

## FOUR-STEP PLAN FOR PROBLEM SOLVING

- 1. Explore** Read the problem carefully.  
Ask yourself questions like, “What facts do I know?” and “What do I need to find out?”
- 2. Plan** See how the facts relate to each other.  
Make a plan for solving the problem.  
Estimate the answer.
- 3. Solve** Use your plan to solve the problem.  
If your plan does not work, revise it or make a new plan.
- 4. Examine** Reread the problem.  
Ask, “Is my answer close to my estimate?”  
Ask, “Does my answer make sense for the problem?” If not, solve another way.

**Example**

How many buses are needed to transport all 238 seventh-grade students at Oak Heights Middle School to the state museum? Each bus seats 56 students.

- Explore** You know how many seventh-graders there are. You also know how many students each bus will hold.
- Plan** To find the number of buses needed, divide 238 by 56.
- Solve**  $238 \div 56 = 4.25$   
The answer is a little more than 4. Since you can't have *part* of a bus, 5 buses are needed.
- Examine** With 4 buses, there would be room for  $4 \times 56$  or only 224 students. With 5 buses, there would be room for  $5 \times 56$  or 280 students. This is enough room.

**AN ALGEBRAIC EXPRESSION INCLUDES AT LEAST ONE OF EACH OF THE FOLLOWING:**

- numbers
- variables (such as  $a$ ,  $n$ , or  $x$ )
- operations ( $+$ ,  $-$ ,  $\times$ , or  $\div$ )

To **evaluate** an expression, replace all of the variables with numbers and find the value.

### Examples

① Evaluate  $7 - n$  if  $n = 4$ .

$$\begin{aligned} 7 - n &= 7 - 4 \\ &= 3 \end{aligned}$$

*Replace  $n$  with 4.  
Subtract.*

② Evaluate  $7x - 2y$  if  $x = 3$  and  $y = 5$ .

$$\begin{aligned} 7x - 2y &= 7(3) - 2(5) \\ &= 21 - 10 \\ &= 11 \end{aligned}$$

*Replace  $x$  with 3 and  $y$  with 5.  
Multiply.  
Subtract.*

③ Evaluate  $\frac{12}{cd}$  if  $c = 2$  and  $d = 3$ .

$$\begin{aligned} \frac{12}{cd} &= \frac{12}{2 \times 3} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

*$cd$  means  $c \times d$ . Replace  $c$  with 2 and  $d$  with 3.  
Multiply.  
 $\frac{12}{6}$  means  $12 \div 6$ . Divide.*

The statements at the right are **equations**.  
An equation is a mathematical statement that contains an equals sign.

$$165 - 10 = 155$$

$$9y = 297$$

$$\frac{54}{b} = 27$$

You **solve** an equation when you replace the variable with a number that makes the equation true. That number is called a **solution**.

### Examples

① Which of the numbers 2, 3, or 4 is a solution of  $14x = 42$ ?

Replace  $x$  with 2.

$$14x = 42$$

$$14(2) \stackrel{?}{=} 42$$

$$28 = 42 \text{ false}$$

Replace  $x$  with 3.

$$14x = 42$$

$$14(3) \stackrel{?}{=} 42$$

$$42 = 42 \text{ ✓ true}$$

Replace  $x$  with 4.

$$14x = 42$$

$$14(4) \stackrel{?}{=} 42$$

$$56 = 42 \text{ false}$$

The solution is 3.

② Solve  $45 = a + 30$  mentally.

$$45 = a + 30$$

$$45 = 15 + 30 \quad \text{You know that } 15 + 30 = 45.$$

$$45 = 45 \text{ ✓}$$

The solution is 15.

## CHANGING METRIC UNITS

km	m	cm	mm
kg	g	cg	mg
kL	L	cL	mL

## Examples

Complete.

①  $0.6 \text{ cm} = \underline{\quad ? \quad} \text{ mm}$

*To change from centimeters to millimeters, multiply by 10 since  $1 \text{ cm} = 10 \text{ mm}$ .*

$$0.6 \times 10 = 6$$

$$0.6 \text{ cm} = 6 \text{ mm}$$

②  $714 \text{ mL} = \underline{\quad ? \quad} \text{ L}$

*To change from milliliters to liters, divide by 1,000 since  $1 \text{ L} = 1,000 \text{ mL}$ .*

$$714 \div 1,000 = 0.714$$

$$714 \text{ mL} = 0.714 \text{ L}$$

③  $159 \text{ mg} = \underline{\quad ? \quad} \text{ g}$

*To change from milligrams to grams, divide by 1,000 since  $1 \text{ g} = 1,000 \text{ mg}$ .*

$$159 \div 1,000 = 0.159$$

$$159 \text{ mg} = 0.159 \text{ g}$$

④  $3.12 \text{ kg} = \underline{\quad ? \quad} \text{ g}$

*To change from kilograms to grams, multiply by 1,000 since  $1 \text{ kg} = 1,000 \text{ g}$ .*

$$3.12 \times 1,000 = 3,120$$

$$3.12 \text{ kg} = 3,120 \text{ g}$$

To change from a larger unit  
to a smaller unit, multiply.

**MULTIPLY**

$\times 1,000$

$\times 100$


$\times 10$



To change from a smaller unit to a larger unit, divide.

  
 $\div 1,000$

  
 $\div 100$

  
 $\div 10$

***DIVIDE***

## MAKING A FREQUENCY TABLE

1. Draw a table with three columns.
2. In the first column, list the items or intervals in the set of data.
3. In the second column, mark the tallies.
4. In the third column, write the frequency or number of tallies.

## Example

Find the range for the set of data below. Choose an appropriate scale and interval. Then make a frequency table.

120.2, 121.7, 120.8, 120.9, 121, 122.3, 120.1, 121.5

The range is the difference between the greatest number and the least number in the set of data.

$$\begin{array}{r}
 122.3 \leftarrow \text{greatest number} \\
 - 120.1 \leftarrow \text{least number} \\
 \hline
 2.2 \leftarrow \text{range}
 \end{array}$$

Since the data set includes numbers from 120.1 to 122.3, you might use a scale of 120.1 to 123 with an interval of 1. Therefore, the categories in the first column are 120.1–121, 121.1–122, and 122.1–123. In the second column, tally the data. In the third column, write the number of tallies.

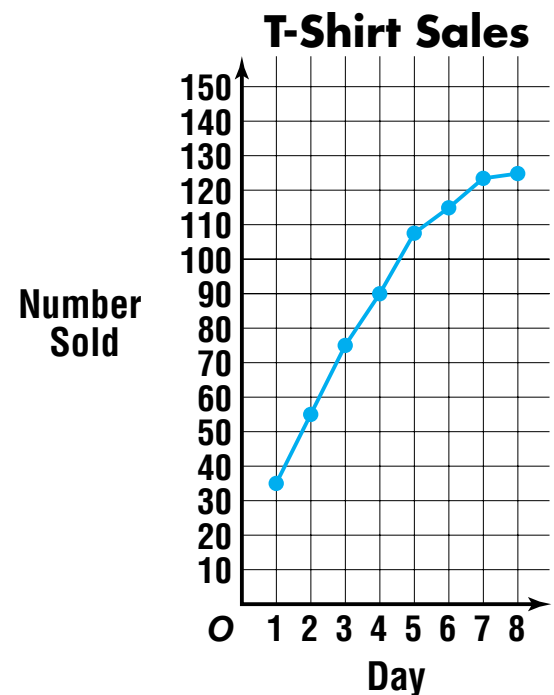
INTERVALS	TALLY	FREQUENCY
120.1–121		5
121.1–122		2
122.1–123		1

You can use line graphs, bar graphs, and scatter plots to predict future events.

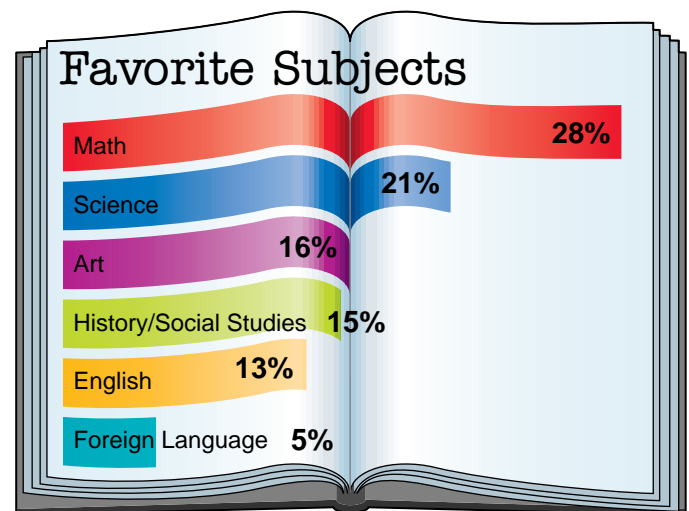
### Examples

- 1 The graph shows the sales of T-shirts designed by the East Middle School Math Club. Use the graph to predict whether the club will reach its goal of selling 200 T-shirts in 10 days.

The line graph flattens out around 120 T-shirts sold. Therefore, it does not seem likely that the club will reach its goal.



- 2 The graph shows the favorite subjects of students ages 10–17. Use the graph to predict the top three subjects if another survey is taken of students in the same age range.



Source: National Science Foundation

The graph shows that math, science, and art, respectively, were the students' favorite subjects. You could predict that those three subjects would be favorites in a larger survey as well.

A **line plot** is a picture of information on a number line.

