Teaching Mathematics with Manipulatives

Mathematics

Applications and Concepts

Course 1

Glencoe

New York, New York Columbus, Ohio Chicago, Illinois Peoria, Illinois Woodland Hills, California

Manipulatives

Glencoe offers three types of kits to enhance the use of manipulatives in your Middle School Mathematics classroom.

- The **Glencoe Mathematics Overhead Manipulative Resources** contains translucent manipulatives designed for use with an overhead projector.
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- The **Glencoe Mathematics Student Manipulative Kit** contains an individual set of manipulatives often used in Student Edition activities.

The manipulatives contained in each of these kits are listed on page vi of this booklet.

Each of these kits can be ordered from Glencoe by calling (800) 334-7344.

Glencoe Mathematics Overhead Manipulative Kit	0-07-830593-4
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ISBN: 0-07-860082-0 *Teaching Mathematics with Manipulatives*

Printed in the United States of America.

1 2 3 4 5 6 7 8 9 10 045 11 10 09 08 07 06 05 04 03 02

Contents

Teacher's Guide to Using *Teaching Mathematics with Manipulatives*

The book contains two sections of masters—Easy-to-Make Manipulatives and activities for Middle School Mathematics. Tabs help you locate the activities for each chapter. A complete list of manipulatives available in each of the three types of Glencoe Mathematics Manipulative Kits appears on the next page.

Easy-to-Make Manipulatives
The first section of this book contains masters for making your own manipulatives. To make more durable manipulatives, consider using card stock. To make algebra tiles similar to those shown in the Student Edition, have students use markers to color the tiles appropriately or use colored card stock.

You can also make transparencies of frequently used items such as grid paper and number lines.

Activity Masters
Each chapter begins with **Teaching Notes and Overview** that summarizes the activities for the chapter and includes sample answers. There are three types of masters.

Mini-Projects are short projects that enable students to independently investigate mathematical concepts.

Using Overhead Manipulatives provides instructions for the teacher to demonstrate an alternate approach to the concepts of the lesson by using manipulatives on the overhead projector.

Student Recording Sheets accompany the Hands-On Lab Activities found in the Student Edition. Students can easily record the results of the activity on prepared grids, charts, and figures.

Glencoe Mathematics Manipulatives

Glencoe Mathematics Overhead Manipulative Resources ISBN: 0-07-830593-4		
Transparencies		**Overhead Manipulatives**
integer mat equation mat product mat inequality mat dot paper isometric dot paper coordinate grids	centimeter grid number lines lined paper regular polygons polynomial models integer models equation models	algebra tiles spinners two-dimensional cups red and yellow counters decimal models (base-ten blocks) compass protractor geoboard/geobands geometric shapes transparency pens in 4 colors

Glencoe Mathematics Classroom Manipulative Kit ISBN: 0-02-833116-8		
Algebra	**Measurement, Probability, and Statistics**	**Geometry**
algebra tiles counters cups centimeter cubes equation mat/product mat coordinate grid stamp and ink pad	base-ten models marbles measuring cups number cubes protractors rulers scissors spinners stopwatches tape measures	compasses geoboards geobands geomirrors isometric dot grid stamp pattern blocks tangrams

Glencoe Mathematics Student Manipulative Kit ISBN: 0-02-833654-2	
algebra tiles red and yellow counters cups equation/product mat compass/ruler	protractor scissors geoboard geobands tape measure

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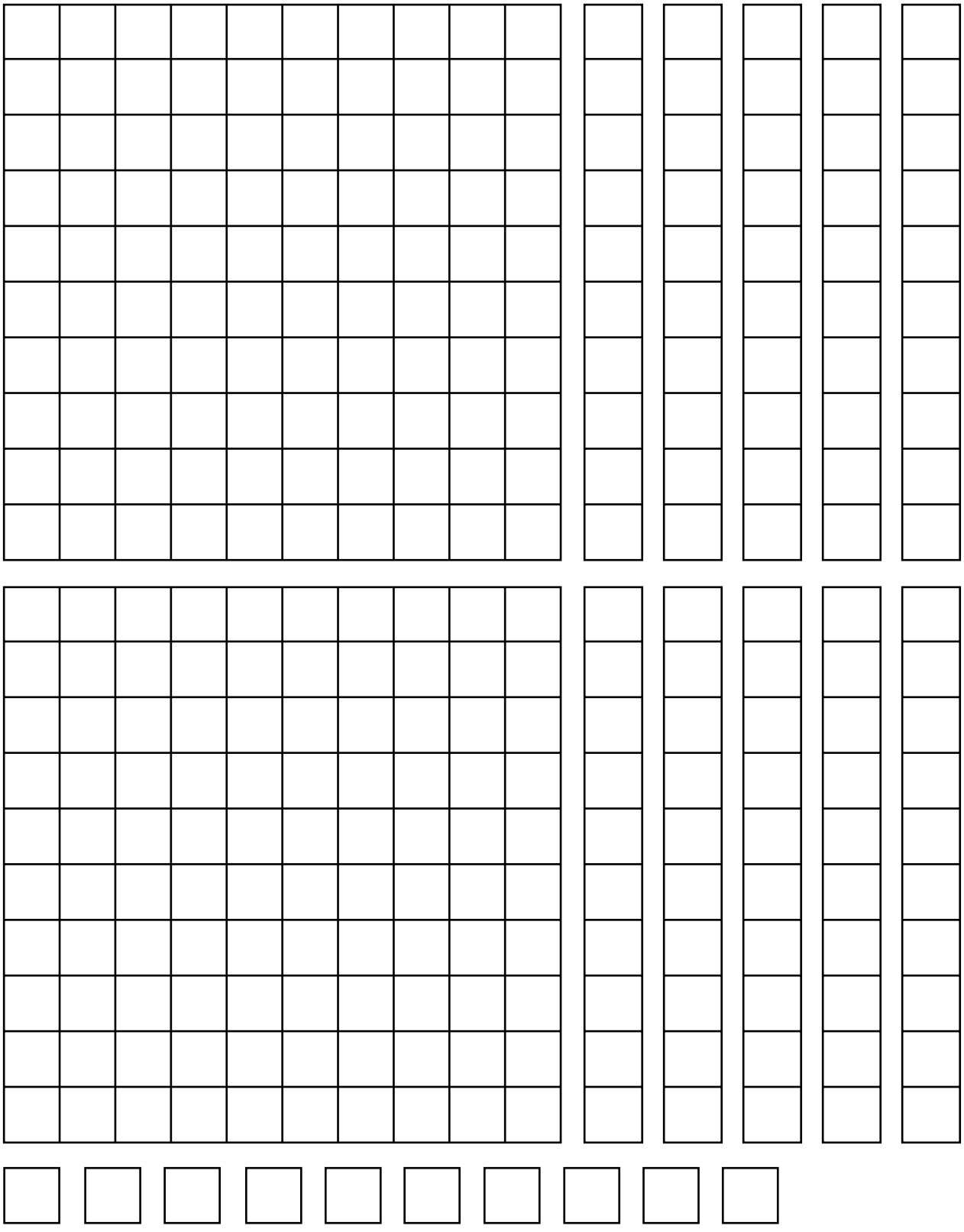
Base-Ten Models



