



CHAPTER

12

Geometry and Measurement

Indiana Academic Standards

Reinforcement of 6.3.5

Develop and use the formulas for the surface area and volume of a cylinder and find the surface area and volume of three-dimensional objects built from rectangular solids and cylinders.

7.1.5 Recognize and use the inverse relationship between squaring and finding the square root of a perfect square integer.

Key Vocabulary

hypotenuse (p. 640)

irrational number (p. 637)

Pythagorean Theorem (p. 640)

surface area (p. 649)

Real-World Link

Spaghetti The shape of many spaghetti boxes are rectangular prisms, and the shape of many cans are cylinders. You can use the formula $S = 2lw + 2lh + 2wh$ to find the surface area of a box of spaghetti given the length l , the width w , and the height h of the box.



FOLDABLES

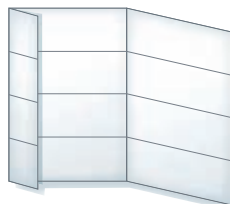
Study Organizer

Geometry and Measurement Make this Foldable to help you organize your notes. Begin with a piece of 11" by 17" paper.

1 **Fold** the paper in fourths lengthwise.



2 **Open** and fold a 2" tab along the short side. Then fold the rest in half.



3 **Draw** lines along the folds and label as shown.

Ch. 12	Rectangular Prisms	Cylinders
Draw Examples		
Find Volume		
Find Surface Area		

GET READY for Chapter 12

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

IN Math Online

Take the Online Readiness Quiz at glencoe.com.

Option 1

Take the Quick Quiz below. Refer to the Quick Review for help.

QUICK Quiz

Evaluate each expression. (Lesson 1-2)

- 4^2
- 7^2
- 13^2
- 24^2
- $5^2 + 8^2$
- $10^2 + 6^2$
- $9^2 + 12^2$
- $15^2 + 17^2$
- AGES** Samuel's mother is 7^2 years old, and his grandmother is 9^2 years old. Find the sum of their ages. (Lesson 1-2)

Evaluate the expression $2ab + 2bc + 2ac$ for each value of the variables indicated. (Lesson 1-6)

- $a = 4, b = 5, c = 8$
- $a = 2, b = 7, c = 11$
- $a = 3.1, b = 2.4, c = 9.9$
- $a = 2.1, b = 1.7, c = 4.6$

Use the π button on your calculator to evaluate each expression below. Round to the nearest tenth. (Lesson 11-3)

- $(2)(\pi)(3^2) + (2)(\pi)(3)(8)$
- $(2)(\pi)(7^2) + (2)(\pi)(7)(5)$

QUICK Review

Example 1

Evaluate $3^2 + 5^2$.

$$\begin{aligned} 3^2 + 5^2 &= 9 + 25 && \text{Evaluate } 3^2 \text{ and } 5^2. \\ &= 34 && \text{Add 9 and 25.} \end{aligned}$$

Example 2

Evaluate the expression $2ab + 2bc + 2ac$ for $a = 3, b = 5,$ and $c = 6$.

$$\begin{aligned} 2ab + 2bc + 2ac &= 2(3)(5) + 2(5)(6) + 2(3)(6) && \text{Replace } a \text{ with 3,} \\ &= 30 + 60 + 36 && \text{b with 5, and c} \\ &= 126 && \text{with 6.} \\ & && \text{Multiply.} \\ & && \text{Add.} \end{aligned}$$

Example 3

Use the π button on your calculator to evaluate $(2)(\pi)(4^2) + (2)(\pi)(4)(6)$. Round to the nearest tenth.

$$\begin{aligned} (2)(\pi)(4^2) + (2)(\pi)(4)(6) &= (2)(\pi)(16) + (2)(\pi)(4)(6) && \text{Evaluate } 4^2. \\ &= (32)(\pi) + (48)(\pi) && \text{Multiply.} \\ &\approx 251.3 && \text{Multiply and add.} \end{aligned}$$

12-1

Estimating Square Roots

MAIN IDEA

Estimate square roots.

IN Academic Standards

7.1.5 Recognize and use the inverse relationship between squaring and finding the square root of a perfect square integer.
7.1.6 Identify, write, rename, compare and order rational and common irrational numbers and plot them on a number line. Also addresses 7.2.3, P.6.1, P.6.3.

New Vocabulary

irrational number

IN Math Online

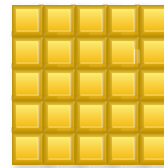
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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Reading in the Content Area

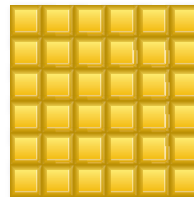
MINI Lab

Estimate the square root of 27.

- Arrange 27 tiles into the largest square possible. You will use 25 tiles and 2 will remain.
- Add tiles to make the next larger square. So, add 9 tiles to make a square with 36 tiles.
- The square root of 27 is between 5 and 6. Since 27 is much closer to 25 than 36, we can expect that the square root of 27 is closer to 5 than 6.



The square root of 25 is 5.



The square root of 36 is 6.

Use algebra tiles to estimate the square root of each number to the nearest whole number.

- 40
- Describe another method that you could use to estimate the square root of a number.



The square root of a perfect square is an integer. You can estimate the square root of a number that is *not* a perfect square.

EXAMPLE Estimate a Square Root

- Estimate $\sqrt{78}$ to the nearest whole number.

List some perfect squares.

1, 4, 9, 16, 25, 36, 49, 64, 81, ...



$$64 < 78 < 81 \quad 78 \text{ is between the perfect squares } 64 \text{ and } 81.$$

$$\sqrt{64} < \sqrt{78} < \sqrt{81} \quad \text{Find the square root of each number.}$$

$$8 < \sqrt{78} < 9 \quad \sqrt{64} = 8 \text{ and } \sqrt{81} = 9$$



So, $\sqrt{78}$ is between 8 and 9. Since 78 is much closer to 81 than to 64, the best whole number estimate is 9. **Verify with a calculator.**

CHECK Your Progress

- Estimate $\sqrt{50}$ to the nearest whole number.



... A number that cannot be expressed as the quotient of two integers is an **irrational number**.



Vocabulary Link

Irrational

Everyday Use lacking usual or normal clarity, as in irrational thinking

Math Use a number that cannot be expressed as the quotient of two integers

Irrational Numbers $\sqrt{2}$, π , 0.636336333...

The square root of any number that is not a perfect square is an irrational number. You can use a calculator to estimate square roots that are irrational numbers.

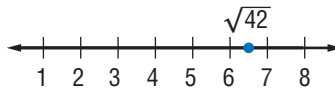
EXAMPLE

Graph Square Roots on a Number Line

2 Graph $\sqrt{42}$ on a number line.

`2nd` `[√]` `42` `ENTER` 6.480740698

$\sqrt{42} \approx 6.5$



Check for Reasonableness

$6^2 = 36$ and $7^2 = 49$. Since 42 is between 36 and 49, the answer, 6.5, is reasonable.



CHECK Your Progress

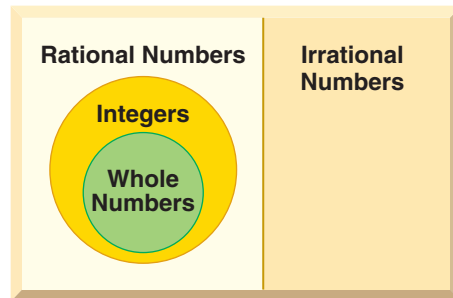
Graph each square root on a number line.

b. $\sqrt{6}$

c. $\sqrt{23}$

d. $\sqrt{309}$

The Venn diagram shows the relationship among sets of numbers.



Whole Numbers: 0, 1, 2, 3, ...

Integers: ..., -2, -1, 0, 1, 2, ...

Rational Numbers: $\frac{1}{2}$, 0.25, -0.2, 0.333...

Irrational Numbers: π , $\sqrt{2}$, 0.124543...



CHECK Your Understanding

Example 1 Estimate each square root to the nearest whole number.

(p. 636)

1. $\sqrt{39}$

2. $\sqrt{106}$

3. $\sqrt{90}$

4. $\sqrt{140}$

Example 2 Graph each square root on a number line.

(p. 637)

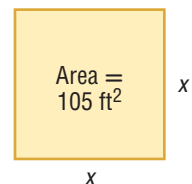
5. $\sqrt{7}$

6. $\sqrt{51}$

7. $\sqrt{135}$

8. $\sqrt{462}$

9. **MEASUREMENT** The diagram at the right shows the floor plan of a square kitchen. What is the approximate length of one side of the kitchen floor to the nearest tenth?





Practice and Problem Solving

HOMework HELP

For Exercises	See Examples
10–17, 26, 27	1
18–25	2

Estimate each square root to the nearest whole number.

10. $\sqrt{11}$ 11. $\sqrt{20}$ 12. $\sqrt{35}$ 13. $\sqrt{65}$
 14. $\sqrt{89}$ 15. $\sqrt{116}$ 16. $\sqrt{137}$ 17. $\sqrt{409}$

Graph each square root on a number line.

18. $\sqrt{15}$ 19. $\sqrt{8}$ 20. $\sqrt{44}$ 21. $\sqrt{89}$
 22. $\sqrt{160}$ 23. $\sqrt{573}$ 24. $\sqrt{645}$ 25. $\sqrt{2,798}$

26. **MEASUREMENT** The bottom of the square baking pan has an area of 67 square inches. What is the approximate length of one side of the pan?



27. **ALGEBRA** What whole number is closest to $\sqrt{m - n}$ if $m = 45$ and $n = 8$?

Estimate each square root to the nearest whole number.

28. $\sqrt{925}$ 29. $\sqrt{2,480}$ 30. $\sqrt{1,610}$ 31. $\sqrt{6,500}$

Find each square root to the nearest tenth.

32. $\sqrt{0.25}$ 33. $\sqrt{0.49}$ 34. $\sqrt{1.96}$ 35. $\sqrt{2.89}$

ALGEBRA For Exercises 36 and 37, estimate each expression to the nearest tenth if $a = 8$ and $b = 3.7$.

36. $\sqrt{a + b}$ 37. $\sqrt{6b - a}$

STAMPS For Exercises 38 and 39, use the information below.