### MAKING A LINE PLOT

1. Draw a number line.
2. Choose a scale and interval. Remember that the line plot does not need to start at 0.
3. Place an “×” above or close to the number that represents each item of data.

### Example

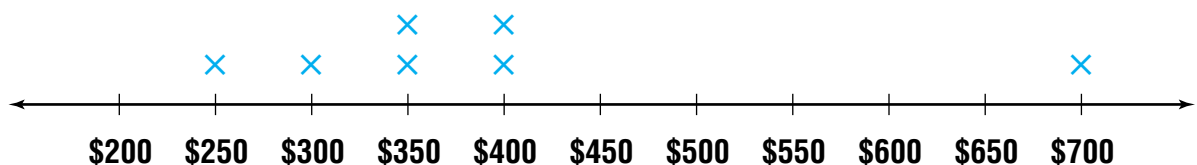
The table shows the average price for several types of exercise equipment. Make a line plot of these data. Round all prices to the nearest fifty dollars.

TYPE	PRICE
treadmill	\$699
home gym	402
ski machine	394
free weights	350
stair climber	349
aerobic rider	282
stationary bike	229

**Step 1** Draw a number line. Since the lowest price is \$229 and the highest price is \$699, you can use a scale of \$200 to \$700 and an interval of \$50.



**Step 2** Put an “×” above the number that represents each price.



## MAKING A STEM-AND-LEAF PLOT

1. Find the least and greatest data values.
2. Identify the stems and list them on the left side of the plot.
3. Identify the leaves and write them in order on the right side of the plot across from the corresponding stems.
4. Include a key to the data.

## Example

Make a back-to-back stem-and-leaf plot for the data.

Temperatures ( $^{\circ}\text{F}$ ) in Miami and Los Angeles, August 1–10

DATE	MIAMI	L.A.	DATE	MIAMI	L.A.
8/1	83 $^{\circ}$	93 $^{\circ}$	8/6	88 $^{\circ}$	78 $^{\circ}$
8/2	79 $^{\circ}$	90 $^{\circ}$	8/7	88 $^{\circ}$	79 $^{\circ}$
8/3	85 $^{\circ}$	91 $^{\circ}$	8/8	77 $^{\circ}$	87 $^{\circ}$
8/4	90 $^{\circ}$	90 $^{\circ}$	8/9	85 $^{\circ}$	91 $^{\circ}$
8/5	91 $^{\circ}$	81 $^{\circ}$	8/10	93 $^{\circ}$	92 $^{\circ}$

Since the tens place is the greatest place value of the data, write the digits in the tens places as the **stems**. List the stems in increasing order.

The numbers on either side of the stems are the **leaves**. Use the digits in the ones place for the leaves. List the leaves in increasing order.

Stem
7
8
9

$$7|9 = 79^{\circ}\text{F}$$

Always include a key to the data.

**Miami**

**9 7**

**8 8 5 5 3**

**3 1 0**

2-5 overlay 1

**Los Angeles**

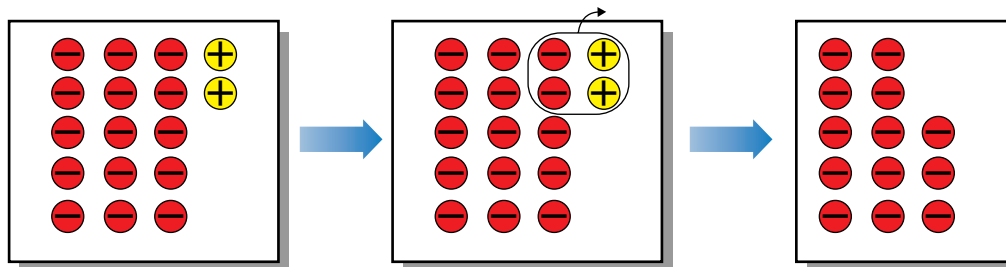
**8 9**

**1 7 8**

**0 0 1 1 2 3**

## Examples

- 1 Use counters to solve  $b = -15 + 2$ .



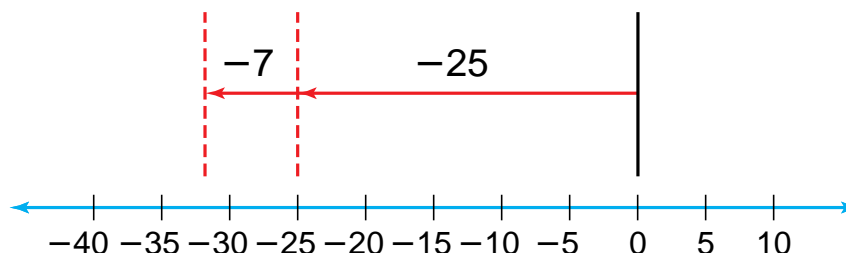
Place 15 negative counters on the mat. Then place 2 positive counters on the mat.

Remove all of the zero pairs.

$$b = -15 + 2$$

$$b = -13$$

- 2 Use a number line to solve  $x = -25 + (-7)$ .



Both addends are negative, so the sum is negative. Start at 0 and move 25 to the left. Then move 7 more units to the left.

$$x = -25 + (-7)$$

$$x = -32$$

- 3 Evaluate  $h + (-9)$  if  $h = 7$ .

$$h + (-9) = 7 + (-9) \quad \text{Replace } h \text{ with } 7.$$

$|7| < |-9|$ , so the sum is negative.

The difference of 7 and 9 is 2. So,  $h + (-9) = -2$ .

Subtracting  
Integers

**Words:** To subtract an integer, add its additive inverse.

**Symbols:**      **Arithmetic**                      **Algebra**  
 $9 - 5 = 9 + (-5)$        $a - b = a + (-b)$

## Examples

① Solve  $y = -4 - 8$ .

$$y = -4 - 8$$

$$y = -4 + (-8)$$

$$y = -12$$

*To subtract 8, add  $-8$ .*

② Solve  $t = 3 - (-2)$ .

$$t = 3 - (-2)$$

$$t = 3 + 2$$

$$t = 5$$

*To subtract  $-2$ , add 2.*

③ Evaluate  $b - a$  if  $a = -12$  and  $b = -2$ .

$$b - a = -2 - (-12)$$

$$= -2 + 12$$

$$= 10$$

*Replace  $b$  with  $-2$  and  $a$  with  $-12$ .*

*To subtract  $-12$ , add 12.*

**Multiplying  
Integers with  
Different Signs**

The product of two integers with different signs is negative.

**Multiplying  
Integers with the  
Same Sign**

The product of two integers with the same sign is positive.

**Examples**

Solve each equation.

$$\textcircled{1} \quad -3(-4) = x$$

The two integers have the same signs. The product will be positive.

$$\begin{aligned} -3(-4) &= x \\ 12 &= x \end{aligned}$$

$$\textcircled{2} \quad -8(6) = t$$

The two integers have different signs. The product will be negative.

$$\begin{aligned} -8(6) &= t \\ -48 &= t \end{aligned}$$

$$\textcircled{3} \quad y = 12(-5)$$

The two integers have different signs. The product will be negative.

$$\begin{aligned} y &= 12(-5) \\ y &= -60 \end{aligned}$$

$$\textcircled{4} \quad g = -3(-7)$$

The two integers have the same signs. The product will be positive.

$$\begin{aligned} g &= -3(-7) \\ g &= 21 \end{aligned}$$

$$\textcircled{5} \quad \text{Evaluate the expression } -3a^2 \text{ if } a = 5.$$

$$\begin{aligned} -3a^2 &= -3(5)^2 && \text{Replace } a \text{ with } 5. \\ &= -3(5 \cdot 5) \\ &= -3(25) \\ &= -75 \end{aligned}$$

## DIVIDING INTEGERS

The quotient of two integers with the *same sign* is *positive*.  
The quotient of two integers with *different signs* is *negative*.

## Examples

① Solve  $k = -36 \div (-9)$ .

$$k = -36 \div (-9) \quad \text{The signs are the same.}$$

$$k = 4 \quad \text{The quotient is positive.}$$

② Solve  $-65 \div 5 = m$ .

$$-65 \div 5 = m \quad \text{The signs are different.}$$

$$-13 = m \quad \text{The quotient is negative.}$$

③ Solve  $d = 81 \div (-9)$ .

$$d = 81 \div (-9) \quad \text{The signs are different.}$$

$$d = -9 \quad \text{The quotient is negative.}$$

④ Evaluate  $\frac{a}{b}$  if  $a = -51$  and  $b = 3$ .

$$\frac{a}{b} = \frac{-51}{3} \quad \text{Replace } a \text{ with } -51 \text{ and } b \text{ with } 3.$$

$$= -17 \quad \text{Since the signs are different, the quotient is negative.}$$

Subtraction  
Property of  
Equality

**Words:** If you subtract the same number from each side of an equation, then the two sides remain equal.

**Symbols:**

Arithmetic	Algebra
$5 = 5$	$a = b$
$5 - 2 = 5 - 2$	$a - c = b - c$
$3 = 3$	

## Examples

① Solve  $x + 60 = 395$ . Check your solution.

$$\begin{aligned} x + 60 &= 395 \\ x + 60 - 60 &= 395 - 60 \\ x &= 335 \end{aligned}$$

**Check:**

$$\begin{aligned} x + 60 &= 395 \\ 335 + 60 &\stackrel{?}{=} 395 \\ 395 &= 395 \quad \checkmark \end{aligned}$$

The solution is 335.

