

## FOUR-STEP PLAN FOR PROBLEM SOLVING

1. **Explore** Read the problem carefully. Ask yourself questions like, “What facts do I know?” and “What do I need to find out?”
2. **Plan** See how the facts relate to each other. Make a plan for solving the problem. Estimate the answer.
3. **Solve** Use your plan to solve the problem. If your plan does not work, revise it or make a new plan.
4. **Examine** Reread the problem. Ask, “Is my answer close to my estimate?” Ask, “Does my answer make sense for the problem?” If not, solve the problem another way.

## Example

The table shows the average number of meals and snacks children prepare for themselves each week. How many times per *month* does the average 11-year-old prepare his or her own meal?

AGES	TIMES PER WEEK
6-8	2
9-11	3-4
12-14	5 or more

Source: KIDTRENDS

- Explore** The table shows that children ages 9-11 prepare their own meals 3-4 times per week.
- Plan** There are about 4 weeks in a month. Multiply 4 by the number of times an 11-year-old prepares his or her own meal.
- Solve**  $4 \times 3 = 12$                        $4 \times 4 = 16$
- Examine** The average 11-year-old prepares his or her own meal about 12-16 times each month.

**A NUMBER IS DIVISIBLE BY:**

- 2 if the ones digit is divisible by 2.
- 3 if the sum of the digits is divisible by 3.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by both 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the ones digit is 0.

**Examples****1 Is 84 divisible by 3?**

The sum of the digits is  $8 + 4$  or 12.

Since 12 is divisible by 3, 84 is divisible by 3.

**2 Is 361 divisible by 9?**

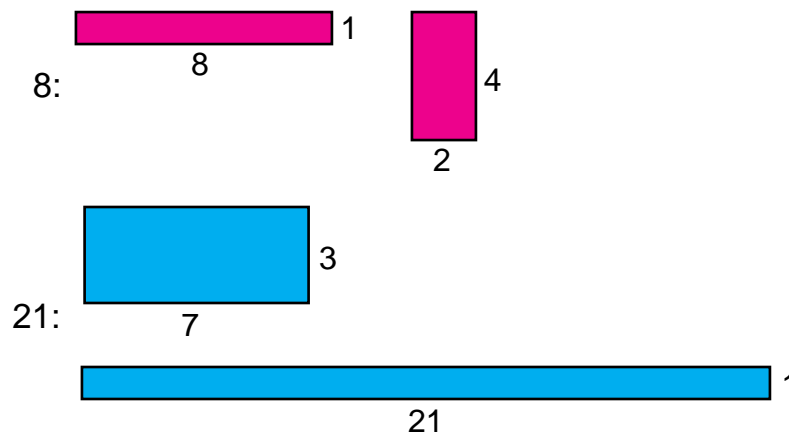
The sum of the digits is  $3 + 6 + 1$  or 10.

Since 10 is not divisible by 9, 361 is not divisible by 9.

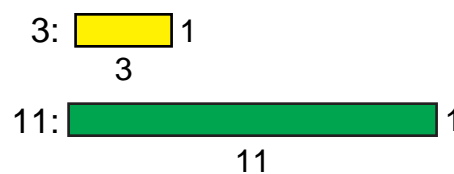
**3 Is 1,455 divisible by 5?**

The ones digit is 5. So, 1,455 is divisible by 5.

The numbers 8 and 21 are examples of **composite numbers**. A composite number has two or more factors.



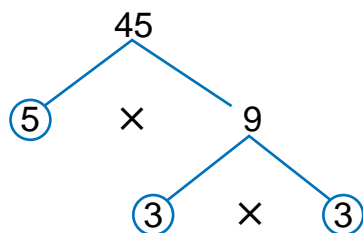
The numbers 3 and 11 are examples of **prime numbers**. A prime number has exactly two factors, 1 and the number itself.



### Example

Find the prime factorization of 45.

**Method 1** Use a factor tree.



*Factor 45. 45 is divisible by 5.*

*Circle the prime number 5. Factor 9.*

*Circle the prime numbers 3.*

**Method 2** Use division.

$$\begin{array}{r} 5 \\ 3 \overline{)15} \\ 3 \overline{)45} \end{array}$$

*Begin with the least prime that is a factor.  
Then divide the quotient by the least possible prime factor.  
Repeat until the quotient is prime.*

The prime factorization of 45 is  $3 \times 3 \times 5$  or  $3^2 \times 5$ .

**AN ALGEBRAIC EXPRESSION INCLUDES AT LEAST ONE OF EACH OF THE FOLLOWING:**

- numbers
- variables (such as  $a$ ,  $n$ , or  $x$ )
- operations ( $+$ ,  $-$ ,  $\times$ , or  $\div$ )

When you evaluate an expression, you replace all of the variables with numbers and find the value.

### Examples

① Evaluate  $d \times 12$  if  $d = 3$ .

$$\begin{aligned}d \times 12 &= 3 \times 12 \\ &= 36\end{aligned}$$

*You know that  $d = 3$ , so replace  $d$  with 3. Then multiply.*

② Evaluate  $a + b$  if  $a = 5$  and  $b = 17$ .

$$\begin{aligned}a + b &= 5 + 17 \\ &= 22\end{aligned}$$

*Replace  $a$  with 5 and  $b$  with 17. Then add.*

③ Evaluate  $s - tu$  if  $s = 47$ ,  $t = 15$ , and  $u = 3$ .

$$\begin{aligned}s - tu &= 47 - 15 \times 3 \\ &= 47 - 45 \\ &= 2\end{aligned}$$

*Replace  $s$  with 47,  $t$  with 15, and  $u$  with 3. Multiply 15 and 3. Then subtract.*

## Examples

Determine the scale and interval for the frequency table for each set of data. Then complete the frequency table.

① 66    95    90    17  
83    98    87    90  
22    89    94    70  
82    70    75    48

② 4,521    4,664    4,555    4,614  
4,907    4,748    4,850    4,880  
4,511    4,617    4,662    4,505  
4,916    4,899    4,740    4,972

SCALE	TALLY	FREQUENCY

SCALE	TALLY	FREQUENCY

The data set includes numbers from 17 to 98.

The data set includes numbers from 4,505 to 4,972.

91-100		3
81-90		6
71-80		1
61-70		3
51-60		0
41-50		1
31-40		0
21-30		1
11-20		1
0-10		0

The  
range is 81. We can use a  
scale of 100 and an interval  
of 10.

4,951-5,000		1
4,901-4,950		2
4,851-4,900		2
4,801-4,850		1
4,751-4,800		0
4,701-4,750		2
4,651-4,700		2
4,601-4,650		2
4,551-4,600		1
4,501-4,550		3

The range is 467. We can use a scale that begins at 4,501 and has intervals of 50.

**Examples**

- ① Use the data in the table to make a bar graph.

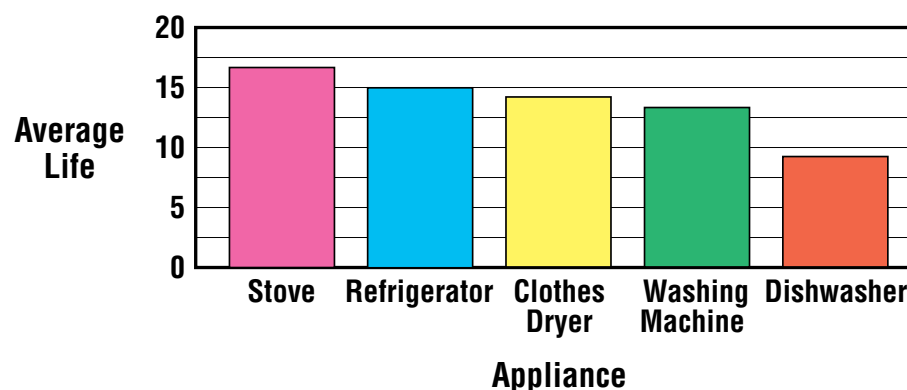
Scale: from 0 to 20

Interval: 2.5

**AVERAGE LIFE OF APPLIANCES**

Appliance	Average Life (years)
stove	17
refrigerator	15
clothes dryer	14
washing machine	13
dishwasher	9

Source: Good Housekeeping

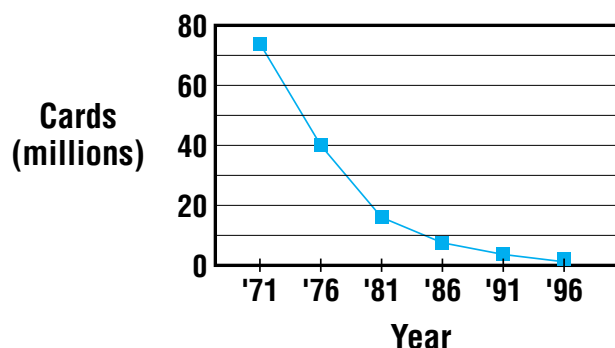
**Average Life of Appliances**

- ② Since libraries now use computers to catalog their materials, the use of card catalogs is declining. Use the data to make a line graph.

**CATALOG CARDS SOLD**

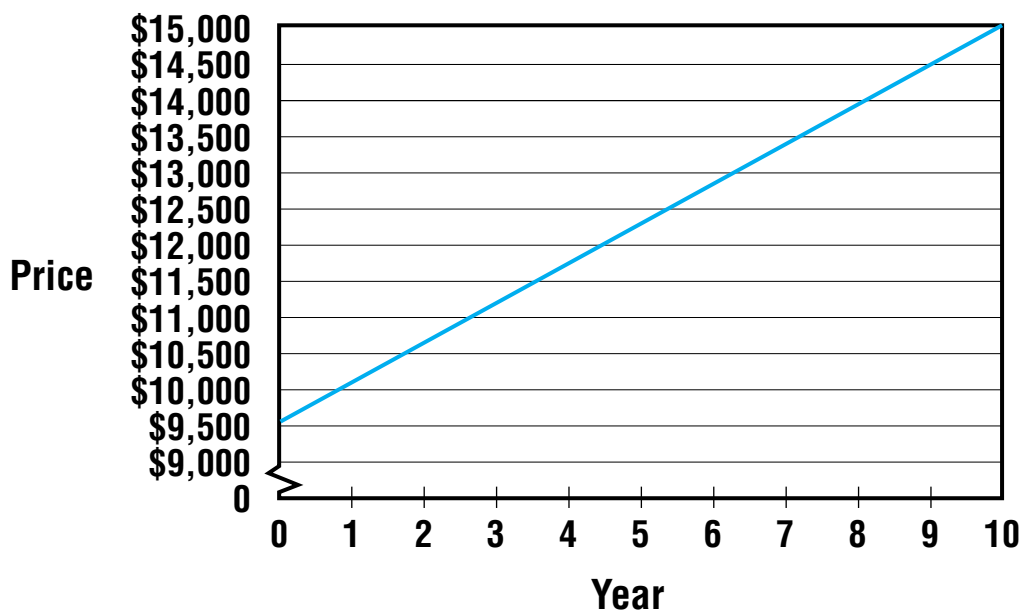
Year	Cards (millions)
1971	74
1976	40
1981	16
1986	8
1991	2
1996	0.6

Source: Library of Congress

**Catalog Cards Sold**

## Example

Price of a New Precisa XT



Assume that the price of a new Precisa XT automobile continues to rise at the same rate. Predict the price of a new Precisa in 10 years. Identify any assumptions you use to make your prediction.

During the first 10 years, the price of a new Precisa rose from \$9,500 to \$15,000. There are two ways to predict the price after another 10 years.

**Method 1**

Draw the graph on a sheet of paper. Extend the graph to predict the price.

**Method 2**

In the first 10 years, the price rose  $\$15,000 - \$9,500$  or  $\$5,500$ . If the price continues to rise at the same rate, it will be  $\$15,000 + \$5,500$  or  $\$20,500$  in another 10 years.

**Example**

Use the table below to make a stem-and-leaf plot that shows the number of losses in the National League in 1997.

TEAM	W	L	TEAM	W	L
Atlanta	100	61	Cincinnati	76	86
Montreal	78	84	Chicago	68	94
Florida	92	70	Pittsburgh	79	83
New York	88	74	San Diego	76	86
Philadelphia	68	94	Los Angeles	88	74
St. Louis	73	89	Colorado	83	79
Houston	84	78	San Francisco	90	72

**Step 1**

Draw a vertical line. To the left of the line, write the digits in the tens place in increasing order. These digits are the **stems**.

Stem	Leaf
6	
7	
8	
9	

**Step 2**

To the right of the line, write the digits in the units place in increasing order. These digits are the **leaves**.