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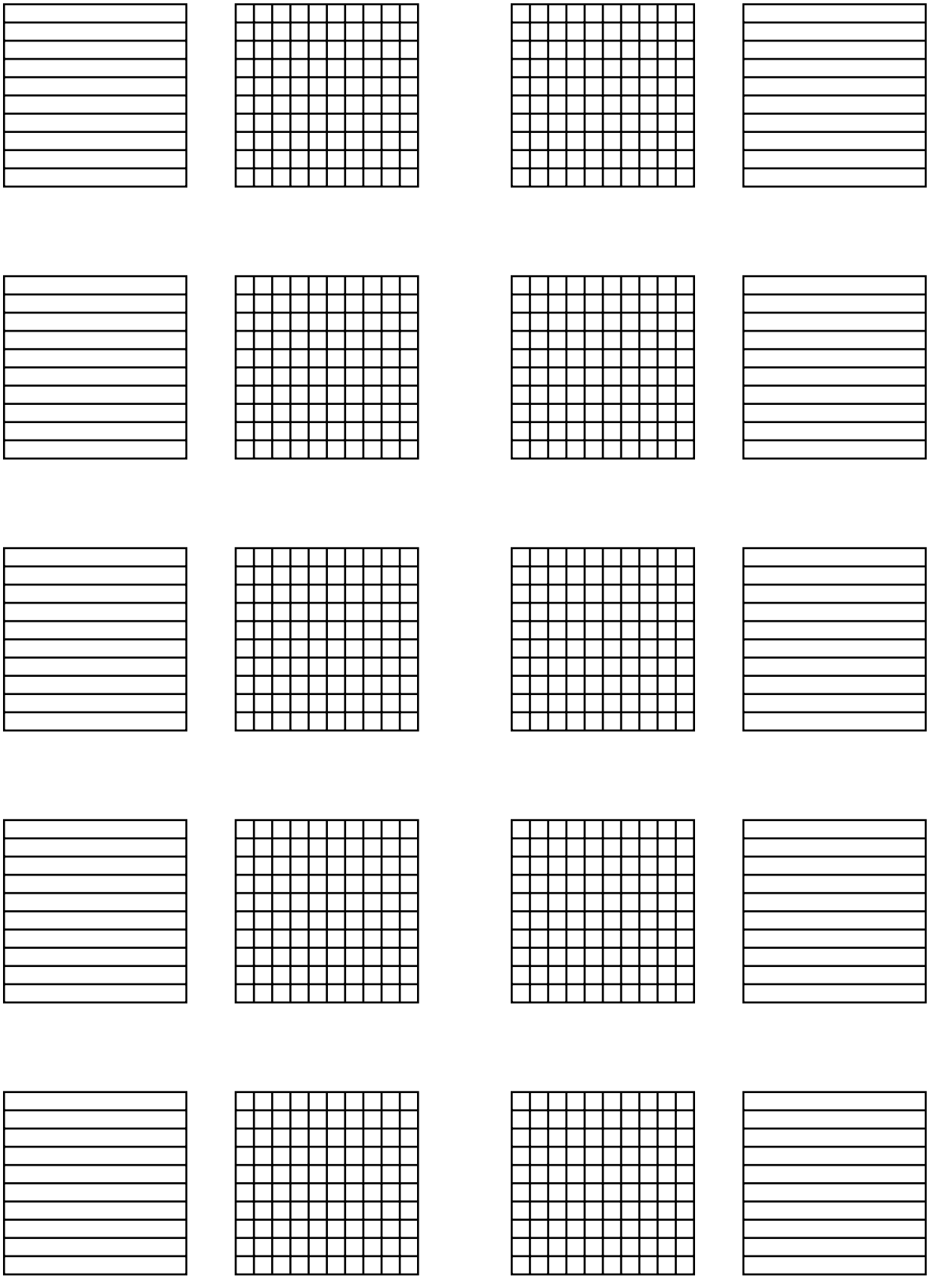
Decimal Models