Addition  
Property of  
Equality

**Words:** If you add the same number to each side of an equation, then the two sides remain equal.

**Symbols:**

Arithmetic	Algebra
$2 = 2$	$a = b$
$2 + 6 = 2 + 6$	$a + c = b + c$
$8 = 8$	

② Solve  $355 = y - 140$ . Check your solution.

$$\begin{aligned} 355 &= y - 140 \\ 355 + 140 &= y - 140 + 140 \\ 495 &= y \end{aligned}$$

**Check:**

$$\begin{aligned} 355 &= y - 140 \\ 355 &\stackrel{?}{=} 495 - 140 \\ 355 &= 355 \quad \checkmark \end{aligned}$$

The solution is 495.

**Division  
Property of  
Equality**

**Words:** If you divide each side of an equation by the same nonzero number, then the two sides remain equal.

**Symbols: Arithmetic**

$$15 = 15$$

$$\frac{15}{3} = \frac{15}{3}$$

$$5 = 5$$

**Algebra**

$$a = b$$

$$\frac{a}{c} = \frac{b}{c}, c \neq 0$$

**Examples**

Solve each equation. Check your solution.

①  $17c = 272$

$$17c = 272$$

$$\frac{17c}{17} = \frac{272}{17}$$

$$c = 16$$

*Divide each side of the equation by 17.*

**Check:**  $17c = 272$

$$17 \cdot 16 \stackrel{?}{=} 272 \quad \text{Replace } c \text{ with } 16.$$

$$272 = 272 \quad \checkmark$$

②  $88 = 5.5a$

$$88 = 5.5a$$

$$\frac{88}{5.5} = \frac{5.5a}{5.5}$$

$$16 = a$$

*Divide each side of the equation by 5.5.*

**Check:**  $88 = 5.5a$

$$88 \stackrel{?}{=} 5.5 \cdot 16 \quad \text{Replace } a \text{ with } 16.$$

$$88 = 88 \quad \checkmark$$

**Examples**

Solve each equation. Check your solution.

①  $2y + 2 = 12$

$$2y + 2 = 12$$

$$2y + 2 - 2 = 12 - 2$$

*Subtract 2 from each side.*

$$2y = 10$$

$$\frac{2y}{2} = \frac{10}{2}$$

*Divide each side by 2.*

$$y = 5$$

Check:  $2y + 2 = 12$

$$2 \cdot 5 + 2 \stackrel{?}{=} 12$$

*Replace y with 5.*

$$10 + 2 \stackrel{?}{=} 12$$

$$12 = 12$$

✓

②  $-10 = -4r - 2$

$$-10 = -4r - 2$$

$$-10 + 2 = -4r - 2 + 2$$

*Add 2 to each side.*

$$-8 = -4r$$

$$\frac{-8}{-4} = \frac{-4r}{-4}$$

*Divide each side by -4.*

$$2 = r$$

Check:  $-10 = -4r - 2$

$$-10 \stackrel{?}{=} -4 \cdot 2 - 2$$

*Replace r with 2.*

$$-10 \stackrel{?}{=} -8 - 2$$

$$-10 = -10$$

✓

An **inequality** is a mathematical sentence that contains one of the symbols at the right.

$>$  greater than  
 $<$  less than  
 $\geq$  greater than or equal to  
 $\leq$  less than or equal to

### GRAPHING A SOLUTION ON A NUMBER LINE

1. Draw a circle at the number that all other numbers in the solution are compared to.
2. Fill in the circle if the number is included in the solution set.
3. Draw a thick arrow to the right or the left to show the numbers that are solutions.

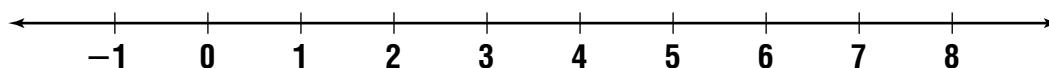
#### Example

Solve  $n - 2 < 5$ . Check your solution. Then graph the solution.

$$\begin{aligned} n - 2 &< 5 \\ n - 2 + 2 &< 5 + 2 && \text{Add 2 to each side.} \\ n &< 7 \end{aligned}$$

Check: Try 6, a number less than 7.

$$\begin{aligned} n - 2 &< 5 \\ 6 - 2 &\stackrel{?}{<} 5 && \text{Replace } n \text{ with } 6. \\ 4 &< 5 && \checkmark \end{aligned}$$





4-5 overlay 1

## FINDING THE GCF

Method 1 Make a list.	Method 2 Use prime factorization.
<ul style="list-style-type: none"> <li>List the factors of each number.</li> <li>Identify the common factors.</li> <li>The greatest of the common factors is the GCF.</li> </ul>	<ul style="list-style-type: none"> <li>Write the prime factorization of each number.</li> <li>Identify all common prime factors.</li> <li>The product of the common prime factors is the GCF.</li> </ul>

## Example

Find the GCF of 44 and 120.

**Method 1** List the factors.

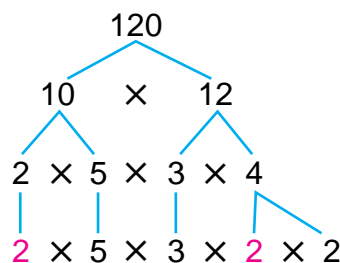
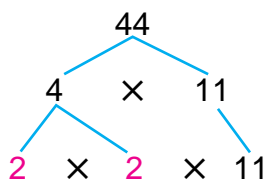
factors of 44: 1, 2, 4, 11, 22, 44

factors of 120: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

common factors: 1, 2, 4

Thus, the GCF of 44 and 120 is 4.

**Method 2** Write the prime factorization.



The common prime factors are 2 and 2. The GCF is  $2 \times 2$  or 4.

In a **terminating decimal**, the division ends or terminates when the remainder is zero.

0.895  
1.62

In a **repeating decimal**, there is a pattern in the digits that repeats forever.

0.2828...  
7.933...

### WRITING DECIMALS AS FRACTIONS

#### Method 1

Use paper and pencil to divide the numerator by the denominator.

#### Method 2

Use a calculator to divide the numerator by the denominator.

### Examples

- ① Express  $\frac{1}{4}$  as a decimal.

#### Method 1

Use paper and pencil.

$$\begin{array}{r} 0.25 \\ 4 \overline{)1.00} \\ \underline{-8} \phantom{0} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

#### Method 2

Use a calculator.

$$1 \div 4 = 0.25$$

$$\text{So, } \frac{1}{4} = 0.25.$$

- ② Express  $3\frac{2}{9}$  as a decimal.

#### Method 1

Use paper and pencil.

$$\begin{array}{r} 0.22 \\ 9 \overline{)2.00} \\ \underline{-18} \phantom{0} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

*The digit 2 will repeat since 2 will continue to be the remainder.*

*So,  $3\frac{2}{9} = 3 + 0.\overline{2}$  or  $3.\overline{2}$*

#### Method 2

Use a calculator.

$$2 \div 9 + 3 = 3.22222222$$

Use bar notation to indicate that the digit 2 repeats.

Percent

37%

Fraction

 $\frac{37}{100}$ 

Decimal

0.37

**TO WRITE A DECIMAL  
AS A PERCENT:**Multiply the decimal  
by 100 and add the  
percent symbol.**TO WRITE A PERCENT  
AS A DECIMAL:**Divide the percent by  
100 and remove the  
percent symbol.**Examples**

Express each decimal as a percent.

**1** 0.3

$$0.3 = \frac{30}{100} \text{ Write as a fraction.}$$

$$= 30\%$$

**2** 0.875

$$0.875 = \frac{875}{1,000} \text{ Write as a fraction.}$$

$$= \frac{875 \div 10}{1,000 \div 10} \text{ Divide to make the denominator 100.}$$

$$= \frac{87.5}{100} \text{ or } 87.5\%$$

Express each percent as a decimal.

**3** 67%

$$67\% = \frac{67}{100} = 0.67$$

$$\text{So, } 67\% = 0.67.$$

**4** 12.5%

$$12.5\% = \frac{12.5}{100} = 0.125$$

$$\text{So, } 12.5\% = 0.125.$$

## FINDING THE LCM

Method 1 Make a list.	Method 2 Use prime factorization.
<ul style="list-style-type: none"> <li>List several multiples of each number.</li> <li>Identify the common multiples.</li> <li>The least of the common multiples is the LCM.</li> </ul>	<ul style="list-style-type: none"> <li>Write the prime factorization of each number.</li> <li>Identify all common prime factors.</li> <li>Then find the product of the prime factors using each common prime factor only once and any remaining factors. This product is the LCM.</li> </ul>

**Example**

Find the LCM of 6, 9, and 12.

**Method 1** Make a list.

multiples of 6: 6, 12, 18, 24, 30, **36**, . . .

multiples of 9: 9, 18, 27, **36**, 45, 54, . . .

multiples of 12: 12, 24, **36**, 48, 60, 72, . . .

The LCM of 6, 9, and 12 is 36.

**Method 2** Use prime factorization.

$$6 = 2 \times 3$$

$$9 = 3 \times 3 \text{ or } 3^2$$

$$12 = 2 \times 2 \times 3 \text{ or } 2^2 \times 3$$

The LCM of 6, 9, and 12 is  $2 \times 2 \times 3 \times 3$  or 36.

### ADDING AND SUBTRACTING FRACTIONS

With Like Denominators	With Unlike Denominators
<ol style="list-style-type: none"> <li>1. Add or subtract the numerators.</li> <li>2. Use the same denominator in the sum or difference.</li> </ol>	<ol style="list-style-type: none"> <li>1. Rename the fractions with a common denominator.</li> <li>2. Add or subtract the numerators.</li> <li>3. Simplify.</li> </ol>

#### Examples

Add or subtract. Write each sum or difference in simplest form.