Stem	Leaf
6	1
7	0 2 4 4 8 9
8	3 4 6 6 9
9	4 4

You may have to make a stem-and-leaf plot by just writing the leaves down, and then make another one with the leaves in increasing order.

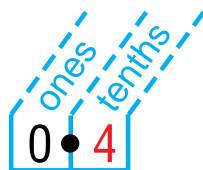
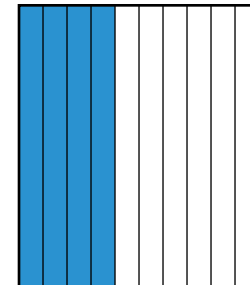
Always include a key or an explanation with your stem-and-leaf-plot.

$$7|1 = 71$$

## Examples

- 1 Write the decimal for the model at the right.

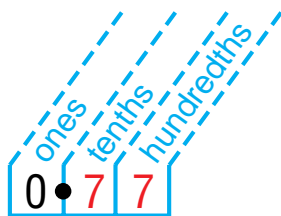
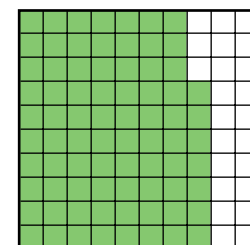
Out of the 10 tenths bars, 4 are shaded.  
Write a 4 in the tenths place on the  
place-value chart.



Tenths are located one place  
to the right of ones on a  
place-value chart.

- 2 Write the decimal for the model at the right.

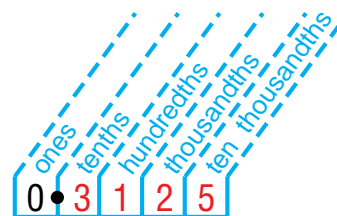
Out of 100 hundredths squares, 77 are  
shaded. There are 77 hundredths.



Hundredths are located one  
place to the right of tenths  
on a place-value chart.

- 3 Write  $\frac{3,125}{10,000}$  as a decimal.

There are 3,125 ten-thousandths.  
The last digit, 5, should be in the  
ten-thousandths place.



## ORDERING DECIMALS

## Method 1

1. Line up the decimal points.
2. Annex zeros so that each decimal has the same number of places.
3. Beginning at the left, compare each place-value position.

## Method 2

Use a number line.

## Examples

- ① Order the following decimals from least to greatest:  
97.27, 97.3, 97.22, 97.21, 97.

The tens digit, 9, is the same.

The ones digit, 7, is the same.

97.27

97.

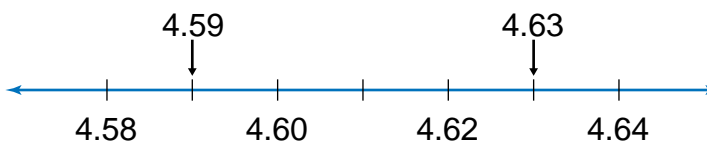
97.

97.

97.

In order, the decimals are 97, 97.21, 97.22, 97.27, and 97.3.

- ② Which is greater, 4.63 or 4.59?



4.63 is to the right of 4.59. So,  $4.63 > 4.59$ .

*The least tenths digit is 0, so 97 is the least decimal.  
The greatest tenths digit is 3, so 97.3 is the greatest decimal.  
All of the other decimals have the same tenths digit, 2.*

↓  
2  
3  
2  
2  
0

7  
0  
2  
1  
0

Since 1 hundredth is less than 2 hundredths, 97.21 is the next greatest decimal. Since 2 hundredths is less than 7 hundredths,  $97.22 < 97.27$ .

## ROUNDING DECIMALS

- Look at the digit to the right of the place being rounded.
- The digit remains the same if the digit to the right is 0, 1, 2, 3, or 4.
- Round up if the digit to the right is 5, 6, 7, 8, or 9.

## Examples

- ① Round 47.075 to the nearest hundredth.

47.075      *The digit to the right of the hundredths place is 5.  
Round up.*  
    ↑  
*hundredths place*

47.075 rounded to the nearest hundredth is 47.08.

- ② Round 5.549 to the nearest tenth.

5.549      *The digit to the right of the tenths place is 4.  
The 5 stays the same.*  
    ↑  
*tenths place*

5.549 rounded to the nearest tenth is 5.5.

- ③ Ashley averaged her science test grades. The result was 82.125. Find her grade to the nearest tenth.

82.125      *The digit to the right of the tenths place is 2.  
The 1 stays the same.*  
    ↑  
*tenths place*

82.125 rounded to the nearest tenth is 82.1.

**Examples**

- ① Add 4.8 and 15.79.

*Estimate:  $5 + 16 = 21$*

$$\begin{array}{r} 4.80 \\ + 15.79 \\ \hline 20.59 \end{array}$$

*Line up the decimal points and annex a zero.  
Add.*

The estimate shows that the answer is reasonable.

- ② Find the difference of 127.905 and 55.621.

*Estimate:  $130 - 60 = 70$*

$$\begin{array}{r} 127.905 \\ - 55.621 \\ \hline 72.284 \end{array}$$

*Line up the decimal points.  
Subtract.*

The estimate shows that the answer is reasonable.

- ③ Evaluate the expression  $x + y$  if  $x = 47.9$  and  $y = 17.3$ .

$x + y = 47.9 + 17.3$     *Replace  $x$  with 47.9 and  $y$  with 17.3.*

$$\begin{array}{r} 47.9 \\ + 17.3 \\ \hline 65.2 \end{array}$$

*Line up the decimal points.  
Add.*

The value is 65.2.

## ESTIMATING PRODUCTS

Method 1 Use rounding.	Method 2 Use compatible numbers.
Round each factor to its greatest place-value position. Then multiply. Do not round 1-digit factors.	Compatible numbers are numbers whose product equals some power of 10.

## Examples

- ① Find  $7.11 \times 2$ .

*Estimate using rounding. Round 7.11 to 7;  $7 \times 2 = 14$ .*

7.11 *Multiply as with whole numbers.*

$$\begin{array}{r} \times \quad 2 \\ \hline \end{array}$$

14.22 *Since the estimate is 14, place the decimal point after 14.*

- ② Find  $6 \times 0.038$ .

0.038 ← *three decimal places*

$$\begin{array}{r} \times \quad 6 \\ \hline \end{array}$$

0.228 The product is 0.228.

- ③ Evaluate the expression  $54a$  if  $a = 135.7$ .

$54a = 54 \times 135.7$  *Replace  $a$  with 135.7.*

135.7 ← *one decimal place*

$$\begin{array}{r} \times \quad 54 \\ \hline \end{array}$$

5428

$$\begin{array}{r} 6785 \\ \hline \end{array}$$

7,327.8 ← *The product also has one decimal place.*

## MULTIPLYING DECIMALS

1. Multiply as you would with whole numbers.
2. Use estimation to place the decimal point in the product.

or

Count the number of decimal places in each factor. The product will have the same number of decimal places as the sum of the number of decimal places in the factors.

## Examples

- ① Multiply 6.27 and 1.1.      *Estimate:  $6 \cdot 1 = 6$*

$$\begin{array}{r} 6.27 \\ \times 1.1 \\ \hline 6.897 \end{array}$$

*Since the estimate was 6, place the decimal point after the 6.*

- ② Multiply 6.36 and 5.98.      *Estimate:  $6 \times 6 = 36$*

$$\begin{array}{r} 6.36 \leftarrow \text{two decimal places} \\ \times 5.98 \leftarrow \text{two decimal places} \\ \hline 38.0328 \end{array}$$

*Since the sum of the number of decimal places in the factors is four, the product has four decimal places.*

- ③ Solve  $0.7 \cdot 5.8 = g$ .

$$\begin{array}{r} 5.8 \leftarrow \text{one decimal place} \\ \times 0.7 \leftarrow \text{one decimal place} \\ \hline 4.06 \leftarrow \text{two decimal places} \end{array}$$

## DIVIDING A DECIMAL BY A WHOLE NUMBER

1. Place the decimal point in the quotient directly above the decimal point in the dividend.
2. Divide as you do with whole numbers.
3. Annex a zero, if necessary, to complete the division.

## Examples

① Find  $14.4 \div 12$ .

*Estimate:*  $14 \div 14 = 1$

$$\begin{array}{r} 1.2 \\ 12 \overline{)14.4} \\ \underline{-12} \phantom{.} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

*Place the decimal point. Divide as with whole numbers.*

$$14.4 \div 12 = 1.2$$

The estimate shows that the answer is reasonable.

② Find  $58.85 \div 25$ .

*Estimate:*  $50 \div 25 = 2$

$$\begin{array}{r} 2.354 \\ 25 \overline{)58.850} \\ \underline{-50} \phantom{.} \\ 88 \\ \underline{-75} \\ 135 \\ \underline{-125} \\ 100 \\ \underline{-100} \\ 0 \end{array}$$

*Place the decimal point. Divide as with whole numbers.*

*Annex a zero to continue dividing.*

$$58.85 \div 25 = 2.354$$

The estimate shows that the answer is reasonable.

## DIVIDING A DECIMAL BY A DECIMAL

1. Multiply both the divisor and the dividend by a power of 10 to change the divisor to a whole number.
2. Divide as with whole numbers.

## Examples

- ① Find  $8.17 \div 4.3$ .

$$4.3 \overline{)8.17} \rightarrow 43 \overline{)81.7}$$

$$\begin{array}{r} 1.9 \\ 43 \overline{)81.7} \\ \underline{-43} \phantom{0} \\ 387 \\ \underline{-387} \\ 0 \end{array}$$

$$8.17 \div 4.3 = 1.9$$

$$\text{Check: } 4.3 \times 1.9 = 8.17 \checkmark$$

$$\text{Estimate: } 8 \div 4 = 2$$

*You need to change 4.3 to a whole number. So multiply 4.3 and 8.17 by 10. Then place the decimal point and divide.*

The estimate shows that the answer is reasonable.

- ② Find  $20.075 \div 3.65$ .

$$3.65 \overline{)20.075} \rightarrow 365 \overline{)2007.5}$$

$$\begin{array}{r} 5.5 \\ 365 \overline{)2007.5} \\ \underline{-1825} \phantom{0} \\ 1825 \\ \underline{-1825} \\ 0 \end{array}$$

$$\text{Check: } 3.65 \times 5.5 = 20.075 \checkmark$$

$$\text{Estimate: } 20 \div 4 = 5$$

## FINDING THE GCF

Method 1 Make a list.	Method 2 Use prime factorization.
<ul style="list-style-type: none"> <li>List all of the factors of each number.</li> <li>Identify the common factors.</li> <li>The greatest of the common factors is the GCF.</li> </ul>	<ul style="list-style-type: none"> <li>Write the prime factorization of each number.</li> <li>Identify all of the common prime factors.</li> <li>The product of the common prime factors is the GCF.</li> </ul>

## Examples

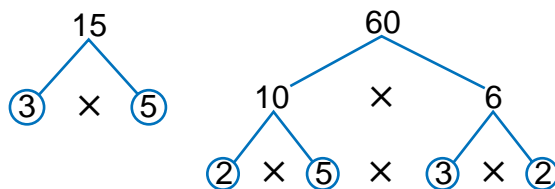
- 1 Find the GCF of 40 and 64 by making a list.

factors of 40: 1, 2, 4, 5, 8, 10, 20, 40  
 factors of 64: 1, 2, 4, 8, 16, 32, 64

*List all of the factors of each number.*

The common factors are 1, 2, 4, and 8.  
 The GCF of 40 and 64 is 8.

- 2 Find the GCF of 15 and 60 by using prime factorization.

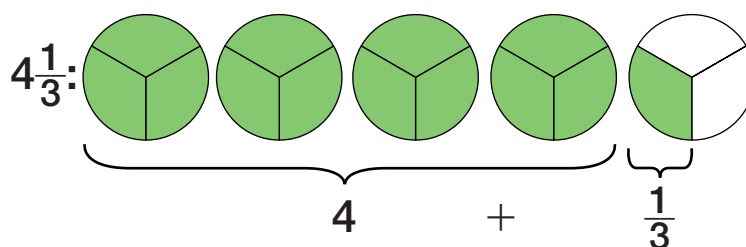
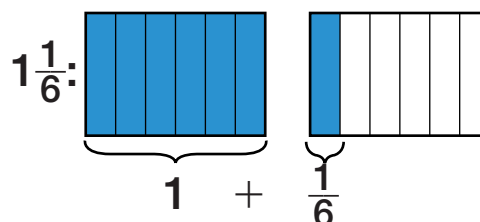


*Write the prime factorization of each number.*

The common prime factors are 3 and 5.  
 The GCF of 15 and 60 is  $3 \times 5$  or 15.