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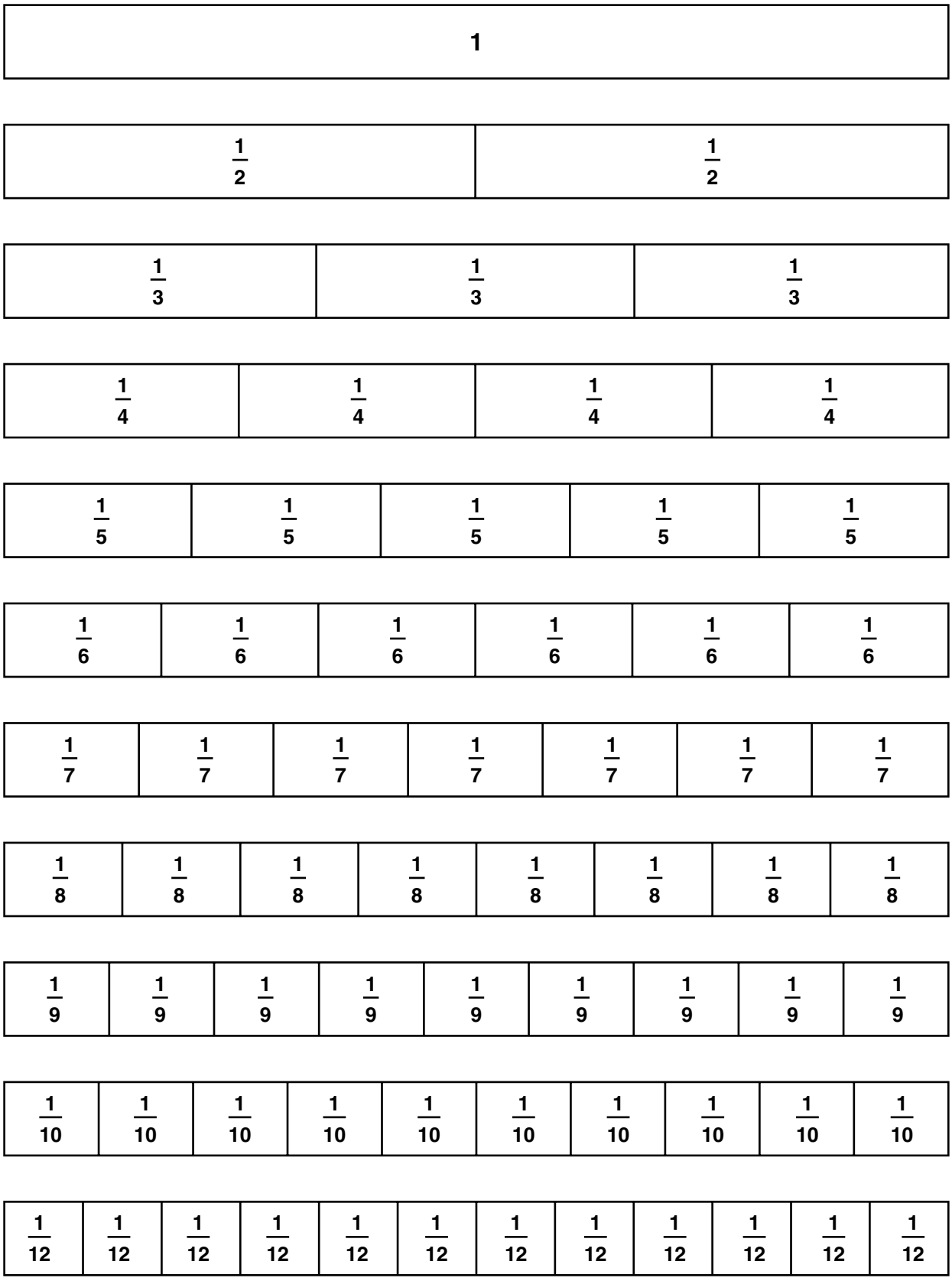
Fraction Models: Bars