①  $\frac{3}{5} + \frac{2}{3}$      *Estimate:*  $\frac{1}{2} + \frac{1}{2} = 1$

$$\begin{array}{r}
 \frac{3}{5} \\
 + \frac{2}{3} \\
 \hline
 \end{array}
 \quad \xrightarrow{\text{LCD: } 15} \quad
 \begin{array}{r}
 \frac{3 \times 3}{5 \times 3} \\
 + \frac{2 \times 5}{3 \times 5} \\
 \hline
 \end{array}
 \quad \rightarrow \quad
 \begin{array}{r}
 \frac{9}{15} \\
 + \frac{10}{15} \\
 \hline
 \frac{19}{15} \text{ or } 1\frac{4}{15}
 \end{array}$$

So,  $\frac{3}{5} + \frac{2}{3} = 1\frac{4}{15}$ .      $1\frac{4}{15}$  is close to the estimate, 1.

②  $\frac{7}{8} - \frac{17}{24}$      *Estimate:*  $1 - \frac{1}{2} = \frac{1}{2}$

$$\begin{array}{r}
 \frac{7}{8} \\
 - \frac{17}{24} \\
 \hline
 \end{array}
 \quad \xrightarrow{\text{LCD: } 24} \quad
 \begin{array}{r}
 \frac{7 \times 3}{8 \times 3} \\
 - \frac{17}{24} \\
 \hline
 \end{array}
 \quad \rightarrow \quad
 \begin{array}{r}
 \frac{21}{24} \\
 - \frac{17}{24} \\
 \hline
 \frac{4}{24} \text{ or } \frac{1}{6}
 \end{array}$$

So,  $\frac{7}{8} - \frac{17}{24} = \frac{1}{6}$ .      $\frac{1}{6}$  is close to the estimate,  $\frac{1}{2}$ .

**Multiplying  
Fractions**

**Words:** To multiply fractions, multiply the numerators and then multiply the denominators.

**Symbols:** **Arithmetic**  $\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$       **Algebra**  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, b, d \neq 0$

**Examples**

**Multiply.** Write each product in simplest form.

①  $\frac{1}{4} \times \frac{3}{5}$       *Estimate:*  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$       *The product is close to the estimate,  $\frac{1}{4}$ .*

②  $1\frac{1}{8} \times 3\frac{3}{5}$       *Estimate:*  $1 \times 3 = 3$

$1\frac{1}{8} \times 3\frac{3}{5} = \frac{9}{8} \times \frac{18}{5}$   
 $= \frac{81}{20}$  or  $4\frac{1}{20}$       *The product is close to the estimate, 3.*

③  $1\frac{17}{18} \times 36$       *Estimate:*  $2 \times 36 = 72$

$1\frac{17}{18} \times 36 = \frac{35}{18} \times \frac{36}{1}$   
 $= 70$       *The product is close to the estimate, 72.*

**CUSTOMARY UNITS OF CAPACITY**

1 cup (c) = 8 fluid ounces (fl oz)  
 1 pint (pt) = 2 cups  
 1 quart (qt) = 2 pints  
 1 gallon (gal) = 4 quarts

**CUSTOMARY UNITS OF WEIGHT**

1 pound (lb) = 16 ounces (oz)  
 1 ton (T) = 2,000 pounds

**Examples**

Complete.

①  $64 \text{ qt} = \underline{\quad ? \quad} \text{ gal}$

*smaller unit → larger unit**Since 4 qt = 1 gal,  
divide by 4.*

$$64 \div 4 = 16$$

$$64 \text{ qt} = 16 \text{ gal}$$

②  $300 \text{ T} = \underline{\quad ? \quad} \text{ lb}$

*larger unit → smaller unit**Since 1 T = 2,000 lb,  
multiply by 2,000.*

$$300 \times 2,000 = 600,000$$

$$300 \text{ T} = 600,000 \text{ lb}$$

③  $6 \text{ pt} = \underline{\quad ? \quad} \text{ fl oz}$

*larger unit → smaller unit**Since 1 c = 8 fl oz and  
1 pt = 2 c, multiply by 16.*

$$6 \times 16 = 96$$

$$6 \text{ pt} = 96 \text{ fl oz}$$

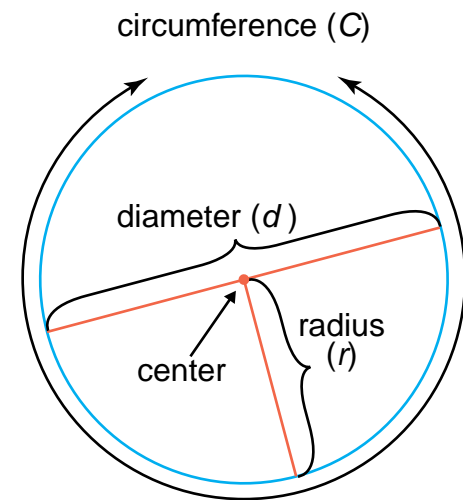
④  $30 \text{ c} = \underline{\quad ? \quad} \text{ qt}$

*smaller unit → larger unit**Since 2 c = 1 pt and  
2 pt = 1 qt, divide by 4.*

$$30 \div 4 = 7\frac{1}{2}$$

$$30 \text{ c} = 7\frac{1}{2} \text{ qt}$$

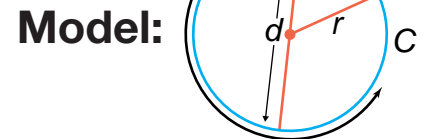
A **circle** is a set of points in a plane, all of which are the same distance from a fixed point in the plane called the **center**.



### Circumference of a Circle

**Words:** The circumference of a circle is equal to  $\pi$  times its diameter or  $\pi$  times twice its radius.

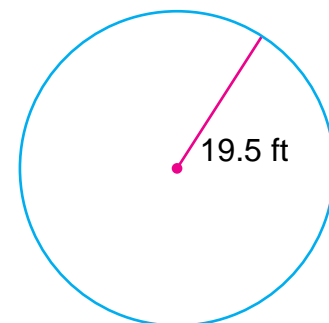
**Symbols:**  $C = \pi d$   
or  
 $C = 2\pi r$



The decimal 3.14 and the fraction  $\frac{22}{7}$  are used as approximations for  $\pi$ .

### Example

Find the circumference of a circle with a radius of 19.5 feet.



$$\begin{aligned} C &= 2\pi r \\ &\approx 2 \cdot 3.14 \cdot 19.5 \\ &\approx 122.46 \end{aligned}$$

*Replace  $\pi$  with 3.14 and  $r$  with 19.5.*  
The circumference is about 122.46 feet.

A **ratio** is a comparison of two numbers by division. At the right are four ways to express the ratio that compares 13 to 25.

13 to 25  
13:25  
13 out of 25  
 $\frac{13}{25}$

### Examples

Express each ratio as a fraction in simplest form.

① 18:30

$$\frac{18}{30} = \frac{18 \div 6}{30 \div 6} \quad \text{The GCF of 18 and 30 is 6.}$$

$$= \frac{3}{5}$$

The ratio in simplest form is  $\frac{3}{5}$  or 3:5.

② 25 books out of 90 books

$$\frac{25}{90} = \frac{25 \div 5}{90 \div 5} \quad \text{The GCF of 25 and 90 is 5.}$$

$$= \frac{5}{18}$$

The ratio in simplest form is  $\frac{5}{18}$  or 5 out of 18.

③ Tell whether 16:40 and 48:120 are equivalent ratios.

Express each ratio as a fraction in simplest form.

$$\frac{16}{40} = \frac{16 \div 8}{40 \div 8} \quad \text{The GCF of 16 and 40 is 8.}$$

$$= \frac{2}{5}$$

$$\frac{48}{120} = \frac{48 \div 24}{120 \div 24} \quad \text{The GCF of 48 and 120 is 24.}$$

$$= \frac{2}{5}$$

The ratios in simplest form are equal.  
So, 16:40 and 48:120 are equivalent ratios.

**Proportion**

**Words:** A proportion is an equation that shows that two ratios are equivalent.

**Symbols:** **Arithmetic**  $\frac{1}{4} = \frac{5}{20}$  **Algebra**  $\frac{a}{b} = \frac{c}{d} (b \neq 0, d \neq 0)$

**Property of Proportions**

**Words:** The cross products of a proportion are equal.

**Symbols:** If  $\frac{a}{b} = \frac{c}{d}$ , then  $ad = bc$ . ( $b \neq 0, d \neq 0$ )

**Examples**

Solve each proportion.

①  $\frac{3}{7} = \frac{6}{n}$

②  $\frac{r}{14} = \frac{5}{7}$

$$\frac{3}{7} = \frac{6}{n}$$

$$3 \times n = 7 \times 6 \quad \text{Cross products}$$

$$3n = 42$$

$$\frac{3n}{3} = \frac{42}{3} \quad \text{Divide.}$$

$$n = 14$$

The solution is 14.

$$\frac{r}{14} = \frac{5}{7}$$

$$r \times 7 = 14 \times 5 \quad \text{Cross products}$$

$$7r = 70$$

$$\frac{7r}{7} = \frac{70}{7} \quad \text{Divide.}$$

$$r = 10$$

The solution is 10.

A percent greater than 100% represents a number greater than 1.  
A percent less than 1% represents a number less than 0.01 or  $\frac{1}{100}$ .

### Examples

Express each percent as a decimal.