The Special Olympics' commemorative stamp is square in shape with an area of 1,008 square millimeters.

38. Find the length of one side of the postage stamp to the nearest tenth.

39. What is the length of one side in centimeters?

40. **ALGEBRA** The formula $D = 1.22 \times \sqrt{h}$ can be used to estimate the distance D in miles you can see from a point h feet above Earth's surface. Use the formula to find the distance D in miles you can see from the top of a 120-foot hill. Round to the nearest tenth.

Academic Standards • ISTEP+
 Extra Practice, pp. 701, 715

41. **FIND THE DATA** Refer to the Data File on pages 16–19. Choose some data and write a real-world problem in which you would estimate a square root.

H.O.T. Problems

42. **Which One Doesn't Belong?** Identify the number that does not have the same characteristic as the other three. Explain your reasoning.

$\sqrt{5}$

π

$\sqrt{81}$

0.535335333...



43. **OPEN ENDED** Select three numbers with square roots between 4 and 5.

44. **NUMBER SENSE** Explain why 8 is the best whole number estimate for $\sqrt{71}$.

CHALLENGE A cube root of a number is one of three equal factors of that number. Estimate the cube root of each number to the nearest whole number.

45. $\sqrt[3]{9}$

46. $\sqrt[3]{26}$

47. $\sqrt[3]{120}$

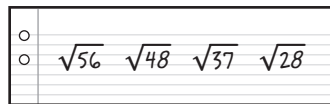
48. $\sqrt[3]{500}$

49. **WRITING IN MATH** Apply what you know about numbers to explain why $\sqrt{30}$ is an irrational number.

ISTEP+ PRACTICE

7.1.5, 7.1.6

50. Reina wrote four numbers on a piece of paper. She then asked her friend Tyron to select the number closest to 5. Which number should he select?



A $\sqrt{56}$

B $\sqrt{48}$

C $\sqrt{37}$

D $\sqrt{28}$

51. Which of the following is an irrational number?

F $\sqrt{25}$

H -13

G $\sqrt{7}$

J $\frac{4}{5}$

52. **SHORT RESPONSE** If the area of a square is 169 square inches, what is the length of the side of the square?

Spiral Review

53. **MEASUREMENT** Find the volume of a can of vegetables with a diameter of 3 inches and a height of 4 inches. Round to the nearest tenth. (Lesson 11-10)

54. **MEASUREMENT** A rectangular prism is 14 inches long, 4.5 inches wide, and 1 inch high. What is the volume of the prism? (Lesson 11-9)

GEOMETRY For Exercises 55–58, use the graph at the right. Classify the angle that represents each category as *acute*, *obtuse*, *right*, or *straight*. (Lesson 10-1)

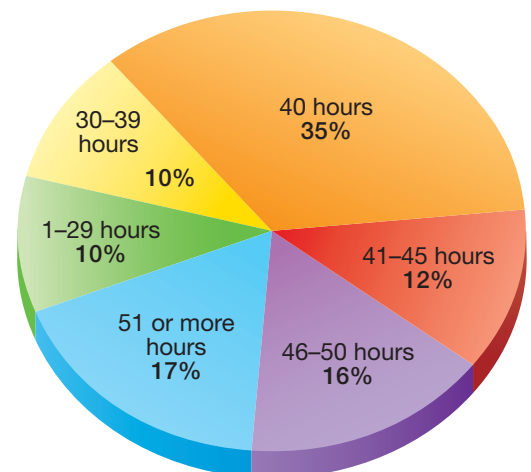
55. 30–39 hours

56. 1–29 hours

57. 40 hours

58. 41–50 hours

Hours Worked in a Typical Week



Source: Heldrich Work Trends Survey

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation.

(Lesson 1-7)

59. $7^2 + 5^2 = c$

60. $4^2 + b = 36$

61. $3^2 + a = 25$

62. $9^2 + 2^2 = c$

12-2

The Pythagorean Theorem

MAIN IDEA

Find length using the Pythagorean Theorem.

IN Academic Standards

Preparation for

8.3.3 Explain why the Pythagorean Theorem is valid using a variety of methods and use the Pythagorean Theorem and its converse. Calculate lengths of the line segments. *Also addresses 7.1.2, 7.1.5.*

New Vocabulary

leg
hypotenuse
Pythagorean Theorem

IN Math Online

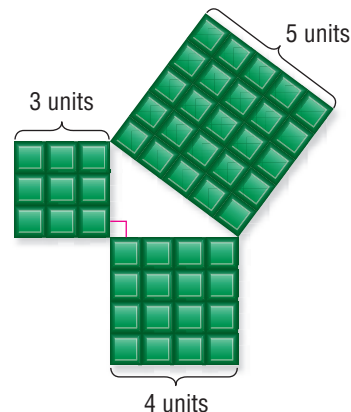
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- Extra Examples
- Personal Tutor
- Self-Check Quiz

MINI Lab

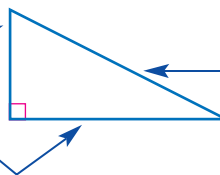
Three squares with sides 3, 4, and 5 units are used to form the right triangle shown.

1. Find the area of each square.
2. How are the squares of the sides related to the areas of the squares?
3. Find the sum of the areas of the two smaller squares. How does the sum compare to the area of the larger square?
4. Use grid paper to cut out three squares with sides 5, 12, and 13 units. Form a right triangle with these squares. Compare the sum of the areas of the two smaller squares with the area of the larger square.



In a right triangle, the sides have special names.

The two sides that form the right angle are the **legs**.



The side opposite the right angle is the **hypotenuse**. It is the longest side of the triangle.



The **Pythagorean Theorem** describes the relationship between the length of the hypotenuse and the lengths of the legs.

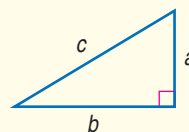
Pythagorean Theorem

Key Concept

Words

In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

Model



Symbols

$$c^2 = a^2 + b^2$$

When using the Pythagorean Theorem, you will encounter equations that involve square roots. Every positive number has both a positive and a negative square root. By the definition of square roots, if $n^2 = a$, then $n = \pm\sqrt{a}$. The notation $\pm\sqrt{\quad}$ indicates both the positive and negative square root of a number. You can use this relationship to solve equations that involve squares.



Study Tip

Check for Reasonableness
You can eliminate -8.9 as a solution because the length of a side of a triangle cannot be a negative number.

EXAMPLE Find the Length of the Hypotenuse

- 1 Find the length of the hypotenuse of the triangle.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

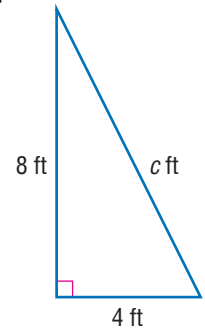
$$c^2 = 8^2 + 4^2 \quad \text{Replace } a \text{ with } 8 \text{ and } b \text{ with } 4.$$

$$c^2 = 64 + 16 \quad \text{Evaluate } 8^2 \text{ and } 4^2.$$

$$c^2 = 80 \quad \text{Add.}$$

$$c = \pm\sqrt{80} \quad \text{Definition of square root}$$

$$c \approx \pm 8.9 \quad \text{Simplify.}$$



The length of the hypotenuse is about 8.9 feet.

CHECK Your Progress

- a. Find the length of the hypotenuse of a right triangle with legs 5 yards and 7 yards. Round to the nearest tenth.



Real-World Career . . .

How Does a Professional Diver Use Math? Professional divers must use formulas to compute pressure and air supply in order to determine safe diving depths and dive times.

IN Math Online

For more information, go to glencoe.com.

Real-World EXAMPLE

- 2 **SCUBA DIVING** A scuba diver dove 14 feet below the surface. Then, he swam 16 feet toward a coral formation. How far is the diver from his boat?

The diver's distance from the boat is the hypotenuse of a right triangle. Write and solve an equation for x .

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

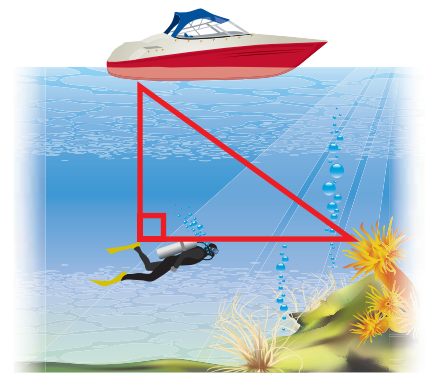
$$x^2 = 14^2 + 16^2 \quad \text{Replace } c \text{ with } x, a \text{ with } 14, \text{ and } b \text{ with } 16.$$

$$x^2 = 196 + 256 \quad \text{Evaluate } 14^2 \text{ and } 16^2.$$

$$x^2 = 452 \quad \text{Add.}$$

$$x = \pm\sqrt{452} \quad \text{Definition of square root}$$

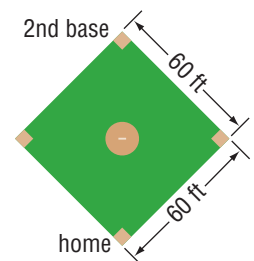
$$x \approx \pm 21.3 \quad \text{Simplify.}$$



The diver's distance from the boat is about 21.3 feet.

CHECK Your Progress

- b. **SOFTBALL** A softball diamond is a square measuring 60 feet on each side. How far does a player on second base throw when she throws from second base to home? Round to the nearest tenth.

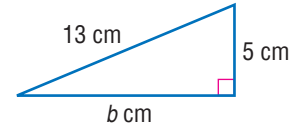




You can also use the Pythagorean Theorem to find the measure of a leg if the measure of the other leg and the hypotenuse are known.

EXAMPLE Find the Length of a Leg

- 3 Find the missing measure of the triangle. Round to the nearest tenth if necessary.



The missing measure is the length of a leg.

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$13^2 = 5^2 + b^2 \quad \text{Replace } a \text{ with } 5 \text{ and } c \text{ with } 13.$$

$$169 = 25 + b^2 \quad \text{Evaluate } 13^2 \text{ and } 5^2.$$

$$\underline{-25 = -25} \quad \text{Subtract 25 from each side.}$$

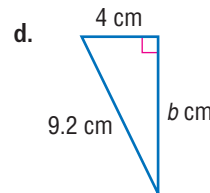
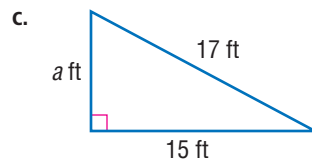
$$144 = b^2 \quad \text{Simplify.}$$

$$\pm\sqrt{144} = b \quad \text{Definition of square root}$$

$$12 = b \quad \text{Simplify.}$$

The length of the leg is 12 centimeters.