The numbers  $1\frac{1}{6}$  and  $4\frac{1}{3}$  are examples of **mixed numbers**.

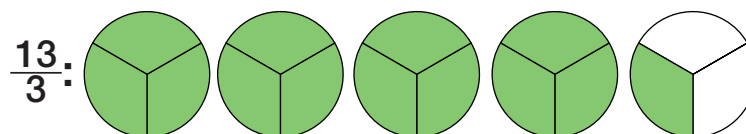
Mixed numbers show the sum of a whole number and a fraction.



The numbers  $\frac{7}{6}$  and

$\frac{13}{3}$  are examples of **improper fractions**.

An improper fraction is a fraction with a numerator that is greater than or equal to the denominator.



### Examples

- ① Express  $2\frac{1}{7}$  as an improper fraction.

$$2\frac{1}{7} = \frac{(2 \times 7) + 1}{7} \text{ or } \frac{15}{7}$$

- ② Express  $\frac{11}{4}$  as a mixed number.

Divide 11 by 4.

$$\begin{array}{r} 2 \\ 4 \overline{)11} \\ \underline{-8} \\ 3 \end{array}$$

$$\frac{11}{4} = 2\frac{3}{4}$$

## Examples

Replace each  $\bullet$  with  $<$ ,  $>$ , or  $=$  to make a true sentence.

①  $\frac{2}{5} \bullet \frac{3}{10}$

The LCM of 5 and 10 is 10. Express  $\frac{2}{5}$  as a fraction with a denominator of 10.

$$\frac{2}{5} = \frac{\blacksquare}{10} \rightarrow \frac{2}{5} = \frac{4}{10}$$

Since  $4 > 3$ ,  $\frac{4}{10} > \frac{3}{10}$ . Therefore,  $\frac{2}{5} > \frac{3}{10}$ .

②  $\frac{8}{15} \bullet \frac{7}{9}$

The LCM of 15 and 9 is 45. Express  $\frac{8}{15}$  and  $\frac{7}{9}$  as fractions with a denominator of 45.

$$\frac{8}{15} = \frac{\blacksquare}{45} \rightarrow \frac{8}{15} = \frac{24}{45}$$

$$\frac{7}{9} = \frac{\blacksquare}{45} \rightarrow \frac{7}{9} = \frac{35}{45}$$

Since  $24 < 35$ ,  $\frac{24}{45} < \frac{35}{45}$ . Therefore,  $\frac{8}{15} < \frac{7}{9}$ .

## WRITING A FRACTION AS A DECIMAL

## Method 1

Use paper and pencil to divide the numerator by the denominator.

## Method 2

Use a calculator to divide the numerator by the denominator.

## Examples

- ① Express  $\frac{5}{16}$  as a decimal.      ② Express  $1\frac{1}{12}$  as a decimal.

Method 1 Divide 5 by 16.

$$\begin{array}{r} \frac{5}{16} \rightarrow 16 \overline{)5.0000} \\ \underline{-0} \\ 50 \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-32} \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

Therefore,  $\frac{5}{16} = 0.3125$ .

## Method 2

$$5 \div 16 = 0.3125$$

Method 1 Since  $1\frac{1}{12} = 1 + \frac{1}{12}$ , divide 1 by 12.

$$\begin{array}{r} \frac{1}{12} \rightarrow 12 \overline{)1.0000} \\ \underline{-0} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-96} \\ 40 \\ \underline{-36} \\ 40 \\ \underline{-36} \\ 4 \end{array}$$

Therefore,  $1\frac{1}{12} = 1.08\overline{3}$ . *The bar indicates that the digit 3 repeats.*

*The pattern will continue.*

## Method 2

$$1 + 1 \div 12 = 1.08333333$$

### ADDING AND SUBTRACTING LIKE FRACTIONS

To add fractions with like denominators, add the numerators. Use the same denominator in the sum.

To subtract fractions with like denominators, subtract the numerators. Use the same denominator in the difference.

#### Examples

- ① Find the sum of  $\frac{2}{3}$  and  $\frac{2}{3}$ .      ② Find the difference of  $\frac{8}{9}$  and  $\frac{5}{9}$ .

*Estimate:*  $\frac{1}{2} + \frac{1}{2} = 1$

*Estimate:*  $1 - \frac{1}{2} = \frac{1}{2}$

$$\frac{2}{3} + \frac{2}{3} = \frac{2+2}{3}$$

$$\frac{8}{9} - \frac{5}{9} = \frac{8-5}{9}$$

$$= \frac{4}{3} \quad \text{Add the numerators.}$$

$$= \frac{3}{9} \quad \text{Subtract the numerators.}$$

$$= 1\frac{1}{3} \quad \text{Rename.}$$

$$= \frac{1}{3} \quad \text{Simplify.}$$

*Compared to the estimate, the answer is reasonable.*

*Compared to the estimate, the answer is reasonable.*

- ③ Solve  $y + \frac{3}{7} = \frac{5}{7}$  mentally.

What can be added to  $\frac{3}{7}$  to make  $\frac{5}{7}$ ?

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}, \text{ so } y = \frac{2}{7}.$$

### ADDING AND SUBTRACTING UNLIKE FRACTIONS

To add fractions with unlike denominators, rename the fractions using the least common denominator (LCD). Then add and simplify.

To subtract fractions with unlike denominators, rename the fractions using the least common denominator (LCD). Then subtract and simplify.

#### Examples

① Find  $\frac{5}{12} + \frac{1}{4}$ .

*The LCD of  $\frac{5}{12}$  and  $\frac{1}{4}$  is 12.*

$$\frac{5}{12} + \frac{1}{4} = \frac{5}{12} + \frac{3}{12}$$

*Rename  $\frac{1}{4}$  as  $\frac{3}{12}$ .*

$$= \frac{5 + 3}{12}$$

$$= \frac{8}{12}$$

*Add the numerators.*

$$= \frac{2}{3}$$

*Simplify.*

② Evaluate  $r - s$  if  $r = \frac{5}{8}$  and  $s = \frac{1}{3}$ .

$$r - s = \frac{5}{8} - \frac{1}{3}$$

*Replace  $r$  with  $\frac{5}{8}$  and  $s$  with  $\frac{1}{3}$ .*

$$= \frac{15}{24} - \frac{8}{24}$$

*Rename  $\frac{5}{8}$  as  $\frac{15}{24}$  and  $\frac{1}{3}$  as  $\frac{8}{24}$ .*

$$= \frac{15 - 8}{24}$$

*Subtract the numerators.*

$$= \frac{7}{24}$$

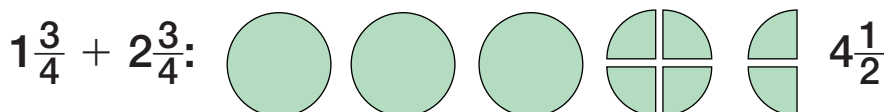
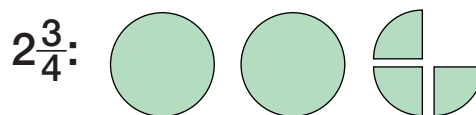
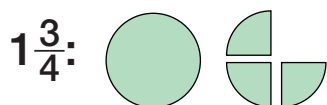
### ADDING AND SUBTRACTING MIXED NUMBERS

To add mixed numbers, add the fractions. Then add the whole numbers. Rename and simplify if necessary.

To subtract mixed numbers, subtract the fractions. Then subtract the whole numbers. Rename and simplify if necessary.

#### Examples

① Find  $1\frac{3}{4} + 2\frac{3}{4}$ .



② Find  $5\frac{1}{4} + 2\frac{3}{4}$ .

*Estimate:*  $5 + 3 = 8$

$$\begin{array}{r} 5\frac{1}{4} \rightarrow 5\frac{1}{4} \\ + 2\frac{3}{4} \rightarrow + 2\frac{3}{4} \\ \hline \frac{4}{4} \qquad 7\frac{4}{4} \end{array}$$

Rename  $7\frac{4}{4}$  as 8.

③ Find  $11\frac{5}{6} - 5\frac{1}{4}$ .

*Estimate:*  $12 - 5 = 7$

$$\begin{array}{r} 11\frac{5}{6} \rightarrow 11\frac{10}{12} \\ - 5\frac{1}{4} \rightarrow - 5\frac{3}{12} \\ \hline 6\frac{7}{12} \end{array}$$

**Example**

Find  $12\frac{1}{5} - 7\frac{4}{5}$ .

$$12\frac{1}{5}$$

$$- 7\frac{4}{5}$$

---

Notice that  $\frac{4}{5}$  cannot be subtracted from  $\frac{1}{5}$ . Rename  $12\frac{1}{5}$  as  $11\frac{6}{5}$ .

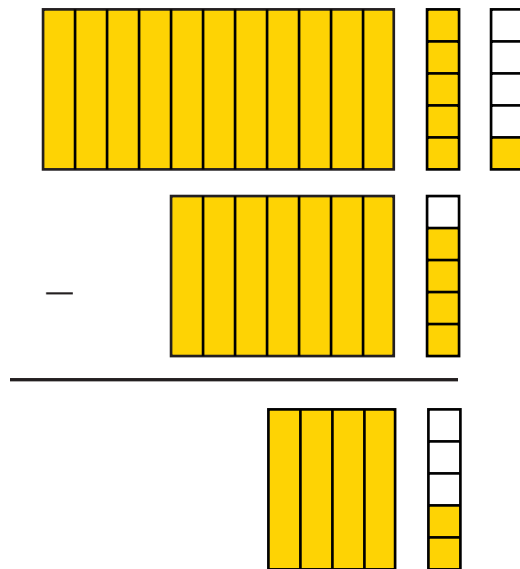
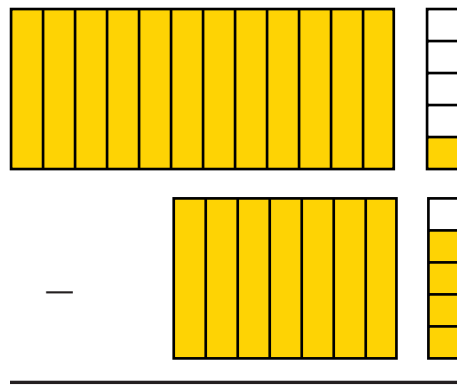
$$11\frac{6}{5}$$

$$- 7\frac{4}{5}$$

---

$$4\frac{2}{5}$$

$$11\frac{6}{5} - 7\frac{4}{5} = 4\frac{2}{5}$$



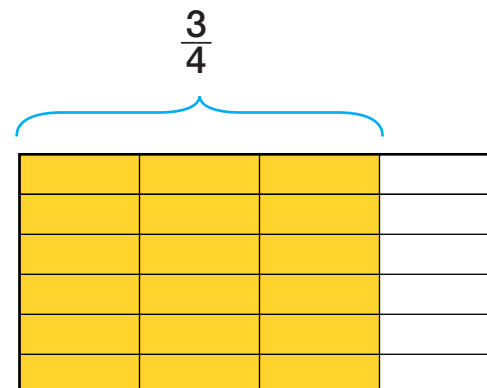
6-6 overlay 1

You can multiply fractions by using a model or using a rule.

### Examples

① Find  $\frac{3}{4} \times \frac{5}{6}$ .

To model  $\frac{3}{4}$ , divide a rectangle into fourths. Shade 3 of the 4 sections.



### MULTIPLYING FRACTIONS

To multiply fractions, multiply the numerators. Then multiply the denominators. Simplify if necessary.

② Find  $\frac{7}{8} \times \frac{2}{3}$ .

$$\frac{7}{8} \times \frac{2}{3} = \frac{7 \cdot 2}{8 \cdot 3}$$

*Multiply the numerators.*

*Multiply the denominators.*

$$= \frac{14}{24} \text{ or } \frac{7}{12}$$

*Simplify.*

To model  $\frac{5}{6}$ , divide the rectangle into sixths. Shade 5 of the 6 sections.



The overlapping area shows the product. Since 15 out of 24 parts are green, the product of  $\frac{3}{4}$  and  $\frac{5}{6}$  is  $\frac{15}{24}$  or  $\frac{5}{8}$ .