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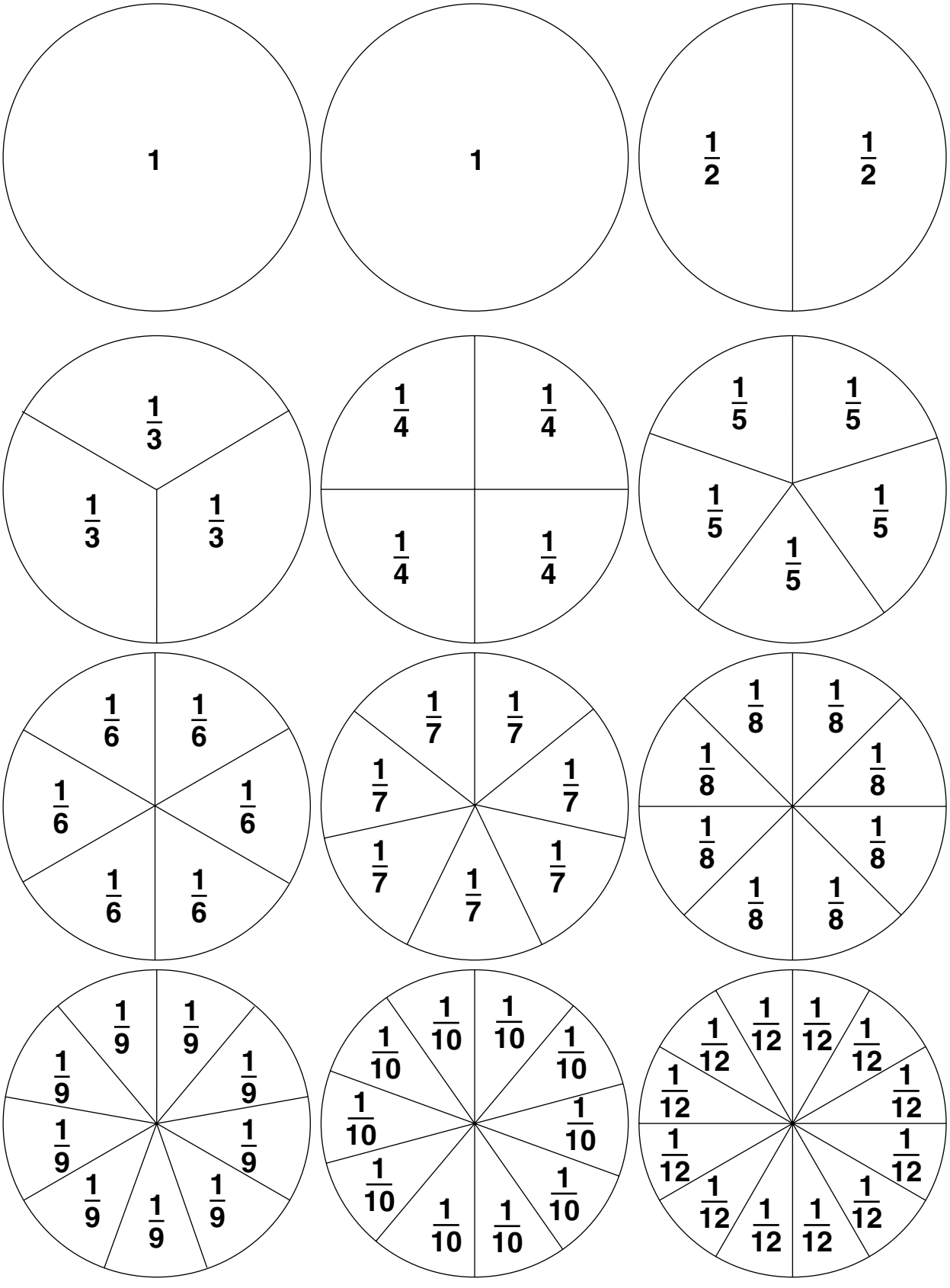
Fraction Models: Circles



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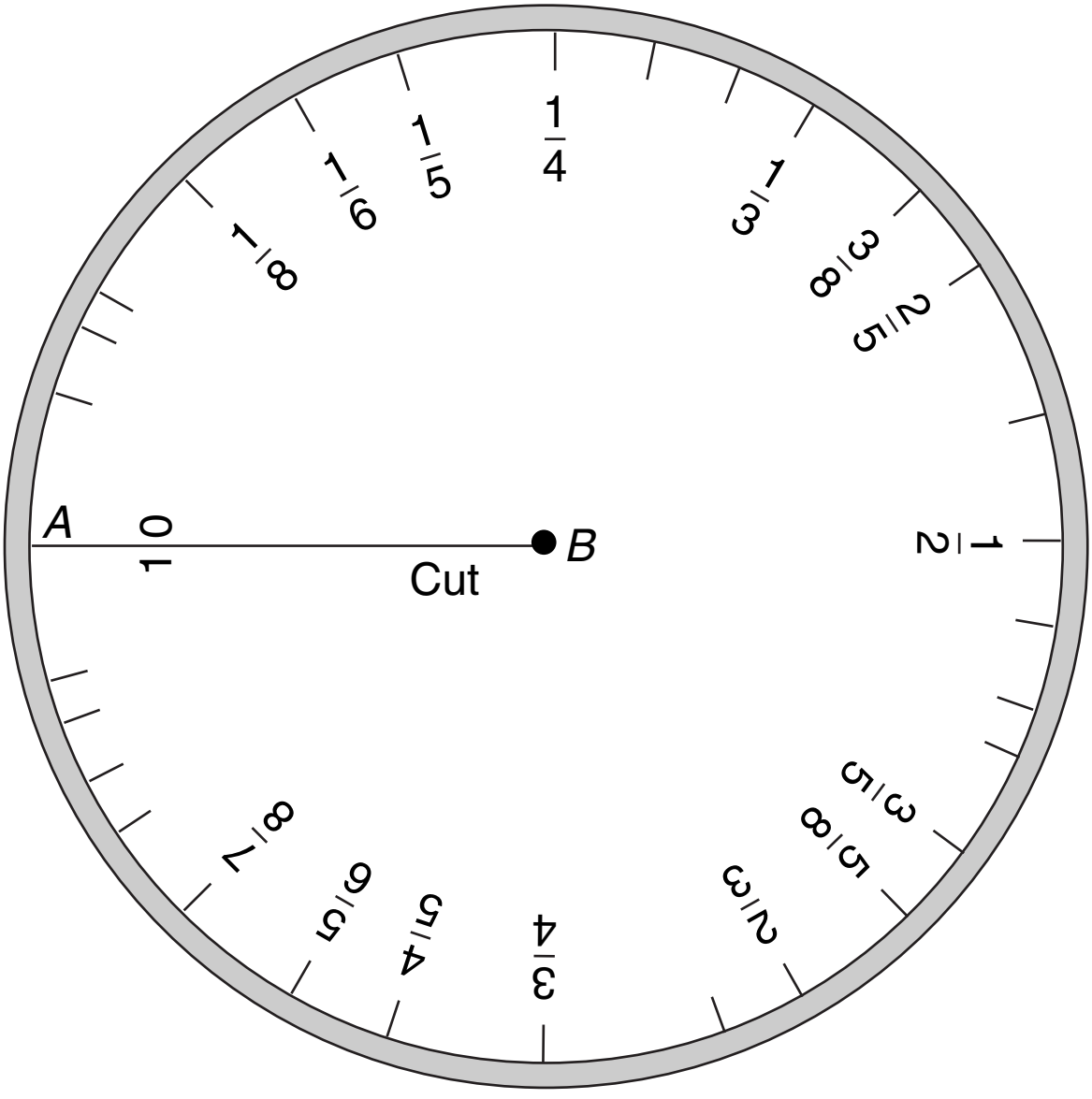
Fraction Wheel