✓ CHECK Your Progress



e. $b = 7 \text{ in.}, c = 25 \text{ in.}$

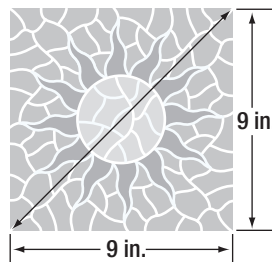
Test-Taking Tip



Formulas Some formulas will be given to you during the test. It is a good idea to familiarize yourself with the formulas before the test.

ISTEP+ EXAMPLE Preparation for 8.3.3

- 4 Mr. Thomson created a mosaic tile in the shape of a square to place in his kitchen.



Which is closest to the length of the diagonal of the tile?

- A 10 in. C 15 in.
B 13 in. D 17 in.

Read the Item

You need to use the Pythagorean Theorem to find the length of the diagonal.



Solve the Item

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 9^2 + 9^2 \quad \text{Replace } a \text{ with } 9 \text{ and } b \text{ with } 9.$$

$$c^2 = 81 + 81 \quad \text{Evaluate } 9^2 \text{ and } 9^2.$$

$$c^2 = 162 \quad \text{Add.}$$

$$c = \pm\sqrt{162} \quad \text{Definition of square root}$$

$$c \approx \pm 12.7 \quad \text{Simplify.}$$



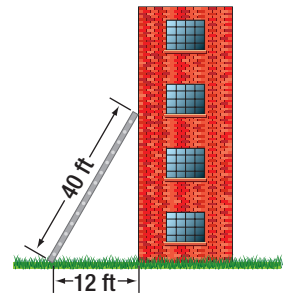
The length is about 12.7 inches.

The answer choice closest to 12.7 inches is 13 inches. So, the answer is B.

CHECK Your Progress

f. A painter leans a ladder against the side of a building. How far from the bottom of the building is the top of the ladder?

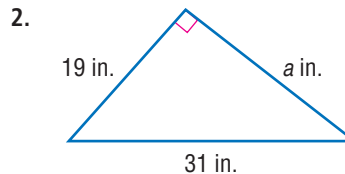
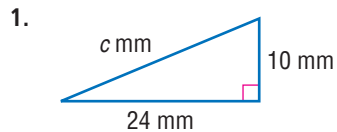
- F 38.2 ft H 21.8 ft
G 28.0 ft J 20.0 ft



CHECK Your Understanding

Examples 1, 3
(pp. 641–642)

Find the missing measure of each triangle. Round to the nearest tenth if necessary.

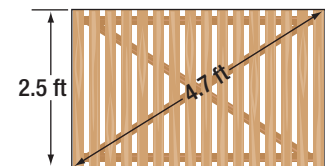


3. $b = 21$ cm, $c = 28$ cm
4. $a = 11$ yd, $b = 12$ yd



Example 2
(p. 641)

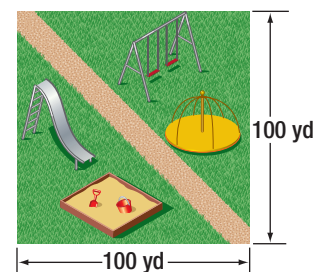
5. **ARCHITECTURE** What is the width of the fence gate shown at the right? Round to the nearest tenth.



Example 4
(pp. 642–643)

6. **MULTIPLE CHOICE** A company designed a public play area in the shape of a square. The play area will include a pathway, as shown. Which is closest to the length of the pathway?

- A 100 yd C 140 yd
B 125 yd D 175 yd

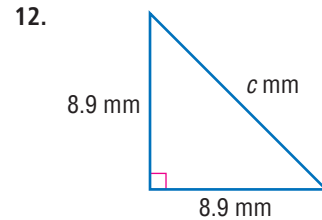
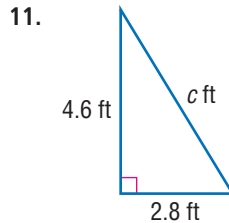
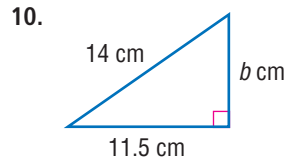
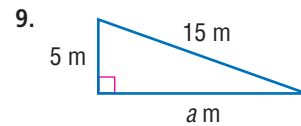
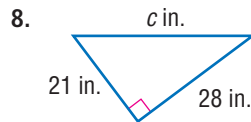
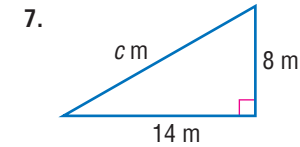




Practice and Problem Solving

HOMEWORK HELP	
For Exercises	See Examples
7–8, 11–12, 15–16	1
17–20	2
9–10, 13–14	3
26–27	4

Find the missing measure of each triangle. Round to the nearest tenth if necessary.



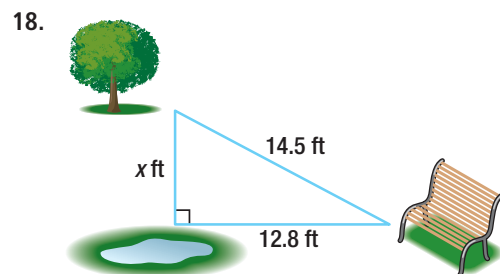
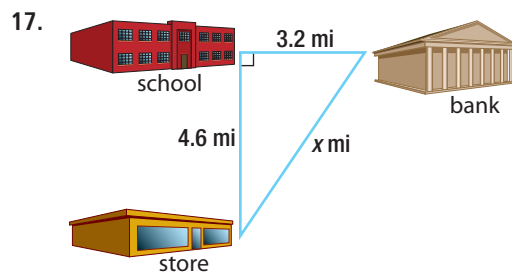
13. $a = 2.4$ yd, $c = 3.7$ yd

14. $b = 8.5$ m, $c = 10.4$ m

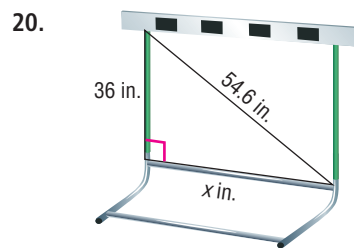
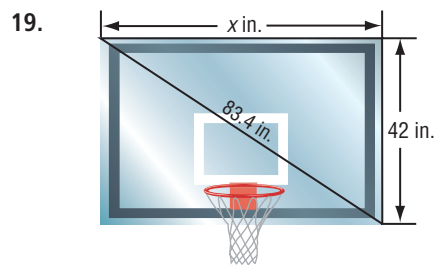
15. $a = 7$ in., $b = 24$ in.

16. $a = 13.5$ mm, $b = 18$ mm

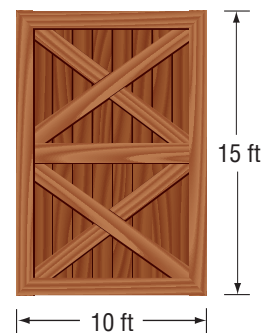
MEASUREMENT For Exercises 17 and 18, find each distance to the nearest tenth.



SPORTS For Exercises 19 and 20, find the length or width of each piece of sports equipment. Round to the nearest tenth.



21. **MEASUREMENT** A barn door is 10 feet wide and 15 feet tall. A square plank 16 feet on each side must be taken through the doorway. Can the plank fit through the doorway? Justify your answer.

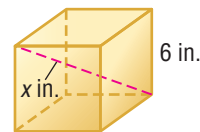


22. **MEASUREMENT** On a weekend trip around California, Sydney left her home in Modesto and drove 75 miles east to Yosemite National Park, then 70 miles south to Fresno, and finally 110 miles west to Monterey Bay. About how far is she from her starting point? Justify your answer with a drawing.

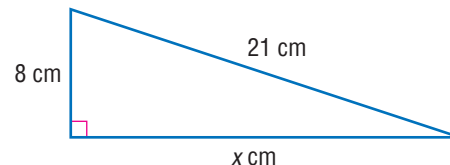
Academic Standards • ISTEP+
Extra Practice, pp. 702, 715

H.O.T. Problems

23. **CHALLENGE** What is the length of the diagonal shown in the cube at the right?



24. **FIND THE ERROR** Marcus and Aisha are writing an equation to find the missing measure of the triangle at the right. Who is correct? Explain.



Marcus

$$21^2 = 8^2 + x^2$$

$$x^2 = 21^2 + 8^2$$

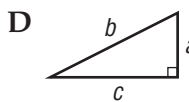
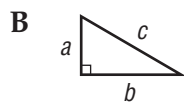
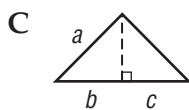
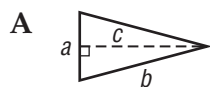


Aisha

25. **WRITING IN MATH** Write a problem about a real-world situation in which you would use the Pythagorean Theorem.

ISTEP+ PRACTICE Preparation for 8.3.3

26. Which triangle has sides a , b , and c so that the relationship $a^2 + b^2 = c^2$ is true?



27. An isosceles right triangle has legs that are each 8 inches long. About how long is the hypotenuse?

- F 12.8 inches
- G 11.3 inches
- H 8 inches
- J 4 inches

Spiral Review

28. **ESTIMATION** Which is closer to $\sqrt{55}$: 7 or 8? (Lesson 12-1)
29. **MEASUREMENT** A cylinder-shaped popcorn tin has a height of 1.5 feet and a diameter of 10 inches. Find the volume to the nearest cubic inch. (Lesson 11-10)

Write each percent as a decimal. (Lesson 4-7)

30. 45% 31. 8% 32. 124% 33. 265%

▶ GET READY for the Next Lesson

34. **PREREQUISITE SKILL** The average person takes about 15 breaths per minute. At this rate, how many breaths does the average person take in one week? Use the *solve a simpler problem* strategy. (Lesson 11-5)

12-3

Problem-Solving Investigation

MAIN IDEA: Solve problems by making a model.