① 250%

$$\begin{aligned} 250\% &= \underbrace{250.}_{\phantom{000}} \\ &= 2.5 \end{aligned}$$

So,  $250\% = 2.5$ .

② 0.08%

$$\begin{aligned} 0.08\% &= \underbrace{000.08}_{\phantom{000}} \\ &= 0.0008 \end{aligned}$$

So,  $0.08\% = 0.0008$ .

Express each number as a percent.

③ 0.004

$$\begin{aligned} 0.004 &= \underbrace{0.004}_{\phantom{000}} \\ &= 0.4\% \end{aligned}$$

So,  $0.004 = 0.4\%$ .

④  $6\frac{4}{5}$

$$\begin{aligned} 6\frac{4}{5} &= \underbrace{6.80}_{\phantom{000}} \\ &= 680\% \end{aligned}$$

So,  $6\frac{4}{5} = 680\%$ .

**Percent  
Proportion**

The percent proportion is  $\frac{P}{B} = \frac{r}{100}$ , where  $P$  represents the percentage,  $B$  represents the base, and  $r$  represents the number per hundred.

**Examples**

Find each number. Round to the nearest tenth if necessary.

- ① What number is 55% of 160?

$$\frac{P}{B} = \frac{r}{100}$$

*Write the percent proportion.*

$$\frac{P}{160} = \frac{55}{100}$$

*Replace  $B$  with 160 and  $r$  with 55.*

$$P \cdot 100 = 160 \cdot 55 \quad \textit{Find the cross products.}$$

$$100P = 8,800$$

$$\frac{100P}{100} = \frac{8,800}{100}$$

*Divide each side by 100.*

$$P \approx 88$$

88 is 55% of 160.

- ② What percent of 65 is 54?

$$\frac{P}{B} = \frac{r}{100}$$

*Write the percent proportion.*

$$\frac{54}{65} = \frac{r}{100}$$

*Replace  $P$  with 54 and  $B$  with 65.*

$$54 \cdot 100 = 65 \cdot r \quad \textit{Find the cross products.}$$

$$\frac{54 \cdot 100}{65} = r$$

$$54 \times 100 \div 65 = 83.07692308$$

$$83.1 \approx r$$

54 is about 83.1% of 65.

**Examples**

- ① What number is 53% of 620? *Estimate:  $\frac{1}{2}$  of 600 = 300.*

$$P = R \cdot B$$

$$P = 0.53 \cdot 620$$

$$P = 328.6$$

*Replace R with 0.53 and B with 620.*

328.6 is 53% of 620.

*328.6 is close to the estimate of 300.*

- ② 12 is what percent of 67? *Estimate:  $\frac{12}{67} \rightarrow \frac{12}{60} = \frac{1}{5}$  or 20%.*

$$P = R \cdot B$$

$$12 = R \cdot 67$$

$$\frac{12}{67} = \frac{67R}{67}$$

$$12 \div 67 \approx 0.179104478$$

$$0.18 \approx R$$

*Replace P with 12 and B with 67.*

*Divide each side by 67.*

*Round to the nearest hundredth.*

12 is about 18% of 67.

*18% is close to the estimate of 20%.*

- ③ 9 is 6% of what number? *Estimate: 9 is 5% or  $\frac{1}{20}$  of 180.*

$$P = R \cdot B$$

$$9 = 0.06 \cdot B$$

$$\frac{9}{0.06} = \frac{0.06B}{0.06}$$

$$9 \div .06 \approx 150$$

*Replace P with 9 and B with 0.06.*

*Divide each side by 0.06.*

9 is 6% of 150.

*150 is close to the estimate of 180.*

**Example**

Dreamy Yogurt expects to sell 5,000 yogurt cups at the first annual Taste of the Town food fair. To determine how much of each flavor to have on hand, Dreamy Yogurt conducted a poll in which 125 yogurt lovers were asked which kind of yogurt they preferred. Use the results given in the table to predict how many people will prefer chocolate and vanilla yogurt if the people who prefer “other” will settle for vanilla.

FLAVOR	NUMBER OF PEOPLE
vanilla	47
chocolate	34
strawberry	28
other	16

You can use the percent proportion to find the number who prefer chocolate.

$$\frac{34}{125} = \frac{n}{5,000}$$

$$34 \cdot 5,000 = 125n$$

$$\frac{34 \cdot 5,000}{125} = \frac{125n}{125}$$

$$34 \times 5000 \div 125 = 1360$$

You can predict that 1,360 people will prefer chocolate.

Use the percent proportion to find the number who prefer vanilla, including those who prefer “other.”

$$47 + 16 = 63$$

$$\frac{63}{125} = \frac{n}{5,000}$$

$$63 \cdot 5,000 = 125n$$

$$\frac{63 \cdot 5,000}{125} = \frac{125n}{125}$$

$$63 \times 5000 \div 125 = 2520$$

You can predict that about 2,520 people will prefer vanilla, including those who prefer “other.”

## Examples

- ① The price of a round-trip ticket between New York and Chicago on an airline is normally \$355 when the ticket is purchased 21 days in advance. For a limited time, the airline is offering a 27% discount on all advance purchase round-trip tickets. How much would you pay for this ticket with the discount?

## Method 1

First, find the amount of the discount.

$$27\% \text{ of } \$355 = d$$

$$.27 \times 355 = 95.85$$

The discount is \$95.85.

Then, subtract to find the discounted price.

$$\$355 - \$95.85 = \$259.15.$$

The discounted price is \$259.15.

## Method 2

First, subtract the percent of discount from 100%.

$$100\% - 27\% = 73\%$$

The discounted price is 73% of the normal price.

Then, multiply to find the discounted price.

$$.73 \times 355 = 259.15$$

- ② Find the total cost of a shirt that costs \$29.95 with 6% sales tax.

## Method 1

$$0.06 \text{ of } \$29.95 = s$$

$$.06 \times 29.95 = 1.797$$

The sales tax is \$1.80.

$$\$29.95 + \$1.80 = \$31.75.$$

The total cost is \$31.75.

## Method 2

$$100\% + 6\% = 106\%$$

The price with sales tax is 106% of the normal price.

$$1.06 \times 29.95 = 31.747$$

**Simple interest** is the amount paid for the use of money. The formula for simple interest is  $I = prt$ , where  $I$  is the interest,  $p$  is the **principal**,  $r$  is the annual interest **rate**, and  $t$  is the **time** in years.

### Examples

- ① Mr. Jones deposited \$800 in his savings account. His account earned 4.5% interest annually. He did not deposit or withdraw any money for 18 months. How much interest did he earn?

$$I = prt$$

$$I = 800 \cdot 0.045 \cdot 1\frac{1}{2} \quad p = \$800, r = 4.5\%, t = 18 \text{ mos or } 1\frac{1}{2} \text{ yr}$$

$$800 \times .045 \times 1.5 = 54$$

$$I = 54$$

The interest earned on \$800 in 18 months was \$54.

- ② Find the interest to the nearest cent on \$750 on a credit card at 14.5% interest for 2 years.

$$I = prt$$

$$I = 750 \cdot 0.145 \cdot 2 \quad p = \$750, r = 14.5\%, t = 2 \text{ years}$$

$$750 \times .145 \times 2 = 217.5$$

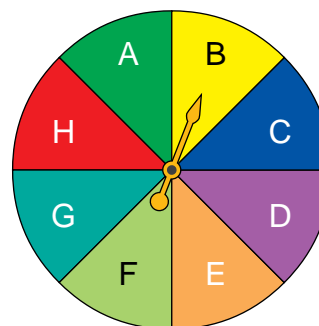
$$I = 217.50$$

The interest paid on a \$750 credit card balance over 2 years was \$217.50.

The **probability** of an event is the ratio of the number of ways an event can occur to the number of possible outcomes.

### Examples

- 1 What is the probability that the spinner will land on a vowel?



$$\begin{aligned}
 P(\text{spinning a vowel}) &= \frac{\text{number of ways to spin a vowel}}{\text{number of ways to spin a letter}} \\
 &= \frac{2}{8} \quad \leftarrow \text{There are 2 vowels on the spinner, A and E.} \\
 &\quad \leftarrow \text{There are 8 letters on the spinner.} \\
 &= \frac{1}{4} \quad \text{The probability is } \frac{1}{4} \text{ or } 25\%.
 \end{aligned}$$

- 2 In a survey of 100 people, 36 said they talked on the telephone in the kitchen the most, 25 said the living room, 13 said the bedroom, and 26 said other rooms. If a person is chosen at random, what is the probability that he or she talks on the telephone in the kitchen the most?

$$\begin{aligned}
 P(\text{kitchen}) &= \frac{\text{number of people who talk in the kitchen the most}}{\text{total number of people}} \\
 &= \frac{36}{100} \quad \leftarrow \text{36 people talk in the kitchen the most.} \\
 &\quad \leftarrow \text{There are 100 people in the survey.} \\
 &= \frac{9}{25} \quad \text{The probability is } \frac{9}{25} \text{ or } 36\%.
 \end{aligned}$$

**The  
Counting  
Principle**

If event  $M$  can occur in  $m$  ways and is followed by event  $N$  that can occur in  $n$  ways, then the event  $M$  followed by  $N$  can occur in  $m \times n$  ways.

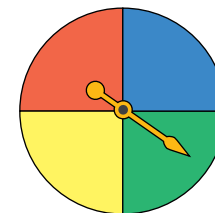
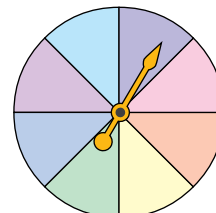
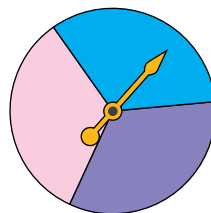
**Examples**

Use the Counting Principle to find the total number of outcomes in each situation.