**Examples**

**1** Find  $4 \times 2\frac{1}{4}$ .

$$4 \times 2\frac{1}{4} = \frac{4}{1} \times \frac{9}{4}$$

$$= \frac{\overset{1}{\cancel{4}} \cdot 9}{1 \cdot \underset{1}{\cancel{4}}}$$

$$= 9$$

*Estimate:  $4 \times 2 = 8$*

*Express 4 and  $2\frac{1}{4}$  as improper fractions.**Divide by the GCF, 4.**Compare with your estimate.*

**2** Find  $\frac{1}{4} \times 7\frac{3}{7}$ .

$$\frac{1}{4} \times 7\frac{3}{7} = \frac{1}{4} \times \frac{52}{7}$$

$$= \frac{1 \cdot \overset{13}{\cancel{52}}}{\underset{1}{\cancel{4}} \cdot 7}$$

$$= \frac{13}{7} \text{ or } 1\frac{6}{7}$$

*Estimate:  $\frac{1}{4} \times 8 = 2$*

*Express  $7\frac{3}{7}$  as an improper fraction.**Divide by the GCF, 4.**Compare with your estimate.*

**3** Solve  $y = 4\frac{1}{2} \times 5\frac{2}{3}$ . *Estimate:  $5 \times 6 = 30$*

$$y = 4\frac{1}{2} \times 5\frac{2}{3}$$

$$y = \frac{9}{2} \times \frac{17}{3}$$

$$y = \frac{\overset{3}{\cancel{9}}}{2} \times \frac{17}{\underset{1}{\cancel{3}}}$$

$$y = \frac{51}{2} \text{ or } 25\frac{1}{2}$$

*Express  $4\frac{1}{2}$  and  $5\frac{2}{3}$  as improper fractions.**Divide by the GCF, 3.**Compare with your estimate.*

Any two numbers whose product is 1 are called **reciprocals**.

### Examples

Find the reciprocal of each number.

① 8

Since  $8 \times \frac{1}{8} = 1$ , the reciprocal of 8 is  $\frac{1}{8}$ .

②  $\frac{7}{9}$

Since  $\frac{7}{9} \times \frac{9}{7} = 1$ , the reciprocal of  $\frac{7}{9}$  is  $\frac{9}{7}$ .

### DIVIDING FRACTIONS

To divide a fraction, multiply by its reciprocal.

### Example

③ Find  $\frac{5}{8} \div \frac{3}{4}$ .

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3}$$

*Multiply by  $\frac{4}{3}$ , the reciprocal of  $\frac{3}{4}$ .*

$$= \frac{5}{\cancel{8}^1} \times \frac{\cancel{4}^1}{3}$$

*Divide 4 and 8 by the GCF, 4.*

$$= \frac{5}{2} \times \frac{1}{3}$$

*Multiply the numerators.  
Multiply the denominators.*

$$= \frac{5}{6}$$

**Examples**

① Find  $18\frac{2}{3} \div 3\frac{1}{3}$ .

*Estimate:  $18 \div 3 = 6$ .*

$$18\frac{2}{3} \div 3\frac{1}{3} = \frac{56}{3} \div \frac{10}{3} \quad \text{Rename } 18\frac{2}{3} \text{ and } 3\frac{1}{3} \text{ as improper fractions.}$$

$$= \frac{56}{3} \times \frac{3}{10} \quad \text{Multiply by the reciprocal.}$$

$$= \frac{28}{1} \times \frac{1}{5} \quad \begin{array}{l} \text{Divide 3 and 3 by the GCF, 3.} \\ \text{Divide 56 and 10 by the GCF, 2.} \end{array}$$

$$= \frac{28}{5} \text{ or } 5\frac{3}{5} \quad \text{Simplify. Compare with your estimate.}$$

② Find  $7\frac{5}{8} \div 1\frac{1}{2}$ .

*Estimate:  $8 \div 2 = 4$*

$$7\frac{5}{8} \div 1\frac{1}{2} = \frac{61}{8} \div \frac{3}{2} \quad \text{Rename } 7\frac{5}{8} \text{ and } 1\frac{1}{2} \text{ as improper fractions.}$$

$$= \frac{61}{8} \times \frac{2}{3} \quad \text{Multiply by the reciprocal.}$$

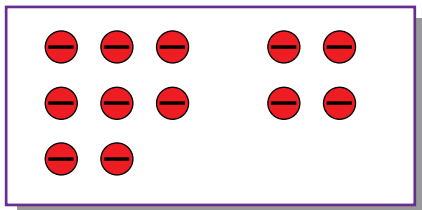
$$= \frac{61}{4} \times \frac{1}{3} \quad \text{Divide 2 and 8 by the GCF, 2.}$$

$$= \frac{61}{12} \text{ or } 5\frac{1}{12} \quad \text{Simplify. Compare with your estimate.}$$

**Examples**

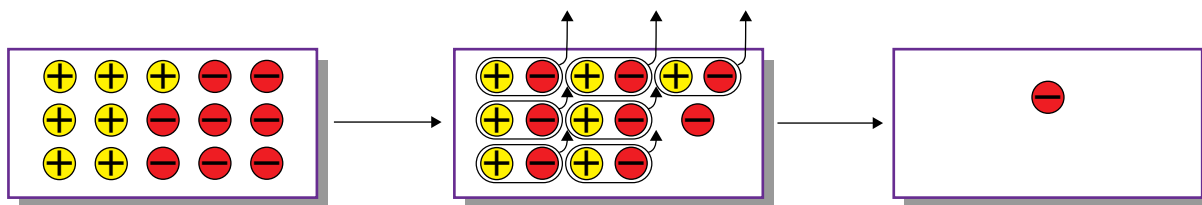
Find each sum.

①  $-8 + (-4)$



Place 8 negative counters on the mat to represent  $-8$ . Place 4 more negative counters on the same mat to represent adding  $-4$ . Since there are no positive counters, you cannot remove any zero pairs. Count the counters on the mat. There are 12 negative counters. So,  $-8 + (-4) = -12$ .

②  $7 + (-8)$



Place 7 positive counters on the mat. Place 8 negative counters on the mat.

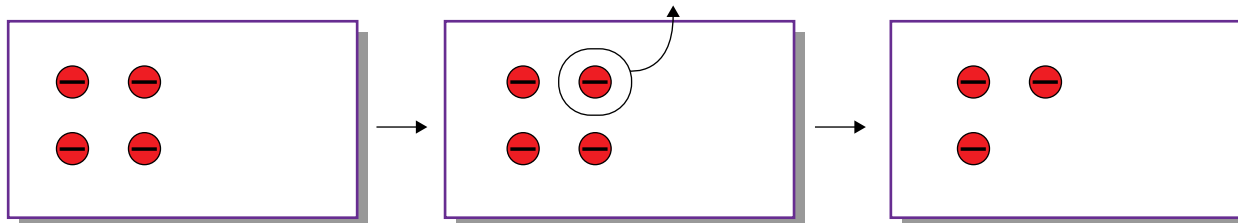
Pair the positive and negative counters. Remove as many zero pairs as possible.

Count the counters left on the mat. There is 1 negative counter. So,  $7 + (-8) = -1$ .

**Examples**

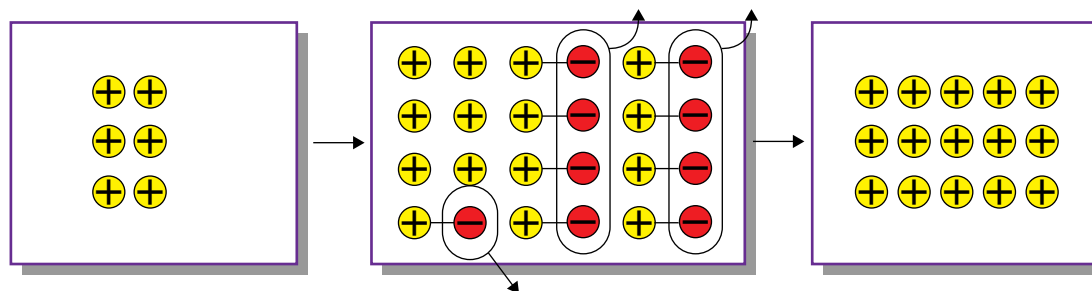
Find each difference.

①  $-4 - (-1)$

**Step 1** Place 4 negative counters on the mat to represent  $-4$ .**Step 2** Remove 1 negative counter to represent subtracting  $-1$ .**Step 3** Count the counters remaining on the mat. There are 3 negative counters. This represents  $-3$ .

So,  $-4 - (-1) = -3$ .

②  $6 - (-9)$

**Step 1** Place 6 positive counters on the mat to represent 6.**Step 2** To subtract  $-9$ , remove 9 negative counters. But there are no negative counters on the mat. So, add 9 zero pairs to the mat. Then remove 9 negative counters.**Step 3** Count the counters remaining on the mat. There are 15 positive counters. This represents 15. So,  $6 - (-9) = 15$ .

## Examples

① Find  $4 \times 3$ .

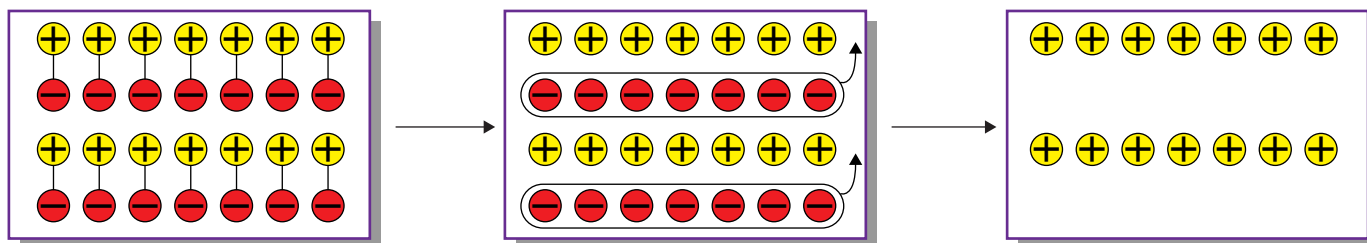
**Step 1**  $4 \times 3$  means to *put in* 4 sets of 3 positive counters. Place these counters on the mat.



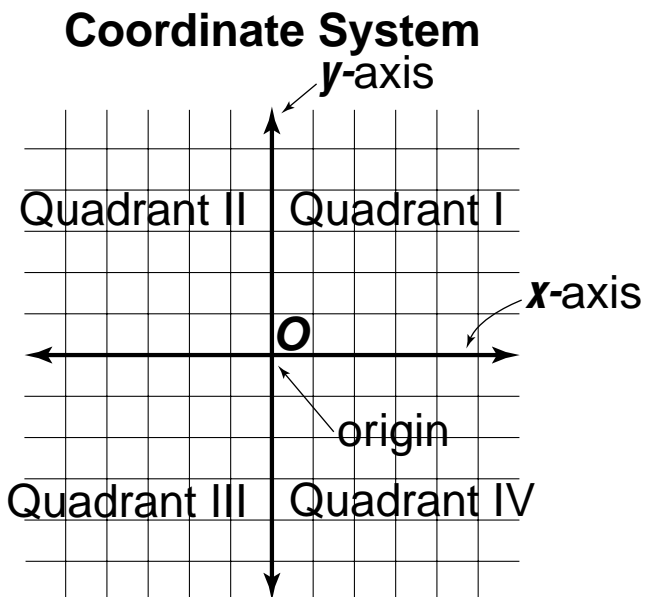
**Step 2** Count the counters on the mat. There are 12 positive counters. This represents 12. So,  $4 \times 3 = 12$ .

② Find  $-2(-7)$ .

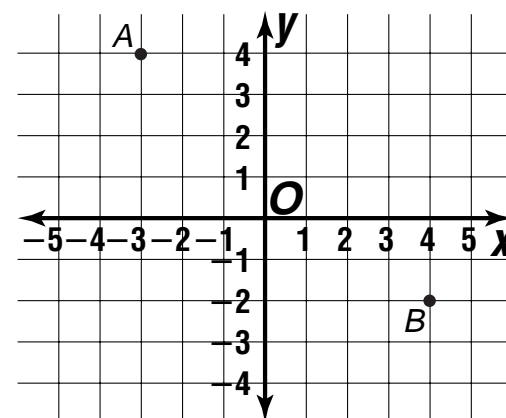
**Step 1** Since  $-2$  is the opposite of 2,  $-2(-7)$  means to *remove* 2 sets of 7 negative counters. First place 2 sets of 7 zero pairs on the mat. Then remove 2 sets of 7 negative counters.



**Step 2** Count the counters remaining on the mat. There are 14 positive counters. This represents 14. So,  $-2(-7) = 14$ .

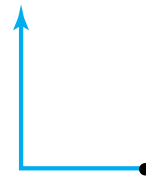
**Example**

Name the ordered pair for points  $A$  and  $B$ .