Academic Standards

P.1.3 Apply and adapt a variety of appropriate strategies to solve problems. **P.5.1** Create and use representations to organize, record, and communicate mathematical ideas.

P.S.I. TEAM +

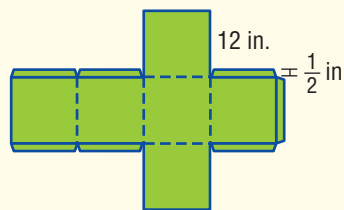
e-Mail: MAKE A MODEL

AYITA: I am decorating the school's gymnasium for the spring dance with cubes that will hang from the ceiling.

YOUR MISSION: Make a model to find how much cardboard will be needed for each cube if the edge of one cube measures 12 inches.



Understand	You know that each cube is 12 inches long.
Plan	Make a cardboard model of a cube with sides 12 inches long. You will also need to determine where to put tabs so that all of the edges are glued together.
Solve	<p>Start with a cube, then unfold it, to show the pattern. You know that 5 of the edges don't need tabs because they are the fold lines. The remaining 7 edges need a tab. Use $\frac{1}{2}$-inch tabs.</p> <p> $7 \times 12 \text{ in.} \times \frac{1}{2} \text{ in.} = 42 \text{ in}^2$ area of 7 tabs $6 \times 12 \text{ in.} \times 12 \text{ in.} = 864 \text{ in}^2$ area of 6 faces 906 in^2 total area </p> <p>So, 906 square inches of cardboard is needed to make one cube.</p>
Check	Make another cube to determine whether all the edges can be glued together using your model.



Analyze The Strategy

- How can making a model be useful when solving a word problem?
- WRITING IN MATH** Write a problem that can be solved by making a model. Then solve the problem.

For Exercises 3–5, make a model to solve the problem.

- CARS** Fiona counted the number of vehicles in the parking lot at a store. She counted a total of 12 cars and motorcycles. If there was a total of 40 wheels, how many cars and motorcycles were there?
- ART** Miguel is making a drawing of his family room for a school project. The room measures 18 feet by 21 feet. If he uses a scale of 1 foot = $\frac{1}{2}$ inch, what are the dimensions of the family room on the drawing?
- MEASUREMENT** Francis has a photo that measures 10 inches by $8\frac{1}{2}$ inches. If the frame he uses is $1\frac{1}{4}$ inches wide, what is the perimeter of the framed picture?



Use any strategy to solve Exercises 6–13. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES

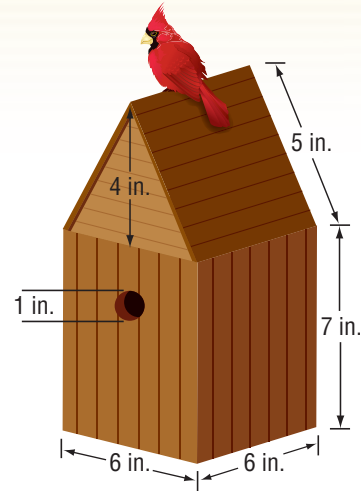
- Draw a diagram.
- Use logical reasoning.
- Make a model.

- DONATIONS** Hickory Point Middle School collected money for a local shelter. The table shows the total amount collected by each grade level. Suppose the school newspaper reported that \$5,000 was collected. Is this estimate reasonable? Explain.

Grade	Dollars Collected
sixth	1,872
seventh	2,146
eighth	1,629

- TRACK** Wei can jog one 400-meter lap in $1\frac{1}{3}$ minutes. How long will it take her to run 1,600 meters at the same rate?

- BIRD HOUSES** About how many square inches of the bird house will be painted if only the outside of the wood is painted?



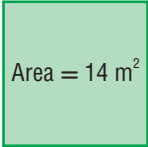
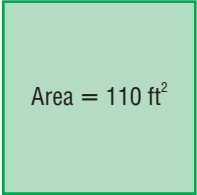
- BOXES** Juliet is placing 20 cereal boxes that measure 8 inches by 2 inches by 12 inches on a shelf that is 3 feet long and 11 inches deep. What is a possible arrangement for the boxes on the shelf?
- MONEY** At the beginning of the week, Marissa had \$45.50. She spent \$2.75 each of five days on lunch, bought a sweater for \$14.95, and Jacob repaid her \$10 that he owed her. How much money does she have at the end of the week?
- MEASUREMENT** How many square feet of wallpaper are needed to cover a wall that measures $15\frac{1}{4}$ feet by $8\frac{3}{4}$ feet and has a window that measures 2 feet by 4 feet?
- BASEBALL** A regulation baseball diamond is a square with an area of 8,100 square feet. If it is laid out on a field that is 172 feet wide and 301 feet long, how much greater is the distance around the whole field than the distance around the diamond?
- DVDs** Marc currently has 68 DVDs in his collection. By the end of the next four months, he wants to have 92 DVDs in his collection. How many DVDs must he buy each month to obtain his goal?



Estimate each square root to the nearest whole number. (Lesson 12-1)

- $\sqrt{32}$
- $\sqrt{80}$
- $\sqrt{105}$
- $\sqrt{230}$

MEASUREMENT Estimate the side length of each square to the nearest whole number. (Lesson 12-1)

- 
- 

Graph each square root on a number line.

(Lesson 12-1)

- $\sqrt{18}$
 - $\sqrt{230}$
9. **MULTIPLE CHOICE** Imani is playing a review game in math class. She needs to pick the card that is labeled with a number closest to 8. Which should she pick? (Lesson 12-1)

$\sqrt{37}$

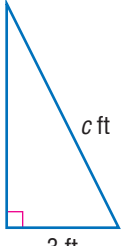
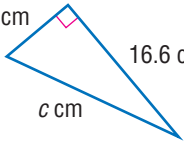
$\sqrt{52}$

$\sqrt{70}$

$\sqrt{83}$

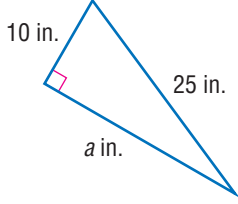
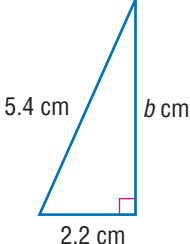
- A $\sqrt{37}$
 B $\sqrt{52}$
 C $\sqrt{70}$
 D $\sqrt{83}$

Find the length of the hypotenuse of each triangle. Round to the nearest tenth if necessary. (Lesson 12-2)

- 
- 

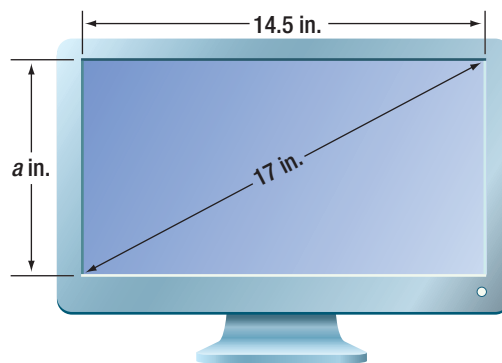
Find the missing measure of each triangle. Round to the nearest tenth if necessary.

(Lesson 12-2)

- 
- 



14. **MEASUREMENT** On a computer monitor, the diagonal measure of the screen is 17 inches.



If the screen length is 14.5 inches, what is the height of the screen to the nearest tenth?

(Lesson 12-2)



15. **MULTIPLE CHOICE** Eduardo jogs 5 kilometers north and 5 kilometers west. To the nearest kilometer, how far is he from his starting point? (Lesson 12-2)
- F 25 km H 7 km
 G 10 km J 5 km



16. **SCIENCE** A certain type of bacteria doubles every hour. If there are two bacteria initially in a sample, how many will be present after five hours? Use the *make a model* strategy. (Lesson 12-3)



17. **SCALE MODELS** A scale model is made of a building measuring 120 feet long, 75 feet wide, and 45 feet high. If the scale is 1 inch = 15 feet, what are the dimensions of the model? Use the *make a model* strategy. (Lesson 12-3)

12-4

Surface Area of Rectangular Prisms

MAIN IDEA

Find the surface areas of rectangular prisms.

IN Academic Standards

Reinforcement of 6.3.5

Develop and use the formulas for the surface area and volume of a cylinder and find the surface area and volume of three-dimensional objects built from rectangular solids and cylinders.

New Vocabulary

surface area

IN Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

MINI Lab

- Use the cubes to build a rectangular prism with a length of 8 centimeters.
- Count the number of squares on the outside of the prism. The sum is the *surface area*.



1. Record the dimensions, volume, and surface area in a table.
2. Build two more prisms using all of the cubes. For each, record the dimensions, volume, and surface area.
3. Describe the prisms with the greatest and least surface areas.

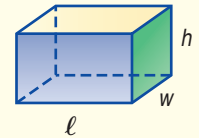
The sum of the areas of all of the surfaces, or faces, of a three-dimensional figure is the **surface area**.

Surface Area of a Rectangular Prism

Key Concept

Words The surface area S of a rectangular prism with length ℓ , width w , and height h is the sum of the areas of its faces.

Model



Symbols $S = 2\ell w + 2\ell h + 2wh$

EXAMPLES Find Surface Area

- 1** Find the surface area of the rectangular prism.

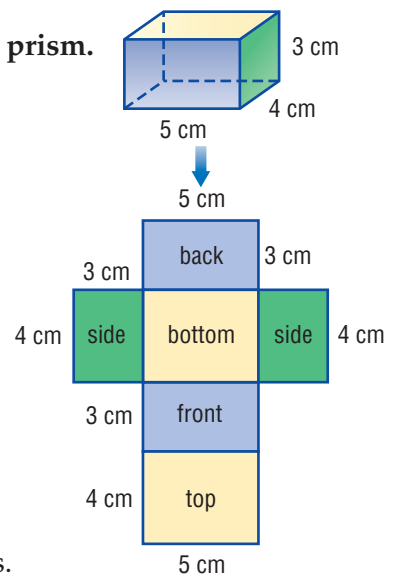
There are three pairs of congruent faces.