- ① choosing a pair of pants if the pants come cuffed or uncuffed, regular fit or relaxed fit, and in the following colors: stone, khaki, wheat, olive, and black

$$\underbrace{\text{choices for cuffs}}_2 \times \underbrace{\text{choices for fit}}_2 \times \underbrace{\text{choices for color}}_5 = \underbrace{\text{total number of choices}}_{20}$$

- ② spinning the spinners shown



$$\underbrace{\text{choices for first spinner}}_3 \times \underbrace{\text{choices for second spinner}}_8 \times \underbrace{\text{choices for third spinner}}_4 = \underbrace{\text{total number of choices}}_{96}$$

A **combination** is an arrangement of objects in which order is not important.

### Examples

- ① In how many ways can the colors of paint shown at the right be mixed two colors at a time?



You can make a list.



You can also use the formula.

$$C = \frac{4 \cdot 3}{2} \quad \leftarrow \text{There are } 4 \cdot 3 \text{ permutations of two colors chosen from four.}$$

$$= \frac{12}{2} \text{ or } 6 \quad \leftarrow \text{There are } 2! \text{ or } 2 \cdot 1 \text{ colors being mixed.}$$

There are six different ways to mix these colors.

- ② Does this problem represent a permutation or a combination? In how many ways can six people pose for a picture if they all stand in a row?

Since order *does* matter, this is a permutation of 6 people.

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 720$$

There are 720 ways for six people to pose for the picture.



The order is not important. So, there is no difference between ●● and ●●. Cross out each package that is the same as another package.

Two or more events in which the outcome of one event *does not* affect the outcome of the other event(s) are called **independent events**.

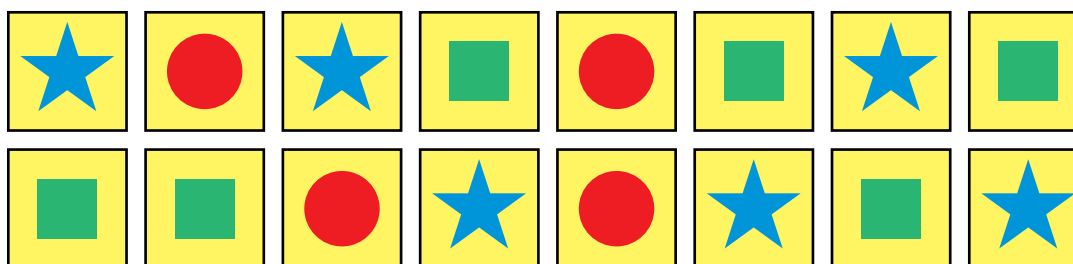
### Probability of Independent Events

The probability of two independent events can be found by multiplying the probability of one event by the probability of the second event.

If the results of one event *does* affect the result of a second event, the events are called **dependent events**.

### Example

Find the probability of selecting a circle card followed by a star card *with* replacement of the first card.



Since the first card is being replaced, the events are independent.

$$P(\text{circle, then star}) = P(\text{circle}) \cdot P(\text{star})$$

$$\begin{aligned}
 &= \frac{4}{16} \cdot \frac{6}{16} \quad \leftarrow \text{number of specified cards} \\
 &\quad \leftarrow \text{total number of cards} \\
 &= \frac{\cancel{4}^1}{\cancel{16}^4} \cdot \frac{\cancel{6}^3}{\cancel{16}^8} \\
 &= \frac{3}{32}
 \end{aligned}$$

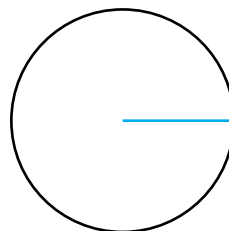
**Example**

According to a recent survey of 500 parents, 33% of the parents said their kids do homework in the bedroom, 25% said the kitchen, 22% said the family or living room, and 20% said the dining room. Make a circle graph to represent the data.

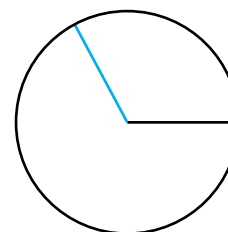
**Step 1** Find the number of degrees for each part. Use  $P = R \cdot B$ .

bedroom:	$33\%$ of $360 = 0.33 \cdot 360 = 118.8^\circ$
kitchen:	$25\%$ of $360 = 0.25 \cdot 360 = 90^\circ$
family or living room:	$22\%$ of $360 = 0.22 \cdot 360 = 79.2^\circ$
dining room:	$20\%$ of $360 = 0.20 \cdot 360 = 72^\circ$

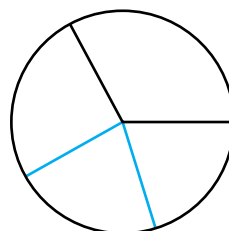
**Step 2** Draw a circle.  
Then draw a  
radius.



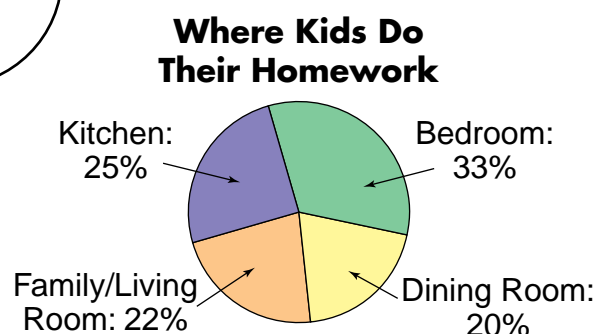
**Step 3** Use a protractor  
to draw an angle  
of  $118.8^\circ$ .



**Step 4** Repeat for the  
other sections.



**Step 5** Label each section of the  
graph with the category  
and percent. Give the  
graph a title.



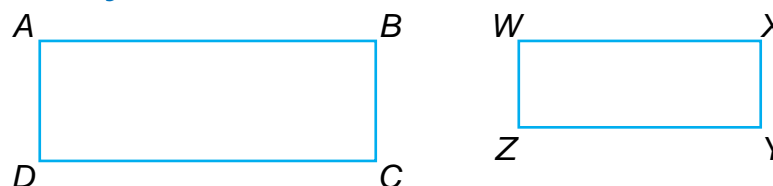
Similar  
Polygons

**Words:** Two polygons are similar if their corresponding angles are congruent and their corresponding sides are in proportion.

**Symbols:**  $ABCD \sim WXYZ$

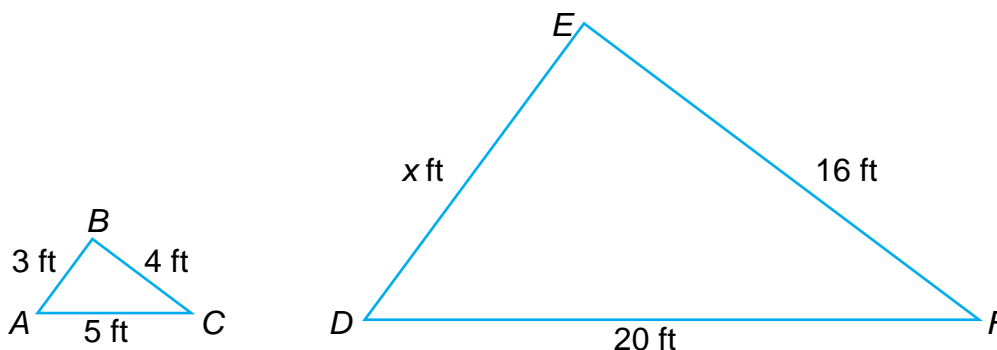
*The symbol  $\sim$  means is similar to.*

**Model:**



## Example

If  $\triangle ABC \sim \triangle DEF$ , find the length of  $\overline{ED}$ .



$\overline{BA}$  and  $\overline{ED}$  are corresponding sides.

$\overline{AC}$  and  $\overline{DF}$  are corresponding sides.

Let  $x$  represent the missing measure.

$$\frac{BA}{ED} \rightarrow \frac{3}{x} = \frac{5}{20} \leftarrow \frac{AC}{DF}$$