**For point A:**

- Start at 0. Move left along the  $x$ -axis until you are under point A. The first coordinate of the ordered pair is  $-3$ .
- Now, move up parallel to the  $y$ -axis until you reach point A. The second coordinate of the ordered pair is 4.
- Thus, the ordered pair for point A is  $(-3, 4)$ .





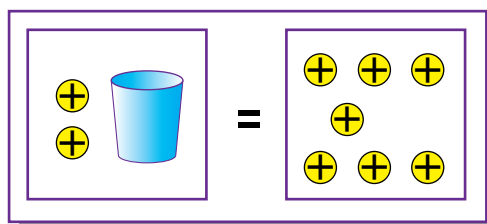
**For point  $B$ :**

- Start at 0. Move right along the  $x$ -axis until you are over point  $B$ . The first coordinate is 4.
- Now, move down parallel to the  $y$ -axis until you reach point  $B$ . The second coordinate of the ordered pair is  $-2$ .
- Thus, the ordered pair for point  $B$  is  $(4, -2)$ .

## Examples

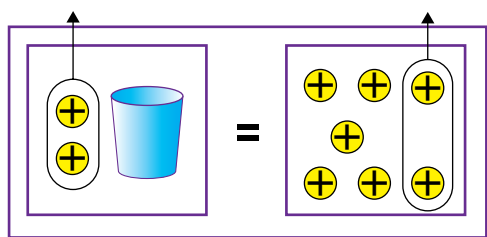
Solve each equation.

①  $n + 2 = 7$



$$n + 2 = 7$$

Place a cup on the left side of the mat to represent  $n$ . Add 2 positive counters to represent  $+2$ . Place 7 positive counters on the right side to represent 7.



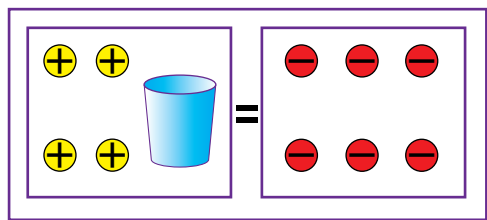
$$n + 2 - 2 = 7 - 2$$

$$n = 5$$

To get the cup by itself, remove 2 positive counters from each side.

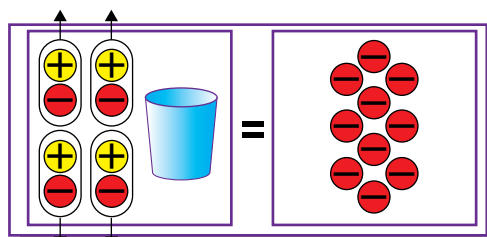
The solution is 5.

②  $c + 4 = -6$



$$c + 4 = -6$$

Place a cup on the left side of the mat to represent  $c$ . Add 4 positive counters to represent  $+4$ . Place 6 negative counters on the right side to represent  $-6$ .



$$c + 4 + (-4) = -6 + (-4)$$

$$c = -10$$

To get the cup by itself, remove 4 positive counters from each side. But there are no positive counters on the right side. So, add 4 negative counters to each side to make 4 zero pairs on the left side.

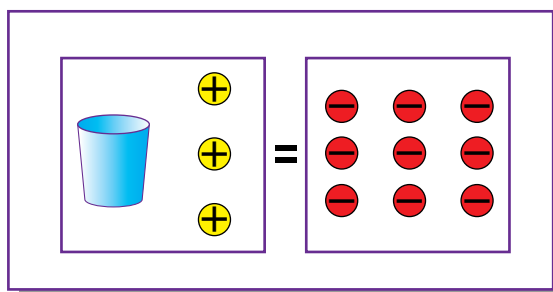
Then remove the zero pairs.

The solution is  $-10$ .

## Examples

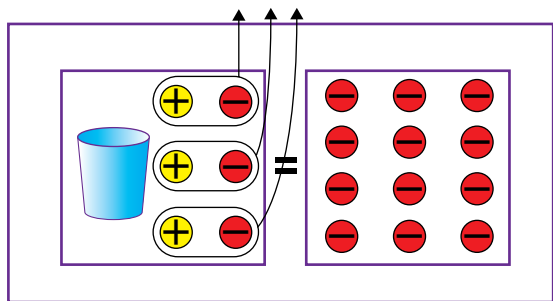
Solve each equation.

①  $f - (-3) = -9$

Rewrite as an addition equation.  $f + 3 = -9$ 

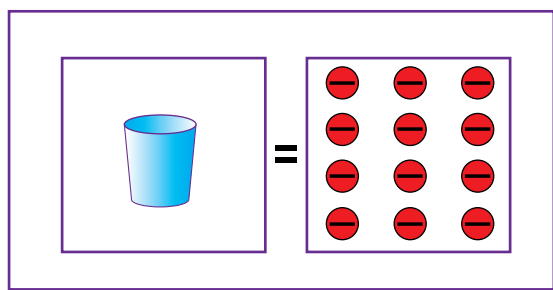
$$f + 3 = -9$$

Place a cup on the left side of the mat to represent  $f$ . Add 3 positive counters to represent  $+3$ . Place 9 negative counters on the right side to represent  $-9$ .



$$f + 3 + (-3) = -9 + (-3)$$

To get the cup by itself, remove 3 positive counters from each side. But there are no positive counters on the right side of the mat. So, add 3 negative counters to each side to make 3 zero pairs on the left side.



$$f = -12$$

Then remove the zero pairs.

The solution is  $-12$ .

②  $w - 5 = -7$

Rewrite as an addition equation.

$$w + (-5) = -7$$

Since  $-2 + (-5) = -7$ ,  $w = -2$ .

Check:  $w - 5 = -7$

$$-2 - 5 \stackrel{?}{=} -7$$

$$-7 = -7 \quad \checkmark$$

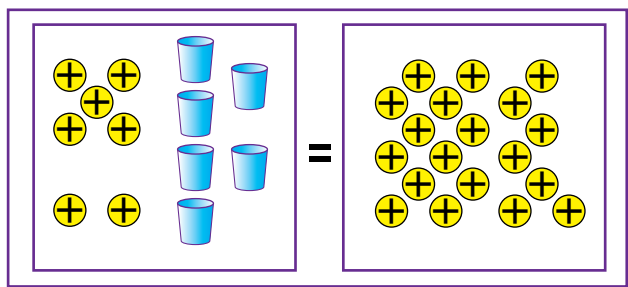
**Example**

Deondra made a gift for her mother at Ceramics by You. She chose a vase that cost \$7. Ceramics by You charges \$6 an hour to paint the piece. If Deondra spent \$19 on her mother's gift, how many hours did it take to paint the vase?

Let  $h$  equal the number of hours. Translate the problem into an equation.

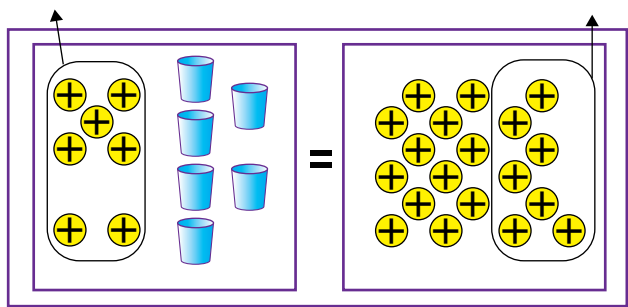
$$\begin{array}{ccccccc} \$7 \text{ for the vase} & \text{plus} & \$6 \text{ per hour} & \text{is} & \text{total cost.} \\ 7 & + & 6h & = & 19 \end{array}$$

Solve the equation  $7 + 6h = 19$ .



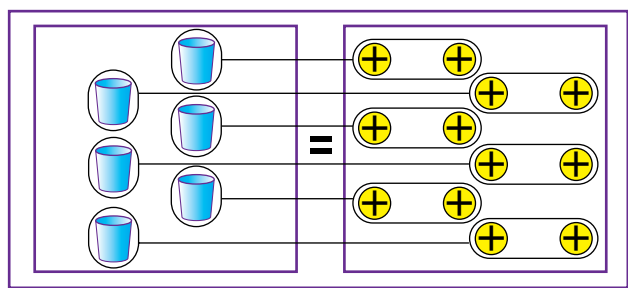
$$7 + 6h = 19$$

*Represent the equation using cups and counters.*



$$\begin{array}{l} 7 + 6h - 7 = 19 - 7 \\ 6h = 12 \end{array}$$

*To get the cups by themselves, remove 7 positive counters from each side.*



$$h = 2$$

*Since there are 6 cups, undo the multiplication by dividing each side by 6. Form 6 equal groups on each side of the mat.*

The solution is 2. It took 2 hours to paint the vase.

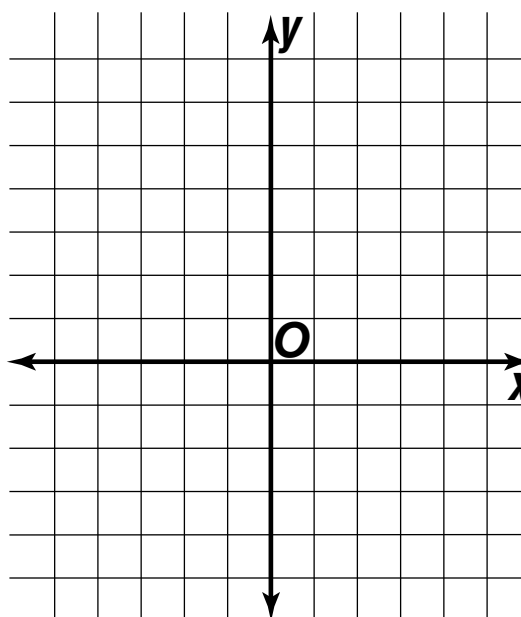
**Example**

Make a function table for the rule  $2x + 2$ . Then graph the function.

**Step 1** Record the input and output in a function table. We chose  $-3$ ,  $-1$ ,  $1$ , and  $2$  for the input.

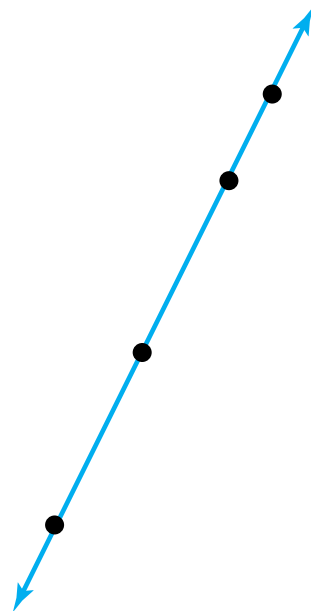
INPUT	FUNCTION RULE	OUTPUT	ORDERED PAIRS
$x$	$2x + 2$	$y$	$(x, y)$
$-3$	$2(-3) + 2$		
$-1$	$2(-1) + 2$		
$1$	$2(1) + 2$		
$2$	$2(2) + 2$		

**Step 2** Graph the ordered pairs from the table in Step 1 on the coordinate plane.



**Step 3** Draw the line that contains these points.

-4	$(-3, -4)$
0	$(-1, 0)$
4	$(1, 4)$
6	$(2, 6)$



9-7 overlay 1

## PROPERTY OF PROPORTIONS

**Words:** The cross products of a proportion are equal.

**Symbols:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $ad = bc$ .

## Examples

Use cross products to determine whether each pair of ratios forms a proportion.

①  $\frac{1}{3}, \frac{3}{9}$

$$1 \times 9 = 3 \times 3$$

$$9 = 9$$

*Cross  
products  
Multiply.*

The cross products are equal. So, the pair of ratios forms a proportion.

②  $\frac{10}{14}, \frac{6}{7}$

$$10 \times 7 = 14 \times 6$$

$$70 \neq 84$$

*Cross  
products  
Multiply.*

The cross products are not equal. So, the pair of ratios does not form a proportion.

Solve each proportion.

③  $\frac{7}{8} = \frac{n}{48}$

$$7 \times 48 = 8 \times n$$

$$336 = 8n$$

*Cross  
products  
Multiply.*

$$\frac{336}{8} = \frac{8n}{8}$$

*Divide.*

$$42 = n$$

The solution is 42.

④  $\frac{2.3}{d} = \frac{4.6}{5}$

$$2.3 \times 5 = d \times 4.6$$

$$11.5 = 4.6d$$

*Cross  
products  
Multiply.*

$$\frac{11.5}{4.6} = \frac{4.6d}{4.6}$$

*Divide.*

$$2.5 = d$$

The solution is 2.5.

A **percent** is a ratio that compares a number to 100.