- top and bottom
- front and back
- two sides

Faces

top and bottom	$2(5 \cdot 4) = 40$
front and back	$2(5 \cdot 3) = 30$
two sides	$2(3 \cdot 4) = 24$
sum of the areas	$40 + 30 + 24 = 94$

The surface area is 94 square centimeters.

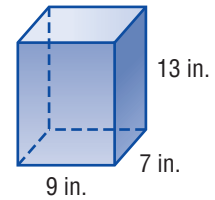




2 Find the surface area of the rectangular prism.

Replace ℓ , with 9, w with 7, and h with 13.

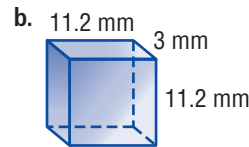
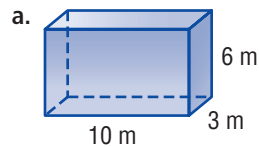
$$\begin{aligned} \text{surface area} &= 2\ell w + 2\ell h + 2wh \\ &= 2 \cdot 9 \cdot 7 + 2 \cdot 9 \cdot 13 + 2 \cdot 7 \cdot 13 \\ &= 126 + 234 + 182 \quad \text{Multiply first. Then add.} \\ &= 542 \end{aligned}$$



The surface area of the prism is 542 square inches.

✓ CHECK Your Progress

Find the surface area of each rectangular prism.



Study Tip

Surface Area

When you find the surface area of a three-dimensional figure, the units are square units, not cubic units.

Real-World EXAMPLE

3 PAINTING Domingo built a toy box 60 inches long, 24 inches wide, and 36 inches high. He has 1 quart of paint that covers about 87 square feet of surface. Does he have enough to paint the toy box? Justify your answer.

STEP 1

Find the surface area of the toy box.

Replace ℓ with 60, w with 24, and h with 36.

$$\begin{aligned} \text{surface area} &= 2\ell w + 2\ell h + 2wh \\ &= 2 \cdot 60 \cdot 24 + 2 \cdot 60 \cdot 36 + 2 \cdot 24 \cdot 36 \\ &= 8,928 \text{ in}^2 \end{aligned}$$

STEP 2

Find the number of square inches the paint will cover.

$$\begin{aligned} 1 \text{ ft}^2 &= 1 \text{ ft} \times 1 \text{ ft} && \text{Replace 1 ft with 12 in.} \\ &= 12 \text{ in.} \times 12 \text{ in.} && \text{Multiply.} \\ &= 144 \text{ in}^2 \end{aligned}$$



So, 87 square feet is equal to 87×144 or 12,528 square inches.

Since $12,528 > 8,928$, Domingo has enough paint.

✓ CHECK Your Progress

- c. **BOXES** The largest corrugated cardboard box ever constructed measured about 23 feet long, 9 feet high, and 8 feet wide. Would 950 square feet of paper be enough to cover the box? Justify your answer.
- d. **BOXES** If 1 foot was added to each dimension of the largest corrugated cardboard box ever constructed, would 950 square feet of paper still be enough to cover the box? Justify your answer.

Study Tip

Consistent Units

Since the surface area of the toy box is expressed in inches, convert 87 ft^2 to square inches so that all measurements are expressed using the same units.

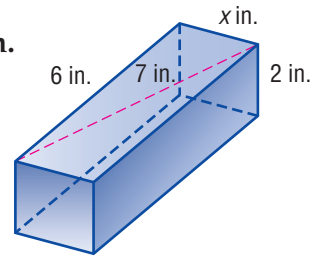


EXAMPLE

Use the Pythagorean Theorem

- 4 Find the surface area of the rectangular prism.

The width and height of the prism are given. To find the surface area, you need to find the length of the prism. Notice that the diagonal, length, and width of the top face of the prism form a right triangle.



Study Tip

Square Roots
The equation $13 = x^2$ has two solutions, 3.6 and -3.6 . However, the length of the prism must be positive, so choose the positive solution.

$$c^2 = a^2 + b^2$$

Pythagorean Theorem

$$7^2 = 6^2 + x^2$$

Replace c with 7, a with 6, and b with x .

$$49 = 36 + x^2$$

Evaluate 7^2 and 6^2 .

$$49 - 36 = 36 + x^2 - 36$$

Subtract 36 from each side.

$$13 = x^2$$

Simplify.

$$\pm\sqrt{13} = x$$

Definition of square root

$$\pm 3.6 \approx x$$

Simplify.

The length of the prism is about 3.6 inches. Find the surface area.

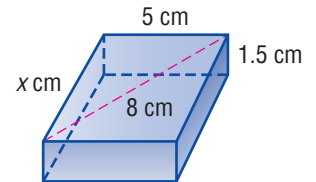
$$\text{surface area} = 2\ell w + 2\ell h + 2wh$$

$$= 2(3.6)(6) + 2(3.6)(2) + 2(6)(2) \text{ or } 81.6$$

The surface area of the prism is about 81.6 square inches.

CHECK Your Progress

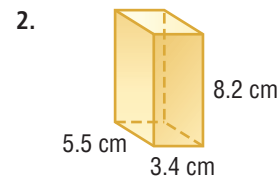
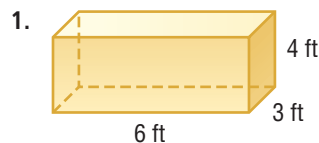
- e. Find the surface area of the rectangular prism to the nearest tenth.



CHECK Your Understanding

Examples 1, 2
(pp. 649–650)

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

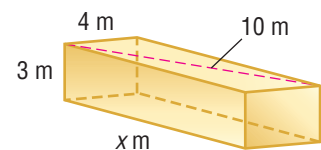


Example 3
(p. 650)

3. **GIFTS** Megan is wrapping a gift. She places it in a box 8 inches long, 2 inches wide, and 11 inches high. If Megan bought a roll of wrapping paper that is 1 foot wide and 2 feet long, did she buy enough paper to wrap the gift? Justify your answer.

Example 4
(p. 651)

4. **MEASUREMENT** Find the surface area of the rectangular prism at the right. Round to the nearest tenth if necessary.



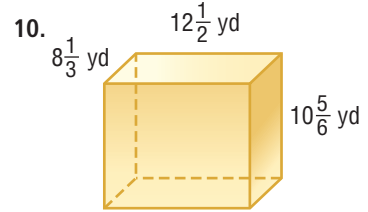
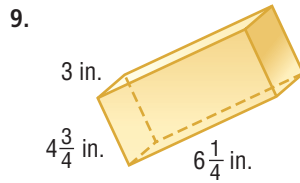
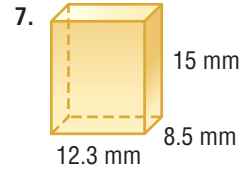
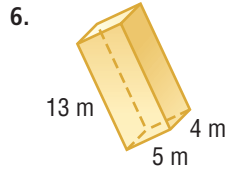
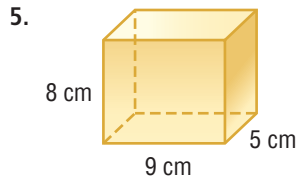


Practice and Problem Solving

HOMEWORK HELP

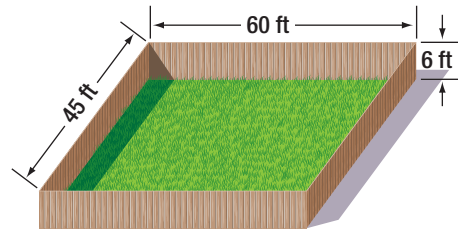
For Exercises	See Examples
5–10	1, 2
11–12	3
13–14	4

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

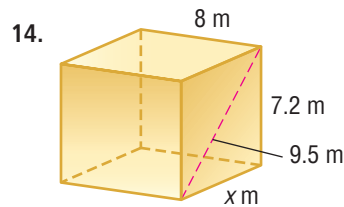
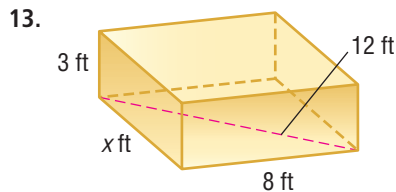


11. **BOOKS** When making a book cover, Anwar adds an additional 20 square inches to the surface area to allow for overlap. How many square inches of paper will Anwar use to make a book cover for a book 11 inches long, 8 inches wide, and 1 inch high?

12. **FENCES** If one gallon of paint covers 350 square feet, will 8 gallons of paint be enough to paint the inside and outside of the fence shown once? Explain.

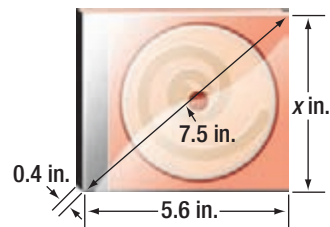


Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.



15. **MUSIC** To the nearest tenth, find the approximate amount of plastic covering the outside of the CD case.

16. **MEASUREMENT** What is the surface area of a rectangular prism that has a length of 6.5 centimeters, a width of 2.8 centimeters, and a height of 9.7 centimeters?



17. **ALGEBRA** Write a formula for the surface area s of a cube in which each side measures x units.

18. **PACKAGING** A company will make a cereal box with whole number dimensions and a volume of 100 cubic centimeters. If cardboard costs \$0.05 per 100 square centimeters, what is the least cost to make 100 boxes?



Real-World Link

In a recent year, 34% of all music sold was rock, 13% was country, and 11% was R&B. Also, 85.6% of all music formats sold were full-length CDs.

Source: Recording Industry Association of America

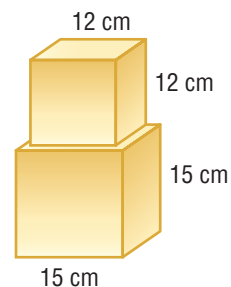
H.O.T. Problems

19. **REASONING** The bottom and sides of a pool in the shape of a rectangular prism will be painted blue. The length, width, and height of the pool are 18 feet, 12 feet, and 6 feet, respectively. Explain why the number of square feet to be painted is *not* equivalent to the expression $2(18)(12) + 2(18)(6) + 2(12)(6)$.

Academic Standards • ISTEP+

Extra Practice, pp. 703, 715

20. **CHALLENGE** The figure at the right is made by placing a cube with 12-centimeter sides on top of another cube with 15-centimeter sides. Find the surface area.