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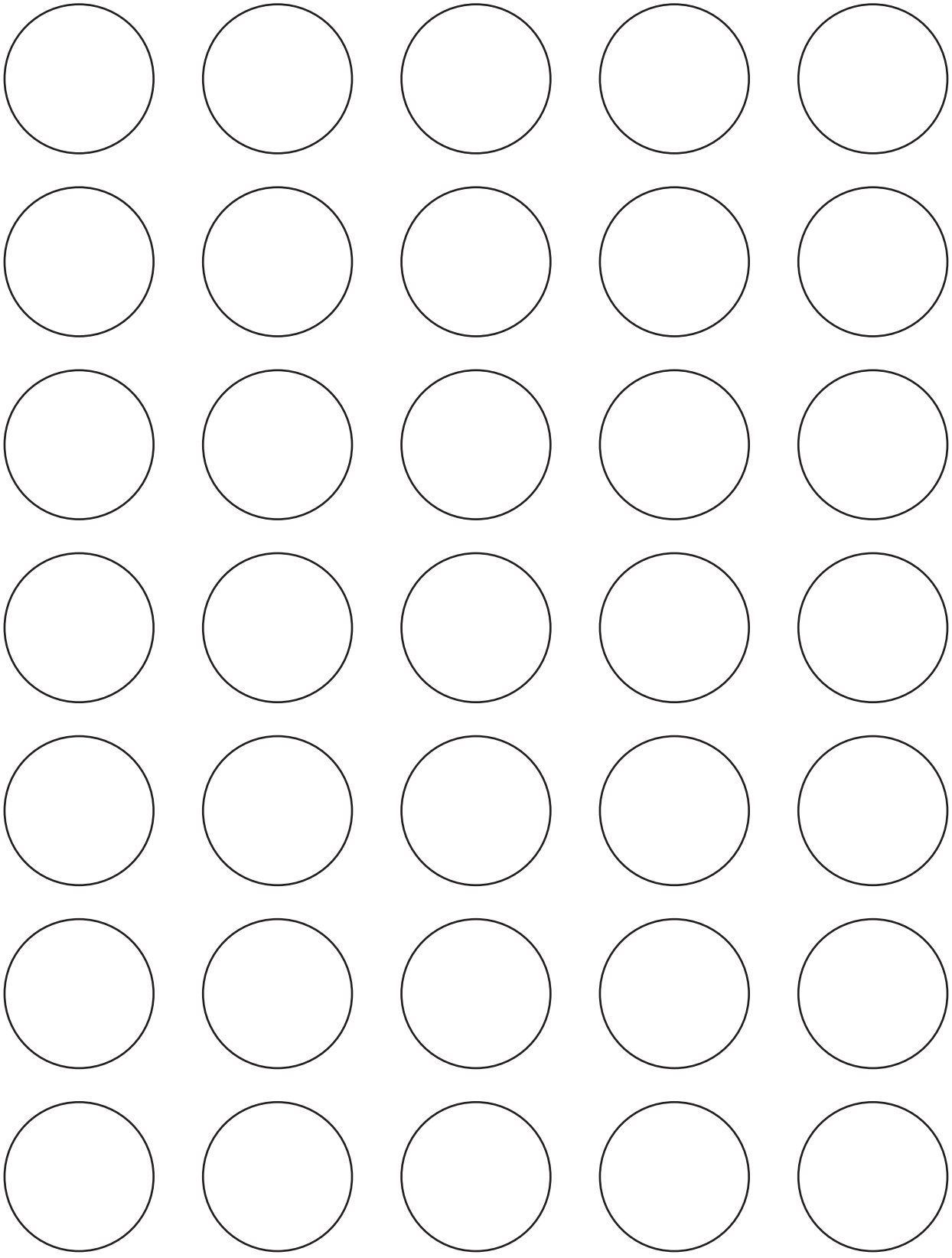
Counters