TO EXPRESS A PERCENT AS A FRACTION:	TO EXPRESS A FRACTION AS A PERCENT:
<ul style="list-style-type: none"> <li>• Write the percent as a fraction with a denominator of 100.</li> <li>• Then simplify the fraction.</li> </ul>	<ul style="list-style-type: none"> <li>• Set up a proportion.</li> <li>• Then solve the proportion using cross products.</li> </ul>

### Examples

Express each percent as a fraction in simplest form.

① 65%

65% means “65 out of 100.”

$$65\% = \frac{65}{100}$$

$$= \frac{13}{20}$$

*Express as a fraction  
with a denominator  
of 100.  
Simplify.*

② 158%

158% means “158 out of 100.”

$$158\% = \frac{158}{100}$$

$$= 1\frac{58}{100} \text{ or } 1\frac{29}{50}$$

Express each fraction as a percent.

③  $\frac{8}{25}$

$$\frac{8}{25} = \frac{n}{100}$$

$$8 \times 100 = 25 \times n$$

$$800 = 25n$$

$$\frac{800}{25} = \frac{25n}{25}$$

$$32 = n$$

So,  $\frac{8}{25}$  is equivalent to 32%.

*Set up a  
proportion.  
Cross  
products  
Multiply.*

*Divide.*

④  $\frac{7}{5}$

$$\frac{7}{5} = \frac{c}{100}$$

$$7 \times 100 = 5 \times c$$

$$700 = 5c$$

$$\frac{700}{5} = \frac{5c}{5}$$

$$140 = c$$

So,  $\frac{7}{5}$  is equivalent to 140%.

Percent

85%

Fraction

 $\frac{85}{100}$ 

Decimal

0.85

TO EXPRESS A PERCENT  
AS A DECIMAL:

- Rewrite the percent as a fraction with a denominator of 100.
- Then express the fraction as a decimal.

TO EXPRESS A DECIMAL  
AS A PERCENT:

- Write the decimal as a fraction with a denominator of 100.
- Then express the fraction as a percent.

## Examples

Express each percent as a decimal.

① 72%

$72\% = \frac{72}{100}$

$= 0.72$

*Rewrite the percent as a fraction.**Express the fraction as a decimal.*

② 105%

$105\% = \frac{105}{100}$

$= 1.05$

*Rewrite the percent as a fraction.**Express the fraction as a decimal.*

Express each decimal as a percent.

③ 0.08

$0.08 = \frac{8}{100}$

$= 8\%$

*Express the decimal as a fraction.**Express the fraction as a percent.*

④ 0.625

$0.625 = \frac{62.5}{100}$

$= 62.5\%$

*Express the decimal as a fraction.**Express the fraction as a percent.*

**Example**

Starship Readers Enterprise reported that 80% of the books it sold last year were superhero comic books. If the company sold 5,000 books last year, how many were superhero comic books?

There are several ways to find the percent of a number.

**Method 1**

Change the percent to a fraction.

$$80\% = \frac{80}{100} \text{ or } \frac{4}{5}$$

$$\begin{aligned} \frac{4}{5} \text{ of } 5,000 &= \frac{4}{5} \times 5,000 \\ &= 4,000 \end{aligned}$$

**Method 2**

Change the percent to a decimal.

$$80\% = \frac{80}{100} \text{ or } 0.8$$

$$\begin{aligned} 0.8 \text{ of } 5,000 &= 0.8 \times 5,000 \\ &= 4,000 \end{aligned}$$

**Method 3 Use a model.**

Since  $80\% = \frac{4}{5}$ , separate a rectangle into fifths. Label the top and bottom in equal intervals as shown.

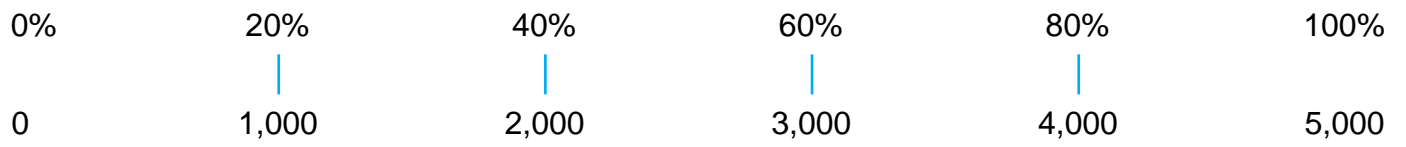


80% of 5,000 is 4,000.

**Method 4 Use a calculator.**

$$80 \text{ [2nd] [%] [×] 5,000 [=] 4000}$$

4,000 out of 5,000 books sold last year by Starship Readers Enterprise were superhero comic books.



10-7 overlay 1

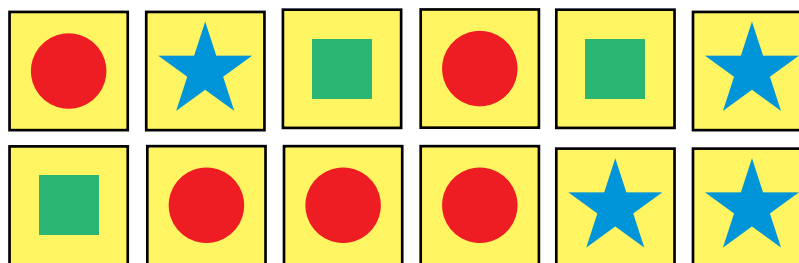
Theoretical  
Probability

**Words:** The theoretical probability of an event is the ratio of the number of ways the event can occur to the number of possible outcomes.

**Symbols:**  $P(\text{event}) = \frac{\text{number of ways the event can occur}}{\text{number of possible outcomes}}$

## Examples

Use the set of cards shown below to find the probability of each event.



- ① Find the probability of drawing a star card.

$$\frac{4}{12} \quad \leftarrow \text{number of ways to draw a star}$$

$$\quad \quad \leftarrow \text{number of possible outcomes}$$

$$\text{Therefore, } P(\text{star}) = \frac{4}{12} \text{ or } \frac{1}{3}.$$

- ② Find the probability of drawing a circle card.

$$\frac{5}{12} \quad \leftarrow \text{number of ways to draw a circle}$$

$$\quad \quad \leftarrow \text{number of possible outcomes}$$

$$\text{Therefore, } P(\text{circle}) = \frac{5}{12}.$$

- ③ Find the probability of drawing a card other than a circle.

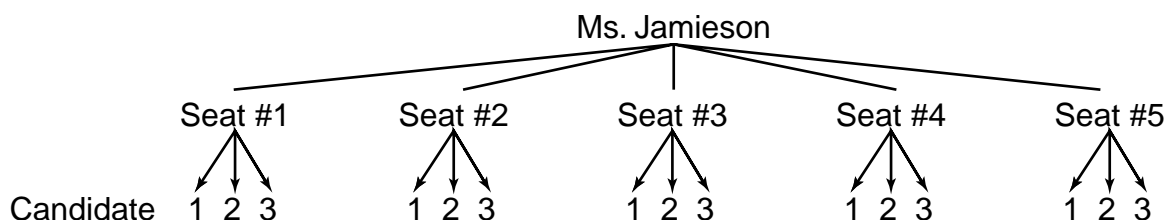
$$\frac{7}{12} \quad \leftarrow \text{number of ways to draw a card other than a circle}$$

$$\quad \quad \leftarrow \text{number of possible outcomes}$$

$$\text{Therefore, } P(\text{other than a circle}) = \frac{7}{12}.$$

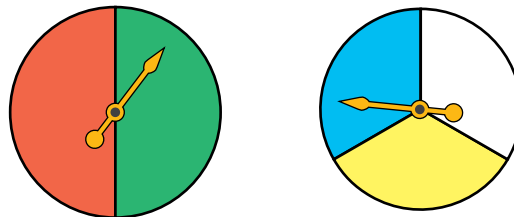
## Examples

- 1 In Pine Bluff, there are 5 council seats and 3 candidates campaigning for each seat. Ms. Jamieson wants to donate \$1,000 to each candidate for city council in Pine Bluff. How much money will she donate?



There are 15 candidates. She will donate \$15,000.

- 2 Each spinner is spun once. Find the probability of spinning red and blue.



Find all of the possible outcomes.

Spinner 1	Spinner 2	Outcome
red (R)	blue (B)	RB
	white (W)	RW
	yellow (Y)	RY
green (G)	blue (B)	GB
	white (W)	GW
	yellow (Y)	GY

One outcome has red and blue. There are six possible outcomes. Therefore,  $P(\text{RB}) = \frac{1}{6}$ .

**Examples**

- ① You want to determine whether today's students like studying mathematics more than students of ten years ago. You sample twenty students in the math club. Is this a random sample?

This is not a random sample because students in the math club are more likely to enjoy mathematics.

- ② Suppose the word *since* appears 8 times in a randomly selected portion of a book. If the portion of the book you are examining has 1,400 words, how many times do you think the word *since* appears in the entire 52,500-word book?

The word *since* appears 8 out of 1,400 words, or  $\frac{1}{175}$ . The probability that any word selected at random will be the word *since* is  $\frac{1}{175}$ . Let  $s$  represent the number of times the word *since* will appear.

$$\frac{1}{175} = \frac{s}{52,500}$$

*Use a proportion.*

$$1 \times 52,500 = 175 \times s$$

*Write the cross products.*

$$52,500 = 175s$$

*Multiply.*

$$\frac{52,500}{175} = \frac{175s}{175}$$

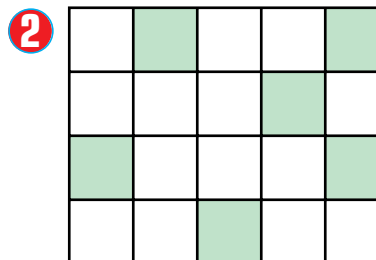
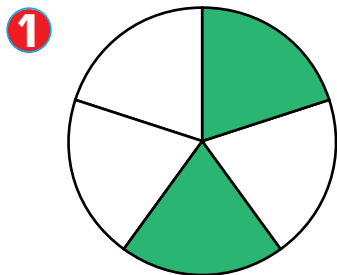
*Divide each side by 175.*

$$300 = s$$

Of the 52,500 words, the word *since* will appear about 300 times.

## Examples

Each figure represents a dartboard. Suppose you threw a dart randomly at the board and it hits the board. Find the probability of the dart landing in the shaded region.



- ③ The figure below represents a dartboard. Suppose you threw a dart at the board 60 times. How many times would you expect it to land in the shaded region?

$$P(\text{shaded region}) = \frac{\text{area of shaded region}}{\text{area of dartboard}}$$

$$= \frac{15}{25} \text{ or } \frac{3}{5}$$

Let  $n$  = times a dart lands in shaded region.

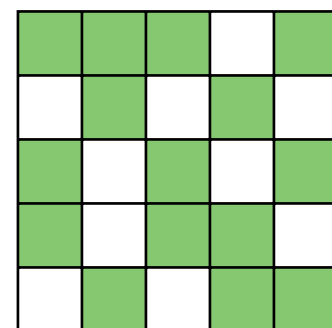
$$\frac{n}{60} = \frac{3}{5} \quad \leftarrow \text{area of shaded region}$$

$$\quad \quad \quad \leftarrow \text{area of dartboard}$$

$$n \times 5 = 60 \times 3 \quad \text{Find the cross products.}$$

$$5n = 180$$

$$n = 36 \quad \text{Divide each side by 5.}$$



So, out of 60 times, the dart should land in the shaded region about 36 times.

There are 5 equal sections.  
2 of the sections are shaded.  
So,  $P(\text{shaded}) = \frac{2}{5}$ .

There are 20 equal sections.  
6 of the sections are shaded.  
So,  $P(\text{shaded}) = \frac{6}{20}$  or  $\frac{3}{10}$ .

## Metric Units of Mass

kilogram (kg)

gram (g)

milligram (mg)

textbook: 1 kg  
bowling ball: 7 kgraisin: 1 g  
penny: 3 ggrain of sand: 1 mg  
postage stamp: 50 mg

$1 \text{ kg} = 1,000 \text{ g}$

$1 \text{ g} = 1,000 \text{ mg}$

$1 \text{ mg} = 0.001 \text{ g}$

$1 \text{ g} = 0.001 \text{ kg}$

## Metric Units of Capacity

liter (L)

milliliter (mL)

bottled water : 1 L  
bathtub: 225 Leyedropper: 1 mL  
teaspoon: 5 mL

$1 \text{ L} = 1,000 \text{ mL}$

$1 \text{ mL} = 0.001 \text{ L}$

## CHANGING METRIC UNITS

km	m	cm	mm
kg	g	cg	mg
kL	L	cL	mL

## Examples

Complete.