21. **WRITING IN MATH** Explain why surface area of a three-dimensional figure is measured in square units rather than in cubic units.

ISTEP+ PRACTICE

Reinforcement of 6.3.5

22. Which of the following expressions represents the surface area of a cube with side length w ?

- A w^3
- B $6w^2$
- C $6w^3$
- D $2w + 4w^2$

23. How much cardboard is needed to make a box with a length of 2.5 feet, a width of 1.6 feet, and a height of 2 feet?

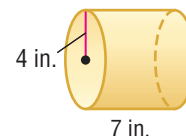
- F 37.5 square feet
- G 24.4 square feet
- H 8 square feet
- J 6.1 square feet

Spiral Review

24. **MEASUREMENT** A rectangular-shaped yard that measures 50 feet by 70 feet is bordered by a flowerbed that is 2 feet wide. What is the perimeter of the entire yard? Use the *make a model* strategy. (Lesson 12-3)

25. **MEASUREMENT** What is the missing measure of a right triangle in which $a = 13$ feet and $c = 18$ feet? Round to the nearest tenth. (Lesson 12-2)

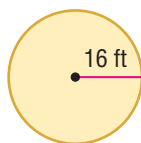
26. **MEASUREMENT** What is the volume of the cylinder shown at the right? Round to the nearest tenth. (Lesson 11-10)



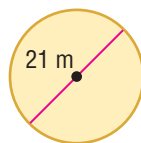
▶ GET READY for the Next Lesson

PREREQUISITE SKILL Find the area of each circle. Round to the nearest tenth. (Lesson 11-4)

27.



28.



29. diameter = 13.6 yd

30. radius = 23 km

Measurement Lab

Changes in Scale

MAIN IDEA

Investigate how changes in scale affect volume and surface area.

IN Academic Standards

7.3.5 Identify, describe and construct similarity relationships and solve problems involving similarity (including similar triangles) and scale drawings by using proportional reasoning. Also addresses P.1.1, P.1.4, P.2.2, P.2.3, P.4.1.

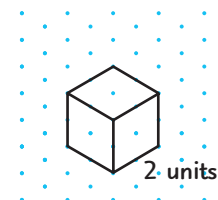
Suppose you have a model of a rectangular prism and you are asked to create a similar model with dimensions that are twice as large. In this lab, you will investigate how the scale factor that relates the lengths in two similar objects affects how the surface areas and volumes are related.

ACTIVITY

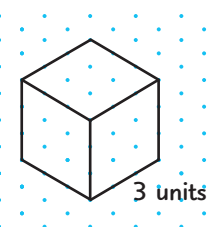
1 **STEP 1** Draw a cube on dot paper that measures 1 unit on each side. Calculate the volume and the surface area of the cube. Then record the data in a table like the one shown below.



STEP 2 Double the side lengths of the cube. Calculate the volume and the surface area of this cube. Record the data in your table.



STEP 3 Triple the side lengths of the original cube. Now each side measures 3 units long. Calculate the volume and the surface area of the cube and record the data.



STEP 4 For each cube, write a ratio comparing the side length and the volume. Then write a ratio comparing the side length and the surface area. The first one is done for you.

Side Length (units)	Volume (units ³)	Surface Area (units ²)	Ratio of Side Length to Volume	Ratio of Side Length to Surface Area
1	$1^3 = 1$	$6(1^2) = 6$	1 : 1	1 : 6
2				
3				
4				
5				
s				

CHECK Your Progress

a. Complete the table above.

ACTIVITY

2

STEP 1

Draw a cube on dot paper that measures 8 units on each side. Calculate the volume and the surface area of the cube. Record the data in a table like the one shown below.

STEP 2

Halve the side lengths of the cube in Step 1. Calculate the volume and the surface area of this cube and record the data.

STEP 3

Halve the side lengths of the cube in Step 2. Calculate the volume and the surface area of the cube and record the data.

STEP 4

For each cube, write a ratio comparing the side length and the volume and a ratio comparing the side length and the surface area. The first one is done for you.

Study Tip

Ratios If you're looking for a pattern among ratios, it is sometimes helpful to reduce each ratio first.

Side Length (units)	Volume (units ³)	Surface Area (units ²)	Ratio of Side Length to Volume	Ratio of Side Length to Surface Area
8	$8^3 = 512$	$6(8^2) = 384$	8 : 512 or 1 : 64	8 : 384 or 1 : 48
4				
2				
s				

✓ CHECK Your Progress

b. Complete the table above.

ANALYZE THE RESULTS

- Write a formula for the volume V of a cube with side length s .
- Write a formula for the surface area A of a cube with side length s .

MAKE A CONJECTURE Complete each sentence.

- If the side length of a cube is doubled, the volume is \blacksquare times greater.
- If the side length of a cube is doubled, the surface area is \blacksquare times greater.
- If the side length of a cube is tripled, the volume increases by \blacksquare times and the surface area increases by \blacksquare times.
- If the side length of a cube decreases by $\frac{1}{2}$, the surface area decreases by \blacksquare .

12-5

Surface Area of Cylinders

MAIN IDEA

Find the surface area of a cylinder.

IN Academic Standards

Reinforcement of 6.3.5 Develop and use the formulas for the surface area and volume of a cylinder and find the surface area and volume of three-dimensional objects built from rectangular solids and cylinders.

IN Math Online

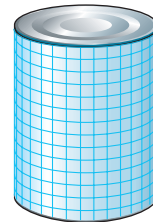
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz

MINI Lab

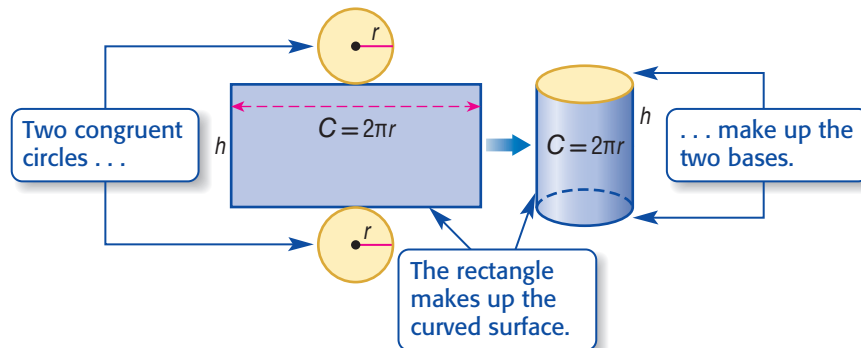
STEP 1 Trace the top and bottom of the can on grid paper. Then cut out the shapes.

STEP 2 Cut a long rectangle from the grid paper. The width of the rectangle should be the same as the height of the can. Wrap the rectangle around the side of the can. Cut off the excess paper so that the edges just meet.



1. Make a net of the cylinder.
2. Name the shapes in the net.
3. How is the length of the rectangle related to the circles?

You can put two circles and a rectangle together to make a cylinder.



In the diagram above, the length of the rectangle is the same as the circumference of the circle. Also, the width of the rectangle is the same as the height of the cylinder.

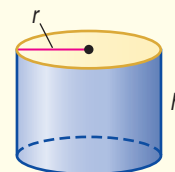
$$\begin{array}{ccc}
 \text{surface area} & & \text{two bases} & & \text{curved surface} \\
 \downarrow & & \downarrow & & \downarrow \\
 S & = & 2(\pi r^2) & + & 2(\pi r)h
 \end{array}$$

Surface Area of a Cylinder

Key Concept

Words The surface area S of a cylinder with height h and radius r is the sum of the areas of the circular bases and the area of the curved surface.

Model



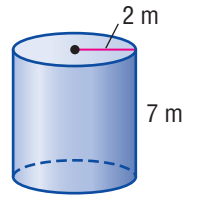
Symbols $S = 2\pi r^2 + 2\pi rh$



EXAMPLE Find the Surface Area of a Cylinder

- 1 Find the surface area of the cylinder.
Round to the nearest tenth.

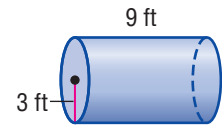
$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh && \text{Surface area of a cylinder} \\ &= 2\pi(2)^2 + 2\pi(2)(7) && \text{Replace } r \text{ with } 2 \text{ and } h \text{ with } 7. \\ &\approx 113.1 && \text{Simplify.} \end{aligned}$$



The surface area is about 113.1 square meters.

CHECK Your Progress

- a. Find the surface area of the cylinder.
Round to the nearest tenth.



Real-World Link

Of the 3,000 to 4,000 wooden carousels carved in America between 1885 and 1930, fewer than 150 operate today.

Source: National Carousel Association



Real-World EXAMPLE

- 2 **CAROUSELS** A circular fence that is 2 feet high is to be built around the outside of a carousel. The distance from the center of the carousel to the edge of the fence will be 35 feet. How much fencing material is needed to make the fence around the carousel?

The radius of the circular fence is 35 feet. The height is 2 feet.

$$\begin{aligned} S &= 2\pi rh && \text{Curved surface of a cylinder} \\ &= 2\pi(35)(2) && \text{Replace } r \text{ with } 35 \text{ and } h \text{ with } 2. \\ &\approx 439.8 && \text{Simplify.} \end{aligned}$$



So, about 439.8 square feet of material is needed to make the fence.

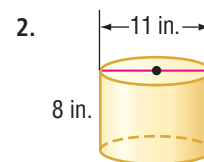
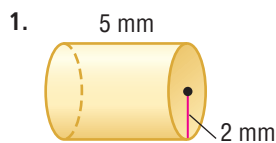
CHECK Your Progress

- b. **DESIGN** Find the area of the label of a can of tuna with a radius of 5.1 centimeters and a height of 2.9 centimeters.



CHECK Your Understanding

- Example 1** (p. 657) Find the surface area of each cylinder. Round to the nearest tenth.



- Example 2** (p. 657)

3. **STORAGE** The height of a water tank is 10 meters, and it has a diameter of 10 meters. What is the surface area of the tank?