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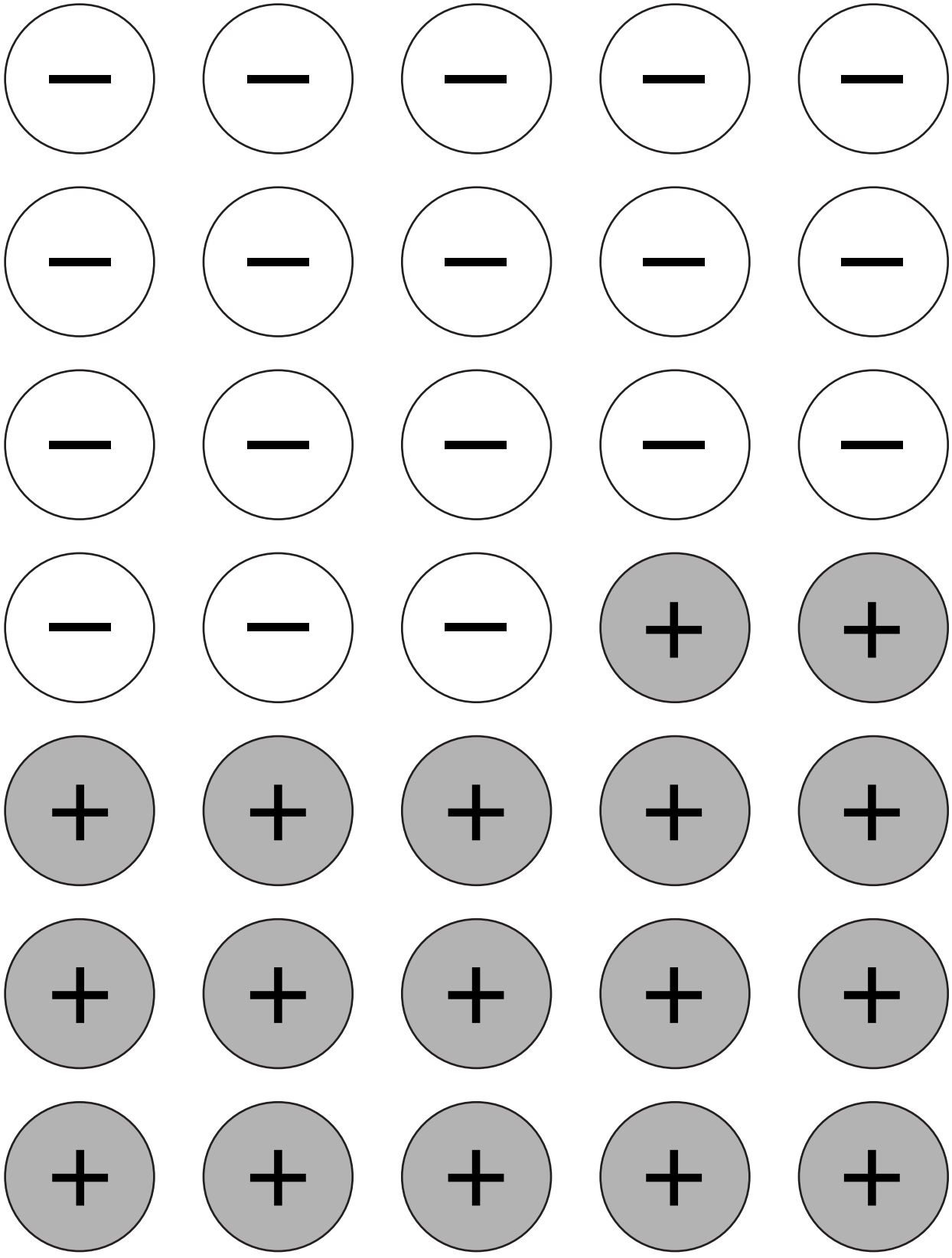
Integer Counters



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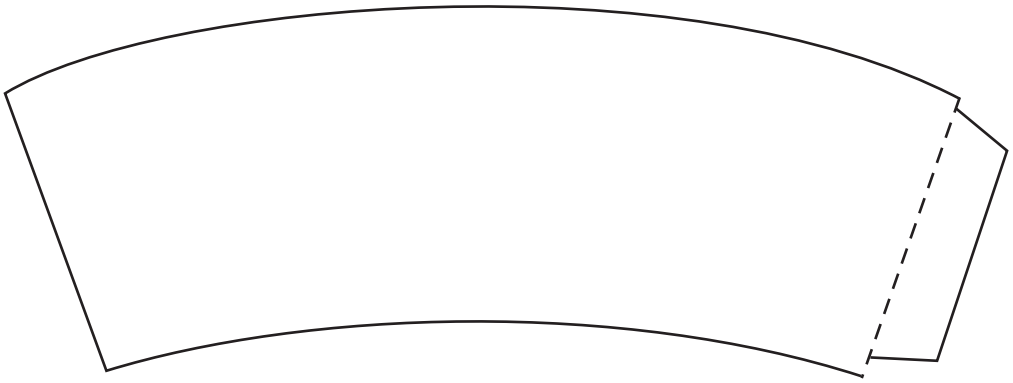
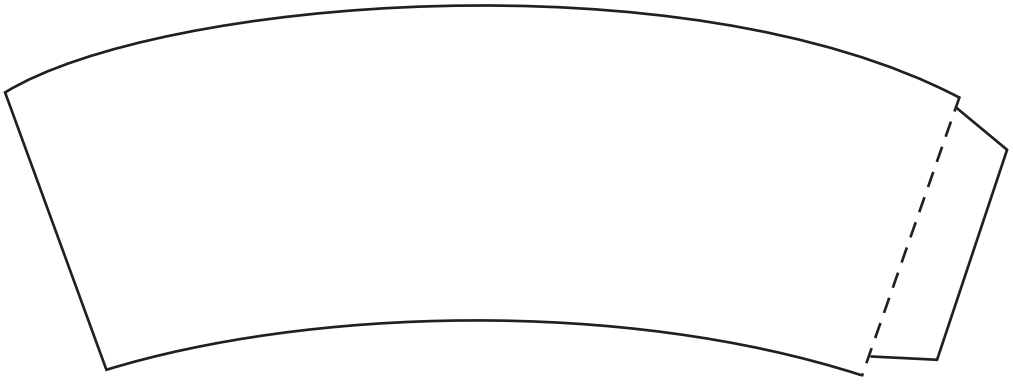
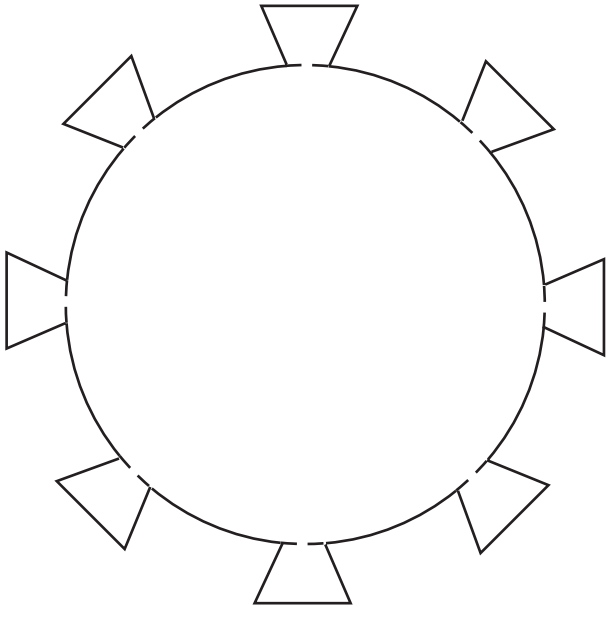
Pattern for Cup







