunch. If the lunch was \$48 and she left a 20% tip, how much money did she spend on lunch?

- F \$68.00                      H \$55.80
- G \$57.60                      J \$38.40

7. What is the measure of  $\angle 1$  in the figure below?



- A  $15^\circ$                               C  $100^\circ$
- B  $25^\circ$                               D  $105^\circ$

8. Josiah found the mean and median of the following list of numbers.

11, 17, 17

If the number 25 is added to this list, then which of the following statements would be true?

- F The mean would increase.
- G The mean would decrease.
- H The median would increase.
- J The median would decrease.

**TEST-TAKING TIP**

**Question 8** Sometimes, it is not necessary to perform any calculations in order to answer the question correctly. In Question 8, you can use number sense to eliminate certain answer choices. Not having to perform calculations can help save time during a test.

**PART 2 Short Response/Grid In**

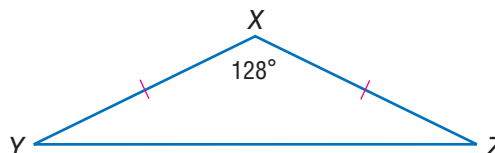
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Three students are to be chosen from 8 auditions to star in the school play. In how many ways can these 3 students be chosen?

**PART 3 Extended Response**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper. Show your work.

10. Use triangle XYZ to answer the following questions.



- a. Classify angle X.
- b. Classify angle Y.
- c. Classify the triangle by its sides and angles.
- d. If  $\angle Y$  is congruent to  $\angle Z$ , find the measure of  $\angle Z$ . Explain.
- e. Can triangle XYZ be used by itself to make a tessellation? If so, include a drawing of the tessellation. If not, explain why not.

**NEED EXTRA HELP?**

If You Missed Question...

Go to Lesson...

**IN Academic Standards**

1	2	3	4	5	6	7	8	9	10
10-1	10-5	6-8	9-1	7-1	7-4	10-2	8-2	9-5	10-5
6.3.1	P.2.1	7.3.5	7.4.5	7.1.8	7.1.8	6.3.1	7.4.3	7.4.5	P.2.1