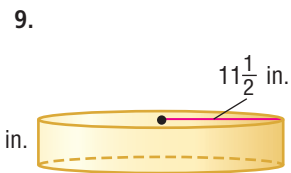
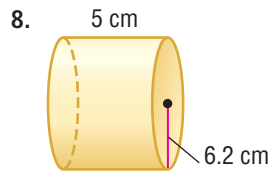
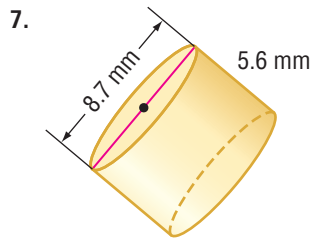
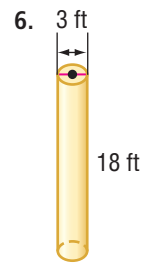
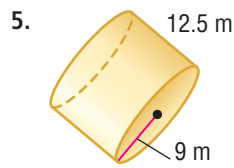
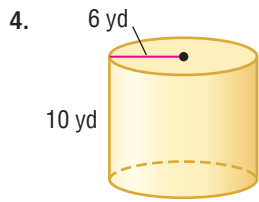


Practice and Problem Solving

HOMEWORK HELP

For Exercises	See Examples
4–9	1
10–11	2

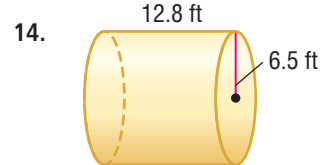
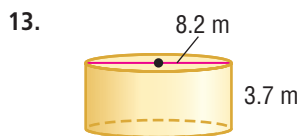
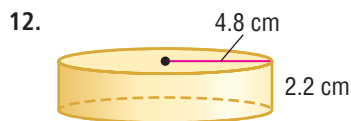
Find the surface area of each cylinder. Round to the nearest tenth.



10. **CANDLES** A cylindrical candle has a diameter of 4 inches and a height of 7 inches. What is the surface area of the candle?

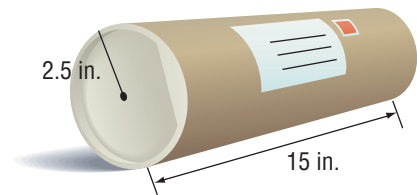
11. **PENCILS** Find the surface area of an unsharpened cylindrical pencil that has a radius of 0.5 centimeter and a height of 19 centimeters.

ESTIMATION Estimate the surface area of each cylinder.



15. **BAKING** Mrs. Jones baked a cake 5 inches high and 9 inches in diameter. If Mrs. Jones covers the top and sides of the cake with frosting, find the area that the frosting covers to the nearest tenth.

16. **PACKAGING** The mail tube shown is made of cardboard and has plastic end caps. Approximately what percent of the surface area of the mail tube is cardboard?



Academic Standards • ISTEP+
Extra Practice, pp. 703, 715



H.O.T. Problems

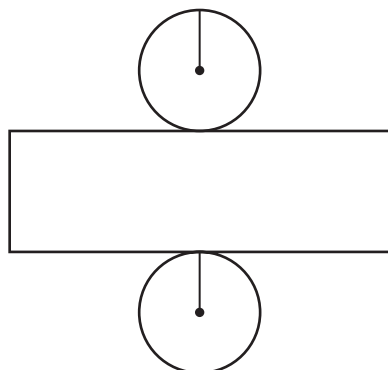
17. **CHALLENGE** If the height of a cylinder is doubled, will its surface area also double? Explain your reasoning.

18. **WRITING IN MATH** Write a problem about a real-world situation in which you would find the surface area of a cylinder. Be sure to include the answer to your problem.

19. **REASONING** Which has more surface area, a cylinder with radius 6 centimeters and height 3 centimeters or a cylinder with radius 3 centimeters and height 6 centimeters? Explain your reasoning.

ISTEP+ PRACTICE Reinforcement of 6.3.5

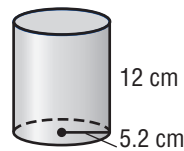
20. Stacey has a cylindrical paper clip holder with the net shown. Use a centimeter ruler to measure the dimensions of the net in centimeters.



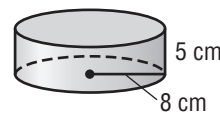
Which is closest to the surface area of the cylindrical paper clip holder?

- A 6.0 cm^2
- B 6.5 cm^2
- C 7.5 cm^2
- D 15.5 cm^2

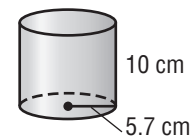
21. The three containers below each hold about 1 liter of liquid. Which container has the greatest surface area?



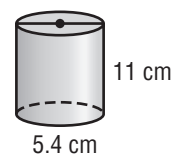
Container I



Container II



Container III

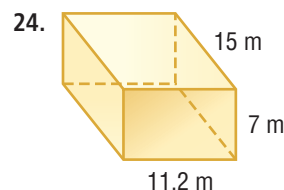
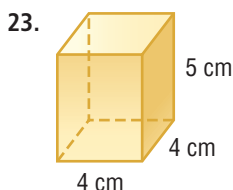
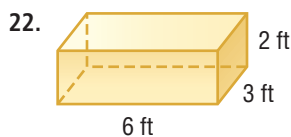


Container IV

- F Container I
- G Container II
- H Container III
- J Container IV

Spiral Review

MEASUREMENT Find the surface area of each rectangular prism. (Lesson 12-4)



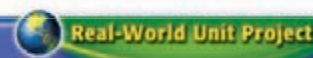
MEASUREMENT Find the missing measure of each right triangle. Round to the nearest tenth if necessary. (Lesson 12-2)

25. $a = 8 \text{ in.}, b = 10 \text{ in.}$

26. $a = 12 \text{ ft}, c = 20 \text{ ft}$

27. $b = 12 \text{ cm}, c = 14 \text{ cm}$

Problem Solving in Life Skills



Design That House It's time to complete your project. Use the measurements you have gathered to create a blueprint of your dream house. Be sure to include the scale and actual measurements of your dream house.

IN Math Online Unit Project at glencoe.com

FOLDABLES GET READY to Study

Study Organizer

Be sure the following Big Ideas are noted in your Foldable.

Ch., IZ	Rectangular Prisms	Cylinders
Unit Overview		
Final Review		
Final Portfolio Review		

BIG Ideas

Irrational Numbers (Lesson 12-1)

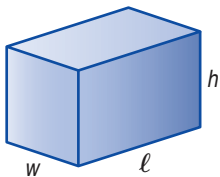
- An irrational number is a number that cannot be written as a fraction.

Pythagorean Theorem (Lesson 12-2)

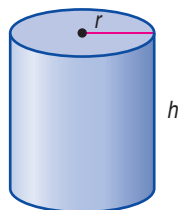
- In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

Surface Area (Lessons 12-4, 12-5)

- The surface area S of a rectangular prism with length ℓ , width w , and height h is the sum of the areas of the faces. $S = 2\ell w + 2\ell h + 2wh$



- The surface area S of a cylinder with height h and a radius r is the sum of the area of the circular bases and the area of the curved surface. $S = 2\pi r^2 + 2\pi rh$



Key Vocabulary

- hypotenuse** (p. 640)
- irrational number** (p. 637)
- leg** (p. 640)
- Pythagorean Theorem** (p. 640)
- surface area** (p. 649)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The side opposite the right angle in a scalene triangle is called a hypotenuse.
2. Either of the two sides that form the right angle of a right triangle is called a hypotenuse.
3. An irrational number is a number that cannot be expressed as the quotient of two integers.
4. In a right triangle, the square of the length of the hypotenuse equals the difference of the squares of the lengths of the legs.
5. The sum of the areas of all the surfaces of a three-dimensional figure is called the surface area.
6. The formula for finding the surface area of a cylinder is $S = 2\ell w + 2\ell h + 2wh$.
7. Rational numbers include only positive numbers.
8. The Pythagorean Theorem can be used to find the length of the hypotenuse of a right triangle if the measures of both legs are known.
9. To find the surface area of a rectangular prism, you must know the measurements of the height and the radius.
10. The square root of a perfect square is a rational number.

Lesson-by-Lesson Review

12-1 Estimating Square Roots (pp. 636–639)



7.1.5,
7.1.6

Estimate each square root to the nearest whole number.

11. $\sqrt{6}$ 12. $\sqrt{99}$ 13. $\sqrt{48}$
14. $\sqrt{76}$ 15. $\sqrt{19}$ 16. $\sqrt{52}$

Graph each square root on a number line.

17. $\sqrt{61}$ 18. $\sqrt{132}$
19. $\sqrt{444}$ 20. $\sqrt{12}$

21. **SWIMMING POOL** The bottom of Marcia's square swimming pool has an area of 118 square feet. What is the approximate length of one of the sides?

Example 1 Estimate $\sqrt{29}$ to the nearest whole number.

$25 < 29 < 36$ 29 is between the perfect squares 25 and 36.

$\sqrt{25} < \sqrt{29} < \sqrt{36}$ Find the square root of each number.

$5 < \sqrt{29} < 6$ $\sqrt{25} = 5$ and $\sqrt{36} = 6$

So, $\sqrt{29}$ is between 5 and 6. Since 29 is closer to 25 than to 36, the best whole number estimate is 5.

12-2 The Pythagorean Theorem (pp. 640–645)



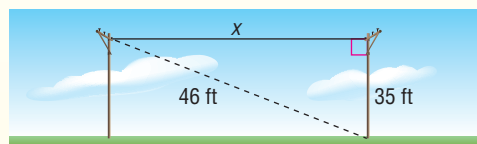
7.1.5,
8.3.4

Find the missing measure of each triangle. Round to the nearest tenth if necessary.

22. 23.

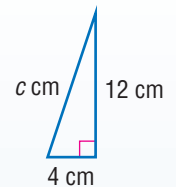
24. $b = 10$ yd, $c = 12$ yd

25. **COMMUNICATION** Find the length of the wire x that is attached to the telephone pole. Round to the nearest tenth.



26. **LADDERS** Bartolo has a 26-foot ladder. He places it 10 feet away from the base of a building. What is the height of the building where the top of ladder rests?

Example 2 Find the missing measure of the triangle shown at the right. Round to the nearest tenth if necessary.



Use the Pythagorean Theorem to solve for c .

$$c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}$$

$$c^2 = 4^2 + 12^2 \quad a = 4 \text{ and } b = 12$$

$$c^2 = 16 + 144 \quad \text{Evaluate.}$$

$$c^2 = 160 \quad \text{Add.}$$

$$c = \pm\sqrt{160} \quad \text{Definition of square root}$$

$$c \approx \pm 12.6 \quad \text{Simplify.}$$

Since length cannot be negative, the length of the hypotenuse is about 12.6 centimeters.