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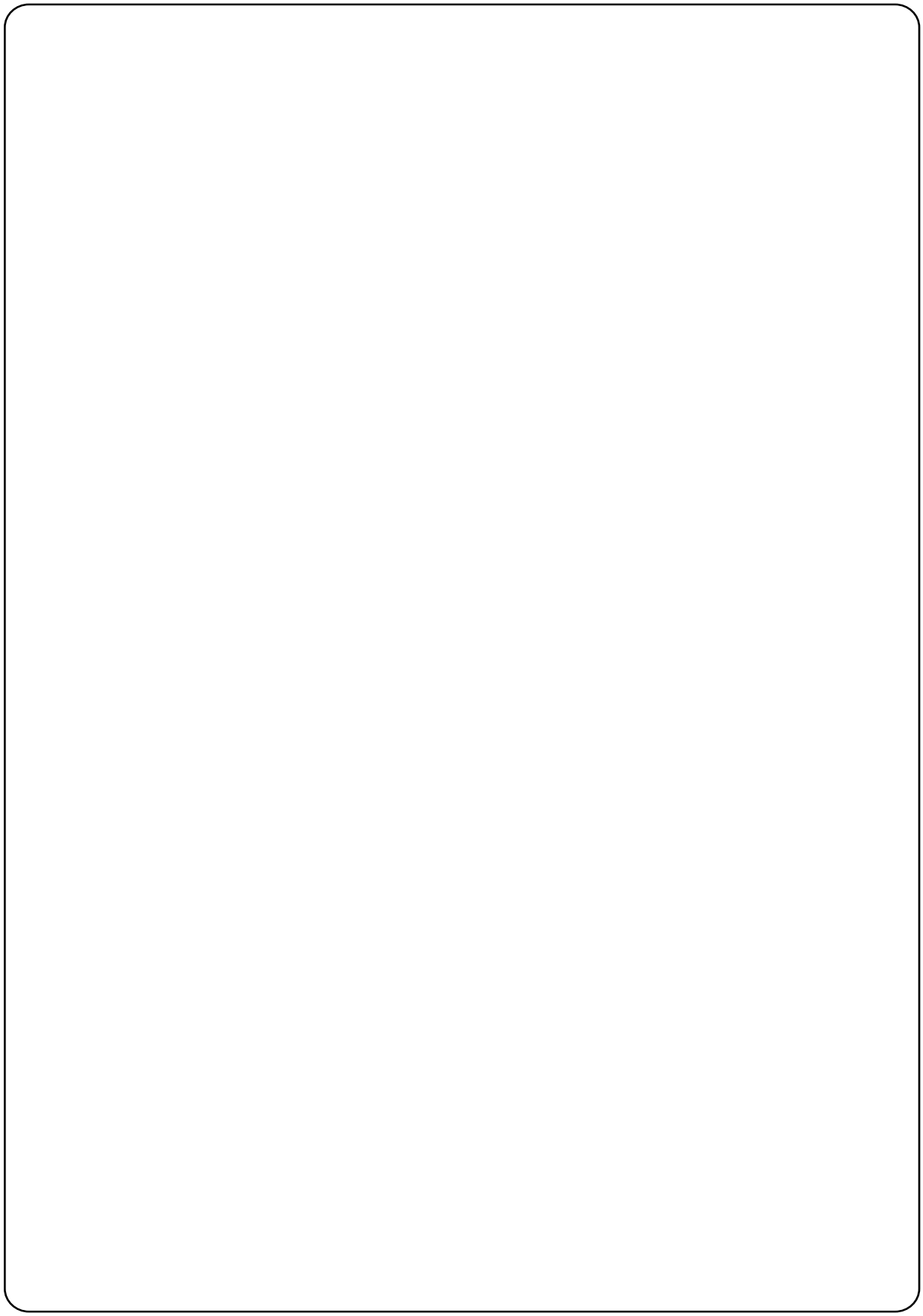
Integer Mat