$$3(20) = x \cdot 5$$

$$60 = 5x$$

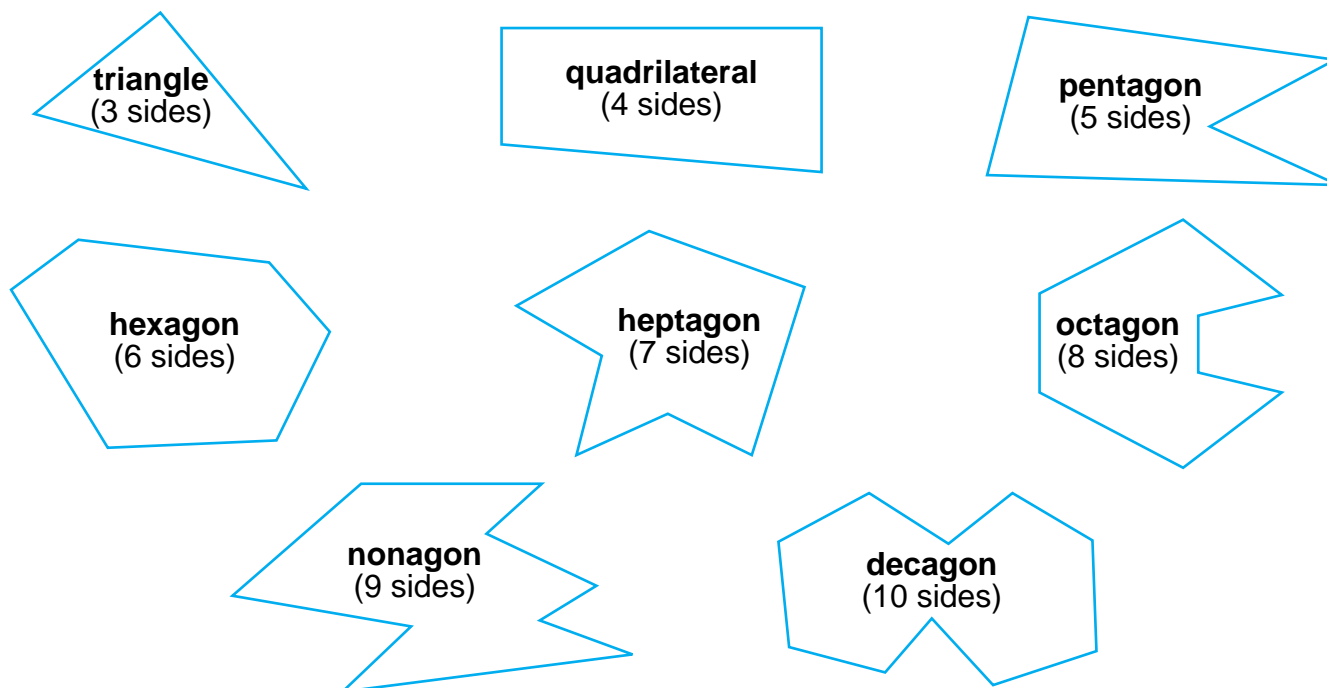
$$12 = x$$

*Write a proportion using the measures of corresponding sides.  
Find the cross products.*

*Divide each side by 5.*

The length of  $\overline{ED}$  is 12 feet.

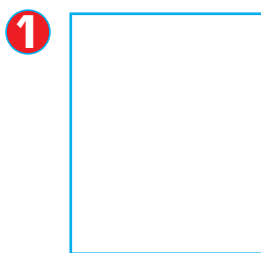
A **polygon** is a simple closed figure formed by three or more line segments.



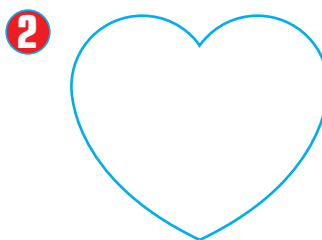
Any polygon with all sides congruent and all angles congruent is called a **regular polygon**.

### Examples

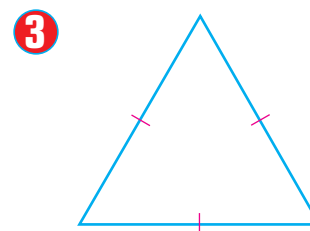
Determine which figures are polygons. If the figure is a polygon, name it and tell whether it is a regular polygon. If the figure is *not* a polygon, explain why.



quadrilateral



not a polygon; no sides are segments



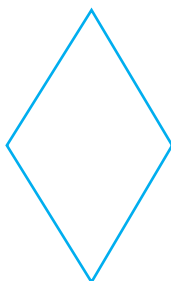
triangle; regular

Figures that match exactly when folded in half have **line symmetry**. Each fold line is called a **line of symmetry**.

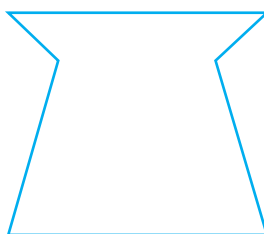
### Examples

Determine which figures have line symmetry. Draw all of the lines of symmetry.

1



2



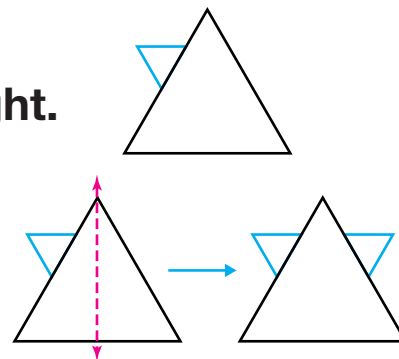
3



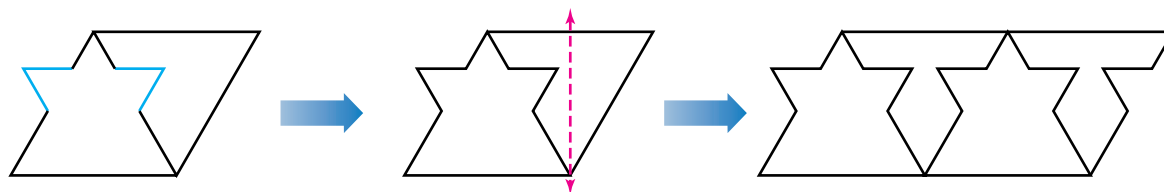
A **reflection** is a mirror image of a figure across a line of symmetry.

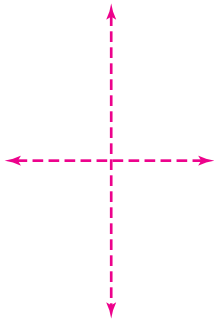
4 Complete an Escher-like drawing using the change shown at the right.

Complete the first pattern unit by drawing the reflection of the design on the other side of the triangle.



Now add another triangle. Reflect the new pattern in the second triangle and complete the tessellation.





no symmetry

square

$$9 \times 9 = 9^2 \text{ or } 81$$

square root

$$\sqrt{81} = 9 \text{ because } 9^2 = 81$$

A radical sign,  $\sqrt{\quad}$ , represents a nonnegative square root.

## Examples

① Evaluate  $8^2$ .

$$8 \times 8 = 64$$

② Evaluate  $15^2$ .

$$15 \boxed{x^2} 255$$

③ Find  $\sqrt{169}$ .

$$\text{Since } 13^2 = 169, \sqrt{169} = 13.$$

④ Find  $\sqrt{324}$ .

$$324 \boxed{2nd} [\sqrt{\quad}] 18$$

⑤ Juan is flying to Europe with his parents for vacation. Since it is a clear day, he can use the formula  $d = 1.4 \times \sqrt{h}$  to estimate the distance in miles that he can see from the airplane. In this formula,  $d$  is the distance from the object in miles, and  $h$  is the height in feet Juan's eyes are above the surface. If Juan's plane is flying at 32,400 feet, how far can he see?

$$d = 1.4 \times \sqrt{h}$$

$$d = 1.4 \times \sqrt{32,400} \quad \text{Replace } h \text{ with } 32,400.$$

$$d = 1.4 \times 180 \quad 32,400 \boxed{2nd} [\sqrt{\quad}] 180$$

$$d = 252$$

Juan can see about 252 miles.

**Examples****1** Estimate  $\sqrt{8}$ .

Since 8 is not a perfect square, estimate  $\sqrt{8}$  by finding the two perfect squares closest to 8. List some perfect squares.

$$1, 4, 9, 16, \dots$$

*8 is between 4 and 9.*

$$4 < 8 < 9$$

$$\sqrt{4} < \sqrt{8} < \sqrt{9}$$

$$2 < \sqrt{8} < 3$$

*Find the square root of each number.*

*This means that  $\sqrt{8}$  is between 2 and 3.*

So,  $\sqrt{8}$  is between 2 and 3. Since 8 is closer to 9 than to 4,  $\sqrt{8}$  is closer to 3 than to 2. The best whole number estimate for  $\sqrt{8}$  is 3.

**2** Estimate  $\sqrt{130}$ .

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, \dots$$

*130 is between 121 and 144.*

$$121 < 130 < 144$$

$$\sqrt{121} < \sqrt{130} < \sqrt{144}$$

$$11 < \sqrt{130} < 12$$

*Find the square root of each number.*

*$\sqrt{130}$  is between 11 and 12.*

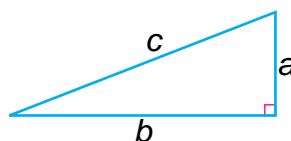
So,  $\sqrt{130}$  is between 11 and 12. Since 130 is closer to 121 than to 144,  $\sqrt{130}$  is closer to 11 than 12. The best whole number estimate for  $\sqrt{130}$  is 11.

Pythagorean  
Theorem

**Words:** In a right triangle, the sum of the squares of the lengths of the legs ( $a$  and  $b$ ) is equal to the square of the length of the hypotenuse ( $c$ ).

**Symbols:**      **Arithmetic**                      **Algebra**  
 $8^2 + 15^2 = 17^2$                        $a^2 + b^2 = c^2$

**Model:**



## Examples

- ① If one side of a triangle measures 6 centimeters and the other side measures 13 centimeters, find the length of the hypotenuse to the nearest tenth.

$$\begin{aligned}
 a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\
 6^2 + 13^2 &= c^2 && \text{Replace } a \text{ with } 6 \text{ and } b \text{ with } 13. \\
 36 + 169 &= c^2 \\
 205 &= c^2 \\
 \sqrt{205} &= c && \text{Definition of square root} \\
 14.3 &\approx c && 205 \text{ } \boxed{2\text{nd}} \text{ } [\sqrt{\quad}] 14.31782106
 \end{aligned}$$

The hypotenuse measures about 14.3 centimeters.

- ② If 5 inches, 12 inches, and 13 inches are the lengths of the sides of a triangle, is the triangle a right triangle?