12-3

PSI: Make a Model (pp. 646–647)

P.1.3,
P.5.1

Solve the problem by using the *make a model* strategy.

27. **FRAMING** A painting 15 inches by 25 inches is bordered by a mat that is 3 inches wide. The frame around the mat is 2 inches wide. Find the area of the picture with the frame and mat.
28. **DVDs** A video store arranges its best-selling DVDs in their front window. In how many different ways can five best-seller DVDs be arranged in a row?

Example 3 The bottom layer of a display of soup cans has 6 cans in it. If there is one less can in each layer above it and there are 4 layers in the display, how many cans are there in the display?



So, based on the model, there are 18 cans.

12-4

Surface Area of Rectangular Prisms (pp. 649–653)



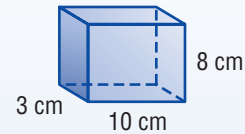
6.3.5

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

29. 30.

31. **MOVING** A large wardrobe box is 2.25 feet long, 2 feet wide, and 4 feet tall. How much cardboard is needed to make the box?

Example 4 Find the surface area of a rectangular prism.



surface area

$$\begin{aligned} &= 2lw + 2lh + 2wh \\ &= 2(10)(3) + 2(10)(8) + 2(3)(8) \\ &= 268 \end{aligned}$$

The surface area is 268 square centimeters.

12-5

Surface Area of Cylinders (pp. 656–659)



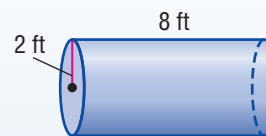
6.3.5

Find the surface area of each cylinder. Round to the nearest tenth.

32. 33.

34. **DESIGN** A can of black beans is $5\frac{1}{2}$ inches high, and its base has a radius of 2 inches. How much paper is needed to make the label on the can?

Example 5 Find the surface area of the cylinder. Round to the nearest tenth.



$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2(\pi)(2^2) + 2(\pi)(2)8 \\ &\approx 125.7 \text{ ft}^2 \end{aligned}$$

The surface area is about 125.7 square feet.

Estimate each square root to the nearest whole number.

1. $\sqrt{500}$ 2. $\sqrt{95}$ 3. $\sqrt{265}$

Graph each square root on the number line.

4. $\sqrt{570}$ 5. $\sqrt{7}$ 6. $\sqrt{84}$

7. **MULTIPLE CHOICE** The length of one side of a square sandbox is 7 feet. Which number is closest to the length of the diagonal of the sandbox?

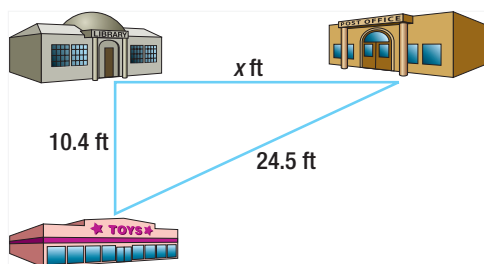
- A $\sqrt{100}$
 B $\sqrt{50}$
 C $\sqrt{14}$
 D $\sqrt{7}$

Find the missing measure of each right triangle. Round to the nearest tenth if necessary.

8. $a = 5$ m, $b = 4$ m
 9. $b = 12$ in., $c = 14$ in.
 10. $a = 7$ in., $c = 13$ in.



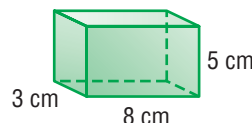
11. **MEASUREMENT** Use the diagram below to find the distance from the library to the post office. Round to the nearest tenth.



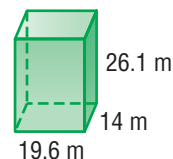
12. **CHAIRS** Chris is responsible for arranging the chairs at the meeting. There are 72 chairs, and he wants to have twice as many chairs in each row as he has in each column. How many chairs should he put in each row? How many rows does he need?

Find the surface area of each rectangular prism and cylinder. Round to the nearest tenth if necessary.

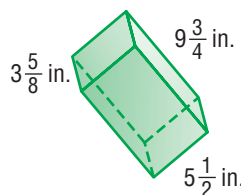
13.



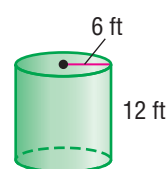
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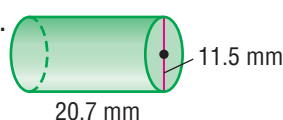
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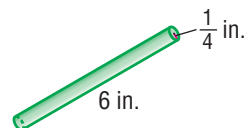
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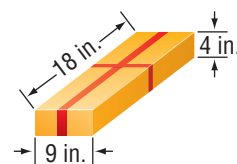
17.



18.

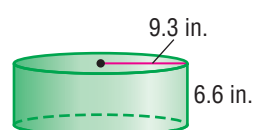


19. **PACKAGING** Mrs. Rodriguez is wrapping a gift. What is the least amount of wrapping paper she will need to wrap the box below?

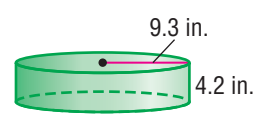


20. **MULTIPLE CHOICE** The dimensions of four containers are given below. Which container has the greatest surface area?

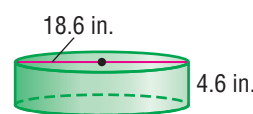
F



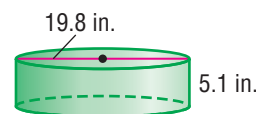
H



G



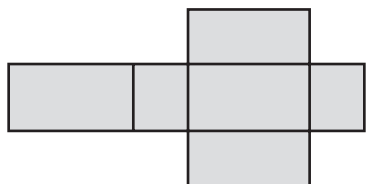
J



PART 1 Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which of the following three-dimensional figures could be formed from this net?



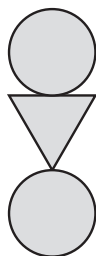
- A cube
- B rectangular pyramid
- C triangular prism
- D rectangular prism

2. Which of the following nets could be used to make a cylinder?

F



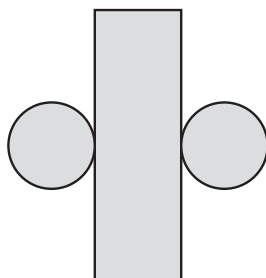
G



H



J



3. Carla has an above-ground swimming pool with a circumference of 20 feet. Which of the following equations could be used to find r , the radius of the pool?

- A $r = \frac{10}{\pi}$
- B $r = \frac{40}{\pi}$
- C $r = \frac{10}{2\pi}$
- D $r = \frac{\pi}{20}$

4. Of the following figures that Ryan drew, which 2 figures have the same area?

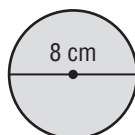


Figure I

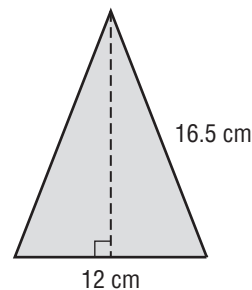


Figure II

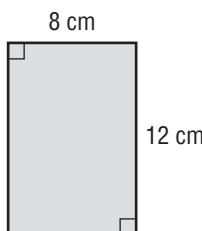


Figure III

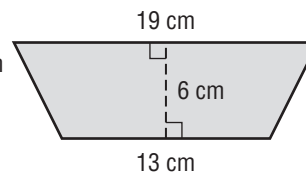


Figure IV

- F Figure I and II
- G Figure II and III
- H Figure II and IV
- J Figure III and IV

5. Cassandra drew a circle with a radius of 12 inches and another circle with a radius of 8 inches. What is the approximate difference between the areas of the 2 circles? Use $\pi = 3.14$.

- A 452.16 in²
- B 251.2 in²
- C 50.24 in²
- D 25.12 in²

6. Which equation could be used to find the area of a circle with a radius of 10 centimeters?
- F $A = 5 \times \pi$
 G $A = \pi \times 5^2$
 H $A = 10 \times \pi$
 J $A = \pi \times 10^2$
7. Dave can run 30 yards in 8.2 seconds. During a race, he ran 120 yards. If Dave's rate of speed remained the same, how long did it take him to run the race?
- A 43 seconds C 24.6 seconds
 B 32.8 seconds D 18.4 seconds
8. Which of the following equations gives the surface area S of a cube with side length m ?
- F $S = m^3$
 G $S = 6m^2$
 H $S = 6m$
 J $S = 2m + 4m^2$

TEST-TAKING TIP

Question 9 Be sure to read each question carefully. In question 9, you are asked to find which statement is *not* true.

9. Which statement is *not* true about an equilateral triangle?
- A The sum of the angles is 180° .
 B It has three congruent angles.
 C It has one right angle.
 D It has exactly three congruent sides.

PART 2 Short Response/Grid In

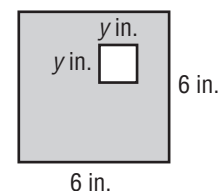
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Bill's Electronics bought 5 computers for a total of \$3,000. The business later bought another computer for \$600. What was the mean price of all the computers?
11. A jar contains 9 yellow marbles and 1 red marble. Ten students will each randomly select one marble to determine who goes first in a game. Whoever picks the red marble goes first. Lily will pick first and keep the marble that she picks. Heath will pick second. What is the probability that Lily will pick a yellow marble and Heath will pick the red marble?

PART 3 Extended Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper. Show your work.

12. A square with a side of y inches is inside a square with a side of 6 inches, as shown below.



- a. Write an expression that can be used to find the area of the shaded region in terms of y .
- b. If the dimensions of both squares are doubled, write an expression that could be used to find the area of the new shaded region.

NEED EXTRA HELP?

If You Missed Question...

Go to Lesson...

IN Academic Standards

1	2	3	4	5	6	7	8	9	10	11	12
11-8	11-8	11-3	11-2	11-4	11-4	6-6	12-4	10-3	8-2	9-8	11-1
8.3.2	8.3.2	8.3.7	8.3.7	8.3.7	8.3.7	7.1.9	6.3.5	7.4.1	7.4.3	7.4.5	8.3.7