①  $0.3 \text{ cm} = \underline{\quad ? \quad} \text{ mm}$

*To change from centimeters to millimeters, multiply by 10 since  $1 \text{ cm} = 10 \text{ mm}$ .*

$$0.3 \times 10 = 3$$

$$0.3 \text{ cm} = 3 \text{ mm}$$

②  $212 \text{ mL} = \underline{\quad ? \quad} \text{ L}$

*To change from milliliters to liters, divide by 1,000 since  $1 \text{ L} = 1,000 \text{ mL}$ .*

$$212 \div 1,000 = 0.212$$

$$212 \text{ mL} = 0.212 \text{ L}$$

③  $200 \text{ mg} = \underline{\quad ? \quad} \text{ g}$

*To change from milligrams to grams, divide by 1,000 since  $1 \text{ g} = 1,000 \text{ mg}$ .*

$$200 \div 1,000 = 0.2$$

$$200 \text{ mg} = 0.2 \text{ g}$$

④  $4.01 \text{ kg} = \underline{\quad ? \quad} \text{ g}$

*To change from kilograms to grams, multiply by 1,000 since  $1 \text{ kg} = 1,000 \text{ g}$ .*

$$4.01 \times 1,000 = 4,010$$

$$4.01 \text{ kg} = 4,010 \text{ g}$$

To change from a larger unit  
to a smaller unit, multiply.

***MULTIPLY***

**$\times 1,000$**

**$\times 100$**

**$\times 10$**



To change from a smaller unit to a larger unit, divide.

  
 $\div 1,000$

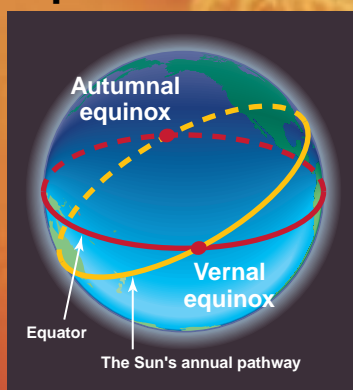
  
 $\div 100$

  
 $\div 10$

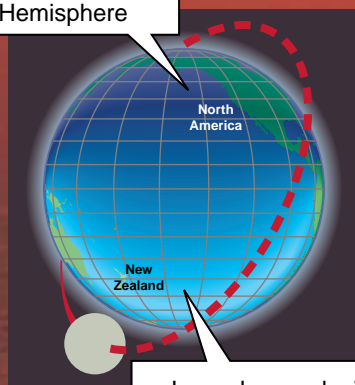
***DIVIDE***

**ADDING AND SUBTRACTING MEASURES OF TIME**

1. Add or subtract the seconds.
  2. Add or subtract the minutes.
  3. Add or subtract the hours.
- Rename if necessary in each step.

**Equinox**

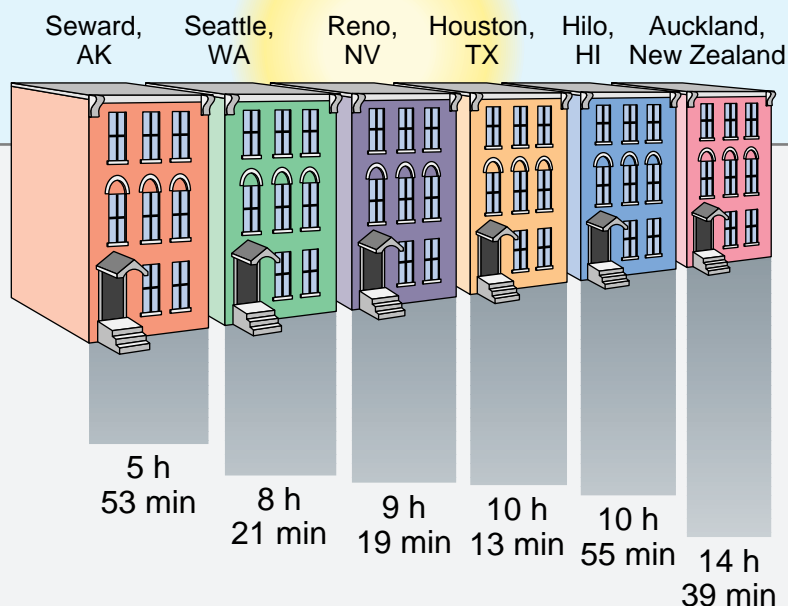
Short days make it  
winter in the  
Northern  
Hemisphere

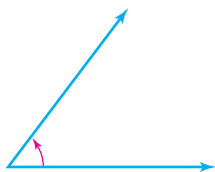
**Solstice**

Long days make it  
summer in the  
Southern Hemisphere

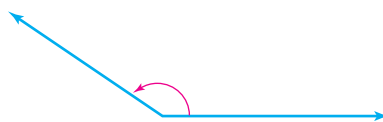
**Winter Solstice**

Time from sunrise to sunset today

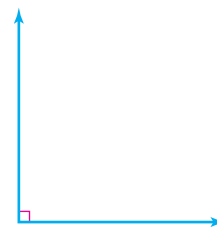




**Acute angles**  
measure between  
 $0^\circ$  and  $90^\circ$ .



**Obtuse angles**  
measure between  
 $90^\circ$  and  $180^\circ$ .

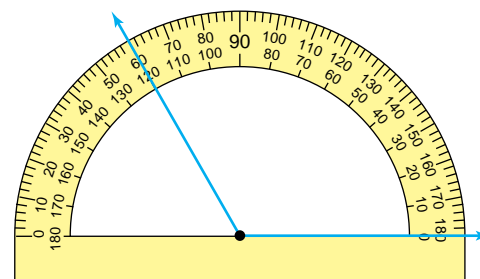


**Right angles**  
measure  $90^\circ$ . The  
symbol  $\square$  indicates  
a right angle.

You can use a protractor to measure angles.

**Step 1** Place the center of the protractor on the vertex of the angle with the straightedge along one side.

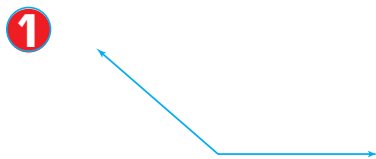
**Step 2** Use the scale that begins with  $0^\circ$  on the side of the angle. Read the angle measure where the other side crosses the same scale. Extend the sides if needed.



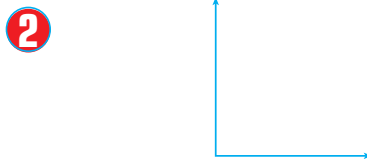
The angle measures  $120^\circ$ .

### Examples

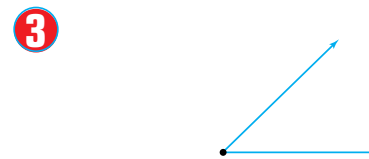
Use a protractor to find the measure of each angle. Classify each angle as *acute*, *right*, or *obtuse*.



The angle  
measures  $140^\circ$ . It  
is an obtuse angle.



The angle  
measures  $90^\circ$ . It  
is a right angle.



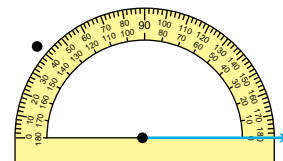
The angle  
measures  $45^\circ$ . It  
is an acute angle.

You can use a protractor and a straightedge to draw an angle.

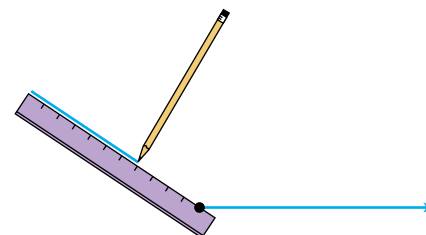
**Step 1** Draw one side of the angle. Then mark the vertex and draw an arrow.



**Step 2** Place the protractor along the side. Find the number of degrees needed for the angle you are drawing and make a pencil mark.



**Step 3** With a straightedge, draw the side that connects the vertex and the pencil mark. Draw an arrow on the end of the other side. The angle drawn is a  $140^\circ$  angle.



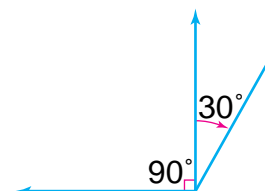
### Example

Estimate the measure of the angle.

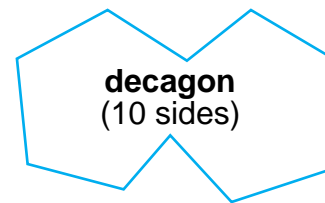
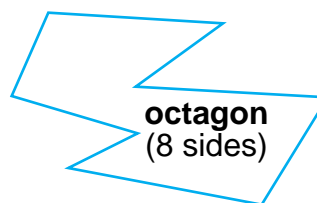
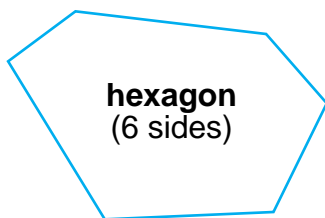
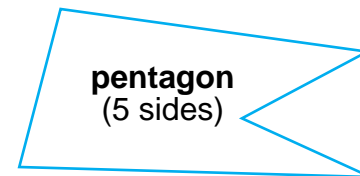
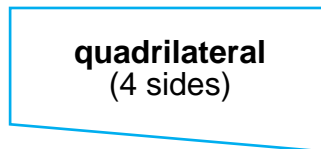
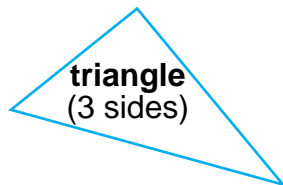
The angle shown is about the same as a  $90^\circ$  angle and a  $30^\circ$  angle.



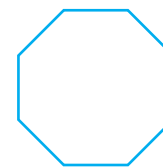
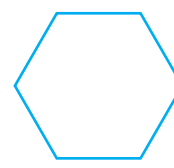
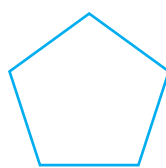
So, the measure of the angle is about  $90^\circ + 30^\circ$  or  $120^\circ$ .



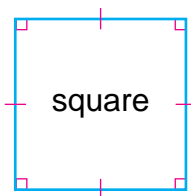
A **polygon** is a simple closed figure formed by three or more sides.



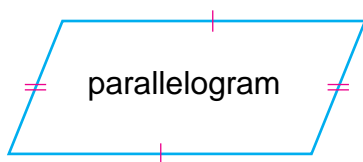
Any polygon with all sides congruent and all angles congruent is called a **regular polygon**.



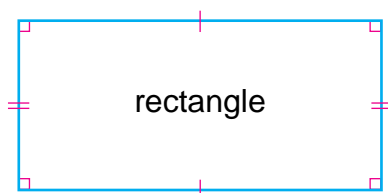
Certain types of quadrilaterals have special characteristics.



- All sides are congruent.
- All angles are right angles.
- Opposite sides are **parallel**. That is, if you extend the lengths of the sides, the opposite sides will never meet.

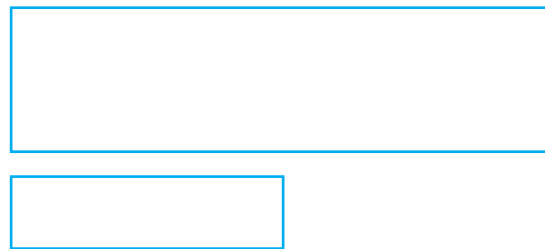


- Opposite sides are congruent.
- Opposite sides are parallel.

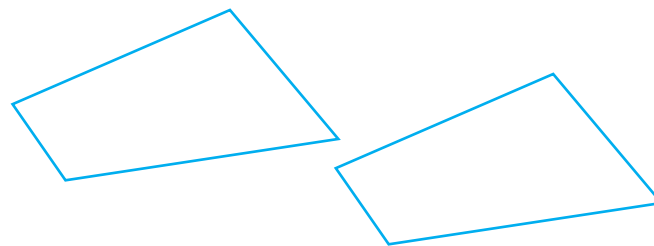


- Opposite sides are congruent.
- Opposite sides are parallel.
- All angles are right angles.

The figures at the right are **similar figures**. Similar figures have the same shape and angles, but different size. The symbol  $\sim$  means *is similar to*.