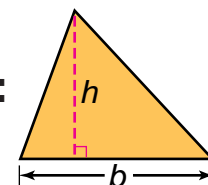
$$\begin{aligned}
 a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\
 5^2 + 12^2 &\stackrel{?}{=} 13^2 && \text{Replace } a \text{ with } 5, b \text{ with } 12, \text{ and } c \text{ with } 13. \\
 25 + 144 &\stackrel{?}{=} 169 \\
 169 &= 169 && \checkmark \text{ It is a right triangle.}
 \end{aligned}$$

Area of a  
Triangle

**Words:** The area ( $A$ ) of a triangle is equal to half the product of its base ( $b$ ) and height ( $h$ ).

**Symbols:**  $A = \frac{1}{2}bh$

**Model:**



## Examples

- ① Find the area of a triangle with a base of 3 inches and a height of 4 inches.

$$A = \frac{1}{2}bh \quad \text{Formula for the area of a triangle}$$

$$A = \frac{1}{2} \times 3 \times 4 \quad \text{Replace } b \text{ with 3 and } h \text{ with 4.}$$

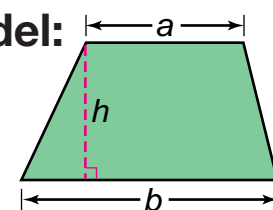
$$A = \frac{1}{2} \times 12 \text{ or } 6 \quad \text{The area is 6 square inches.}$$

Area of a  
Trapezoid

**Words:** The area ( $A$ ) of a trapezoid is equal to half the product of the height ( $h$ ) and the sum of the bases ( $a + b$ ).

**Symbols:**  $A = \frac{1}{2}h(a + b)$

**Model:**

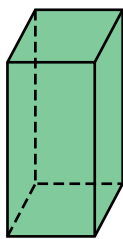
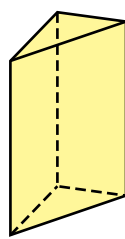
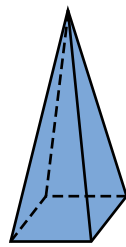


- ② Find the area of a trapezoid with bases of 6 meters and 8 meters and a height of 4 meters.

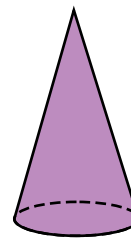
$$A = \frac{1}{2}h(a + b) \quad \text{Formula for the area of a trapezoid}$$

$$A = \frac{1}{2}(4)(6 + 8) \quad \text{Replace } h \text{ with 4, } a \text{ with 6, and } b \text{ with 8.}$$

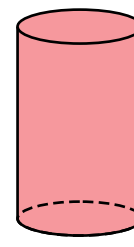
$$A = 2(14) \text{ or } 28 \quad \text{The area is 28 square meters.}$$

rectangular  
prismtriangular  
prism

pyramid



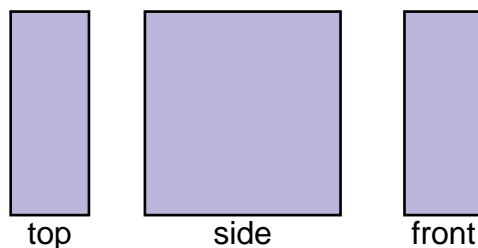
cone



cylinder

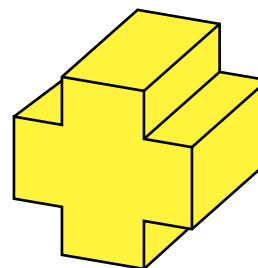
### Examples

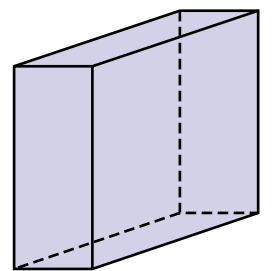
- 1 Make a perspective drawing of a figure by using the top, side, and front views of the figure below.



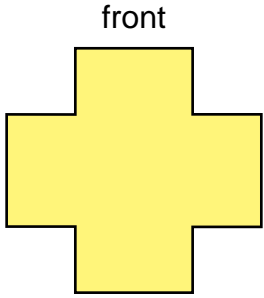
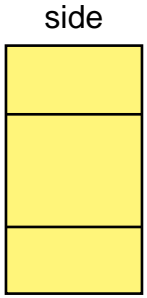
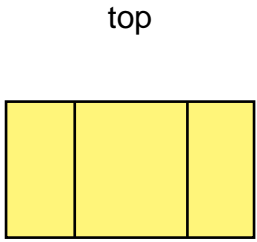
First, sketch a rectangle for the top. Then, add the front and side views. Finally, add dashed lines to show hidden edges.

- 2 Draw a top, a side, and a front view of the figure.





12-1 overlay 1



12-1 overlay 2

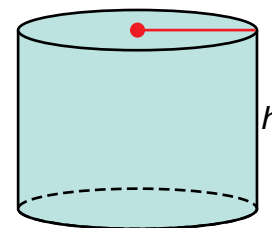
A **cylinder** is a solid figure that has two congruent, parallel circles as its bases.

### Volume of a Cylinder

**Words:** The volume ( $V$ ) of a cylinder is found by multiplying the area of the base ( $\pi r^2$ ) by the height ( $h$ ).

**Symbols:**  $V = \pi r^2 h$

**Model:**



### Examples

- ① Find the volume of a cylinder with a radius of 6 feet and a height of 4 feet. Use 3.14 for  $\pi$ . *Estimate:*  $3 \times 6^2 \times 4 = 432$

$$V = \pi r^2 h$$

*Formula for volume of cylinder*

$$V \approx 3.14 \cdot 6^2 \cdot 4$$

*Replace  $r$  with 6 and  $h$  with 4.*

$$V \approx 452.16$$

The cylinder has a volume of *about* 452 cubic feet.

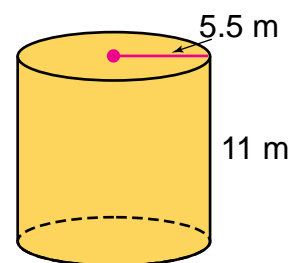
- ② Find the volume of the cylinder.

$$\textit{Estimate: } 3 \times 6^2 \times 10 = 1,080$$

$$V = \pi r^2 h$$

$$V \approx 3.14 \cdot (5.5)^2 \cdot 11$$

$$V \approx 1,044.835$$



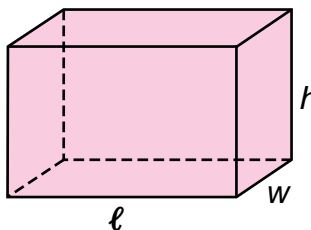
The cylinder has a volume of *about* 1,045 cubic meters.

Surface  
Area of a  
Rectangular  
Prism

**Words:** The surface area of a rectangular prism equals the sum of the areas of the faces.

**Symbols:** surface area =  $2\ell w + 2\ell h + 2wh$

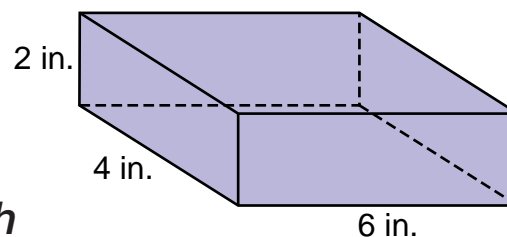
**Model:**



## Examples

- ① Find the surface area of the rectangular prism.

$$\begin{aligned} \text{surface area} &= 2\ell w + 2\ell h + 2wh \\ &= 2 \times 6 \times 4 + 2 \times 6 \times 2 + 2 \times 4 \times 2 \\ &= 48 + 24 + 16 \quad \text{Multiply first. Then add.} \\ &= 88 \end{aligned}$$



The surface area of the rectangular prism is 88 square inches.

- ② Find the surface area of a rectangular prism with a length of 12 meters, a width of 7 meters, and a height of 5 meters.

$$\begin{aligned} \text{surface area} &= 2\ell w + 2\ell h + 2wh \\ &= 2 \times 12 \times 7 + 2 \times 12 \times 5 + 2 \times 7 \times 5 \\ &= 168 + 120 + 70 \quad \text{Multiply first. Then add.} \\ &= 358 \end{aligned}$$

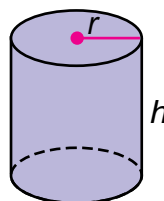
The surface area of the rectangular prism is 358 square meters.

Surface  
Area of a  
Cylinder

**Words:** The surface area of a cylinder equals the sum of the areas of the circular bases ( $2\pi r^2$ ) and the area of the curved surface ( $2\pi rh$ ).

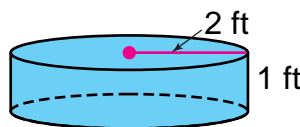
**Symbols:** surface area =  $2\pi r^2 + 2\pi rh$

**Model:**



## Examples

- 1 Find the surface area of the cylinder.



$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 2^2 + 2 \times \pi \times 2 \times 1 \\ 2 \times \pi \times 2^2 + 2 \times \pi \times 2 \times 1 &= 37.69911184 \end{aligned}$$

The surface area of the cylinder is *about* 38 square feet.

- 2 Find the surface area of a cylinder with a diameter of 11 centimeters and a height of 18 centimeters.

The diameter of the cylinder is 11 centimeters. Therefore, the radius is 5.5 centimeters.

$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times (5.5)^2 + 2 \times \pi \times 5.5 \times 18 \\ 2 \times \pi \times 5.5^2 + 2 \times \pi \times 5.5 \times 18 &= 812.10701 \end{aligned}$$

The surface area of the cylinder is *about* 812 square centimeters.