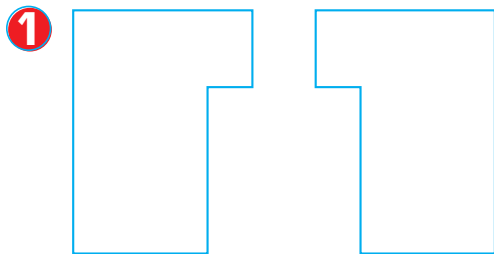


The figures at the right are **congruent figures**. Congruent figures are the same size and shape. The symbol  $\cong$  means *is congruent to*.

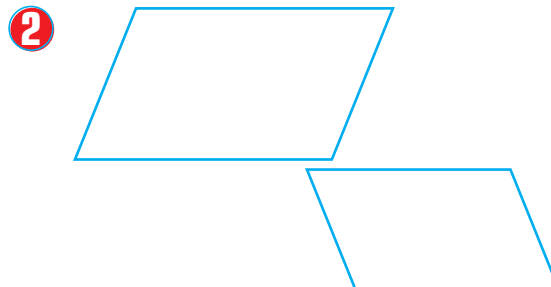


### Examples

Tell whether each pair of polygons is **congruent**, **similar**, or **neither**.

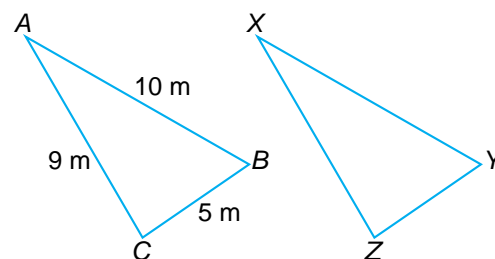


The polygons are the same size and shape. They are congruent.

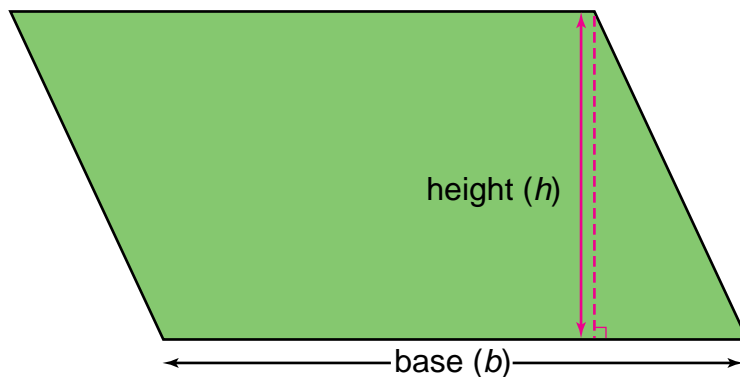


The polygons have the same shape, but are not the same size. They are similar.

- 3  $\triangle ABC$  is congruent to  $\triangle XYZ$ .
- What is the measure of side  $\overline{XZ}$ ?
  - Which side corresponds to  $\overline{XY}$ ?
  - Which side corresponds to  $\overline{YZ}$ ?
  - What is the perimeter of  $\triangle XYZ$ ?



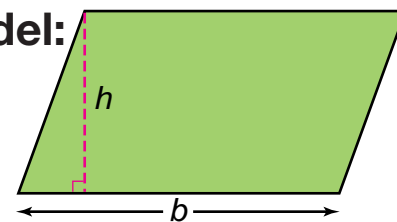
A **parallelogram** is a quadrilateral with two pairs of parallel sides.



### Area of a Parallelogram

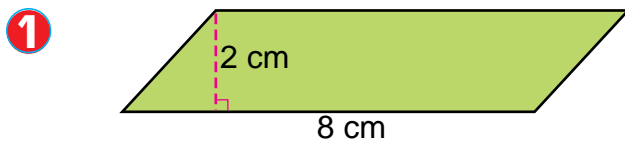
**Words:** The area ( $A$ ) of a parallelogram equals the product of its base ( $b$ ) and height ( $h$ ).

**Symbols:**  $A = bh$  **Model:**



### Examples

Find the area of each parallelogram.

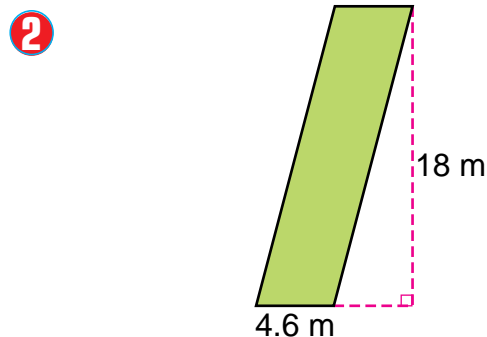


$$A = bh$$

$$A = 8 \cdot 2 \quad \text{Replace } b \text{ with } 8$$

$$A = 16 \quad \text{and } h \text{ with } 2.$$

The area is  
16 square centimeters.



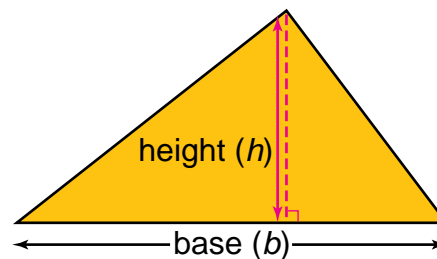
$$A = bh$$

$$A = 4.6 \cdot 18$$

$$A = 82.8$$

The area is  
82.8 square meters.

The base of a triangle is any one of its sides. The height of a triangle is the distance from a base to the opposite vertex.

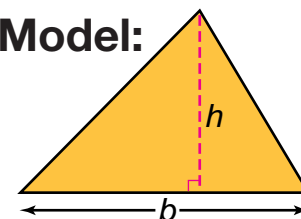


### Area of a Triangle

**Words:** The area ( $A$ ) of a triangle equals half of the product of the length of the base ( $b$ ) and the height ( $h$ ).

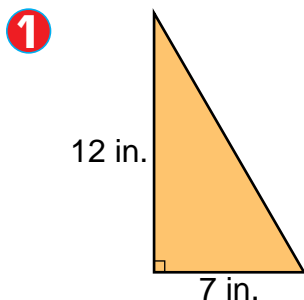
**Symbols:**  $A = \frac{1}{2}bh$

**Model:**



### Examples

Find the area of each triangle.



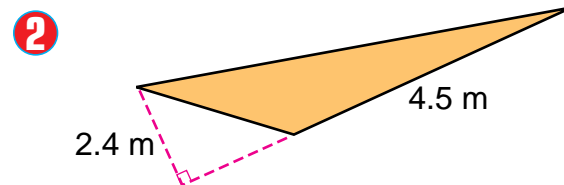
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 7 \cdot 12$$

*Replace  $b$  with 7 and  $h$  with 12.*

$$A = \frac{1}{2} \cdot 84 \text{ or } 42$$

The area is  
42 square inches.



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \cdot 4.5 \cdot 2.4$$

$$A = \frac{1}{2} \cdot 10.8 \text{ or } 5.4$$

The area is  
5.4 square meters.

A **three-dimensional figure** encloses a part of space.

2 rectangular bases  
4 rectangular faces  
12 edges  
8 vertices

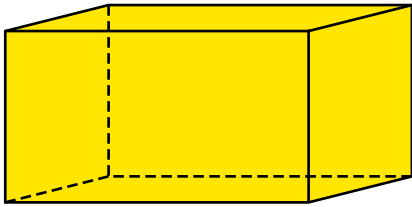
1 square base  
4 triangular faces  
8 edges  
5 vertices

2 circular bases  
curved surface, no faces  
no edges  
no vertices

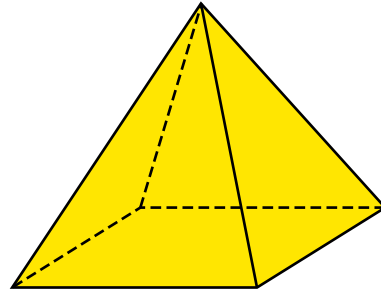
1 circular base  
curved surface, no faces  
no edges  
1 vertex

no bases  
no faces  
no edges  
no vertices

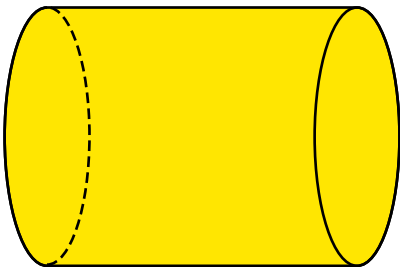
rectangular prism



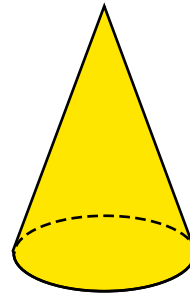
square pyramid



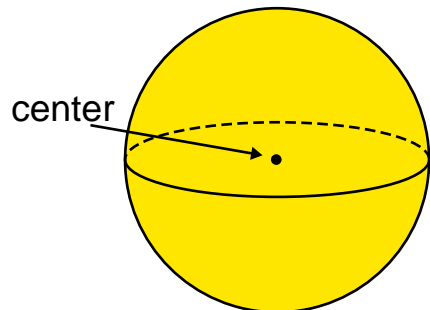
cylinder



cone



sphere



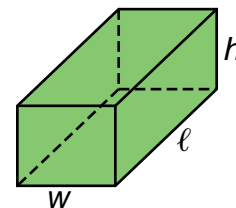
**Volume** is the amount of space that a three-dimensional figure contains. Volume is expressed in cubic units.

### Volume of a Rectangular Prism

**Words:** The volume ( $V$ ) of a rectangular prism equals the product of its length ( $\ell$ ), its width ( $w$ ), and its height ( $h$ ).

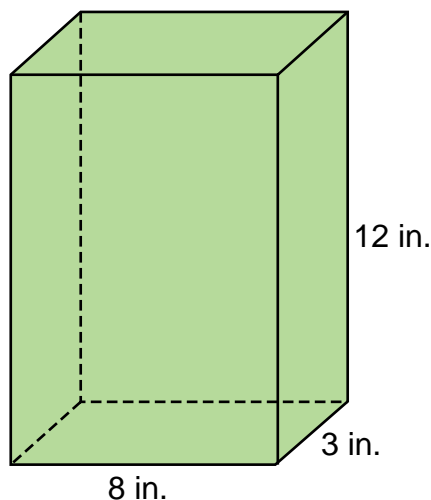
**Symbols:**  $V = \ell wh$

**Model:**



### Examples

- ① Find the volume of the rectangular prism.



$$\begin{aligned}\ell &= 3 \text{ in.} \\ w &= 8 \text{ in.} \\ h &= 12 \text{ in.}\end{aligned}$$

$$\begin{aligned}V &= \ell wh \\ V &= 3 \times 8 \times 12 \\ V &= 288\end{aligned}$$

The volume is  $288 \text{ in}^3$ .

- ② Find the volume of a rectangular prism that is 9 meters wide, 10 meters high, and 4 meters long.

$$\begin{aligned}\ell &= 4 \text{ in.} \\ w &= 9 \text{ in.} \\ h &= 10 \text{ in.}\end{aligned}$$

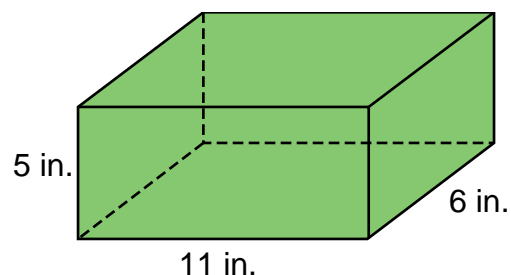
$$\begin{aligned}V &= \ell wh \\ V &= 4 \times 9 \times 10 \\ V &= 360\end{aligned}$$

The volume is  $360 \text{ m}^3$ .

The **surface area** of a three-dimensional object is the total area of its faces and curved surfaces. The surface area of a rectangular prism is the sum of the areas of its faces.

### Examples

- ① Find the surface area of the rectangular prism.



In a rectangular prism, opposite sides have the same dimensions.

*top and bottom*

$$11 \times 6 = 66 \text{ in}^2$$

*front and back*

$$11 \times 5 = 55 \text{ in}^2$$

*right and left sides*

$$5 \times 6 = 30 \text{ in}^2$$

Add the areas.  $2(66) + 2(55) + 2(30) = 302$

The surface area is  $302 \text{ in}^2$ .

- ② Find the surface area of a rectangular prism with a length of 6 centimeters, a width of 8 centimeters, and a height of 7 centimeters.

*top and bottom*

$$6 \times 8 = 48 \text{ cm}^2$$

*front and back*

$$8 \times 7 = 56 \text{ cm}^2$$

*right and left sides*

$$6 \times 7 = 42 \text{ cm}^2$$

Add the areas.  $2(48) + 2(56) + 2(42) = 292$

The surface area is  $292 \text{ cm}^2$ .