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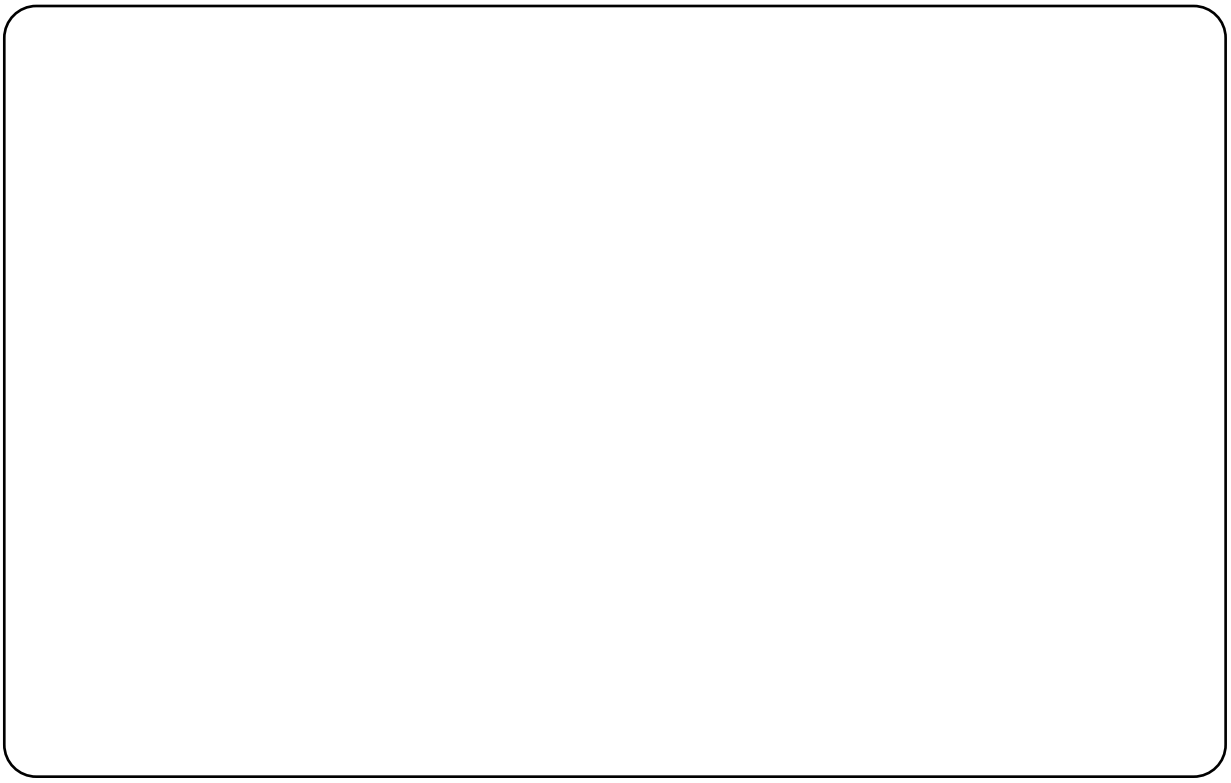
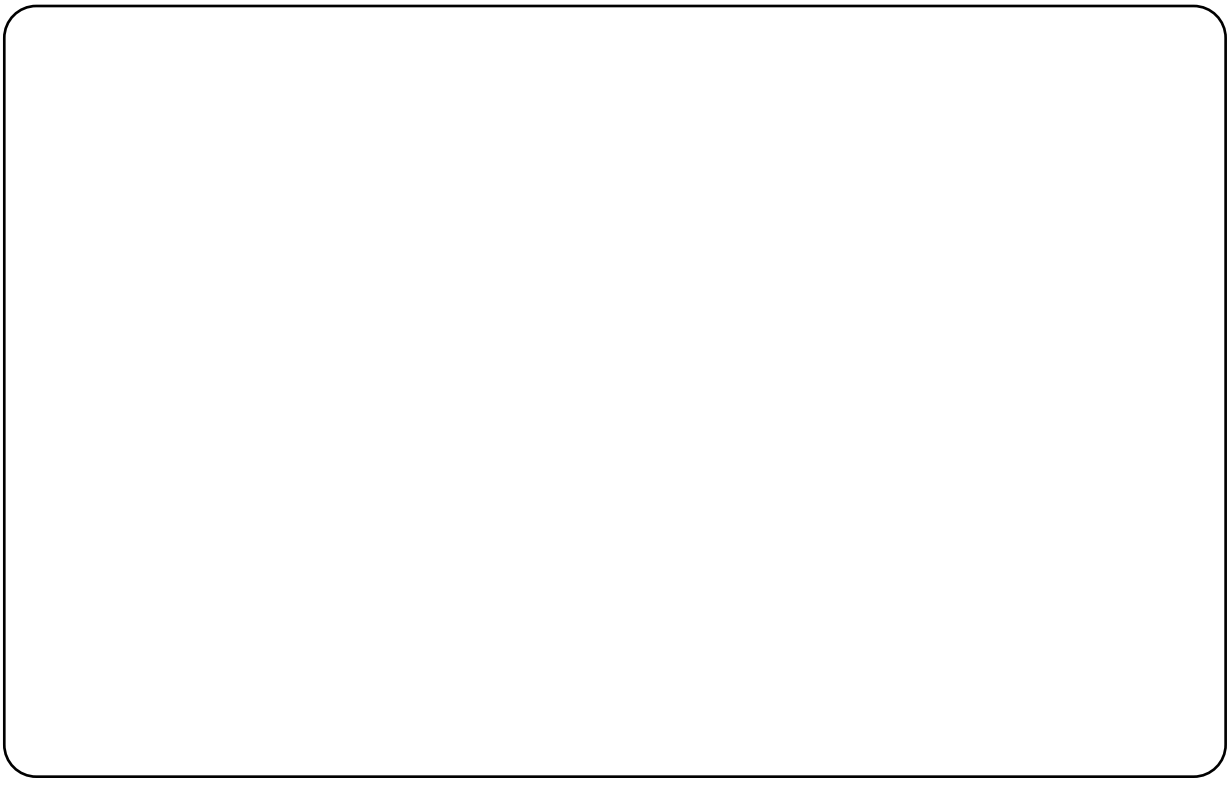
Equation Mat





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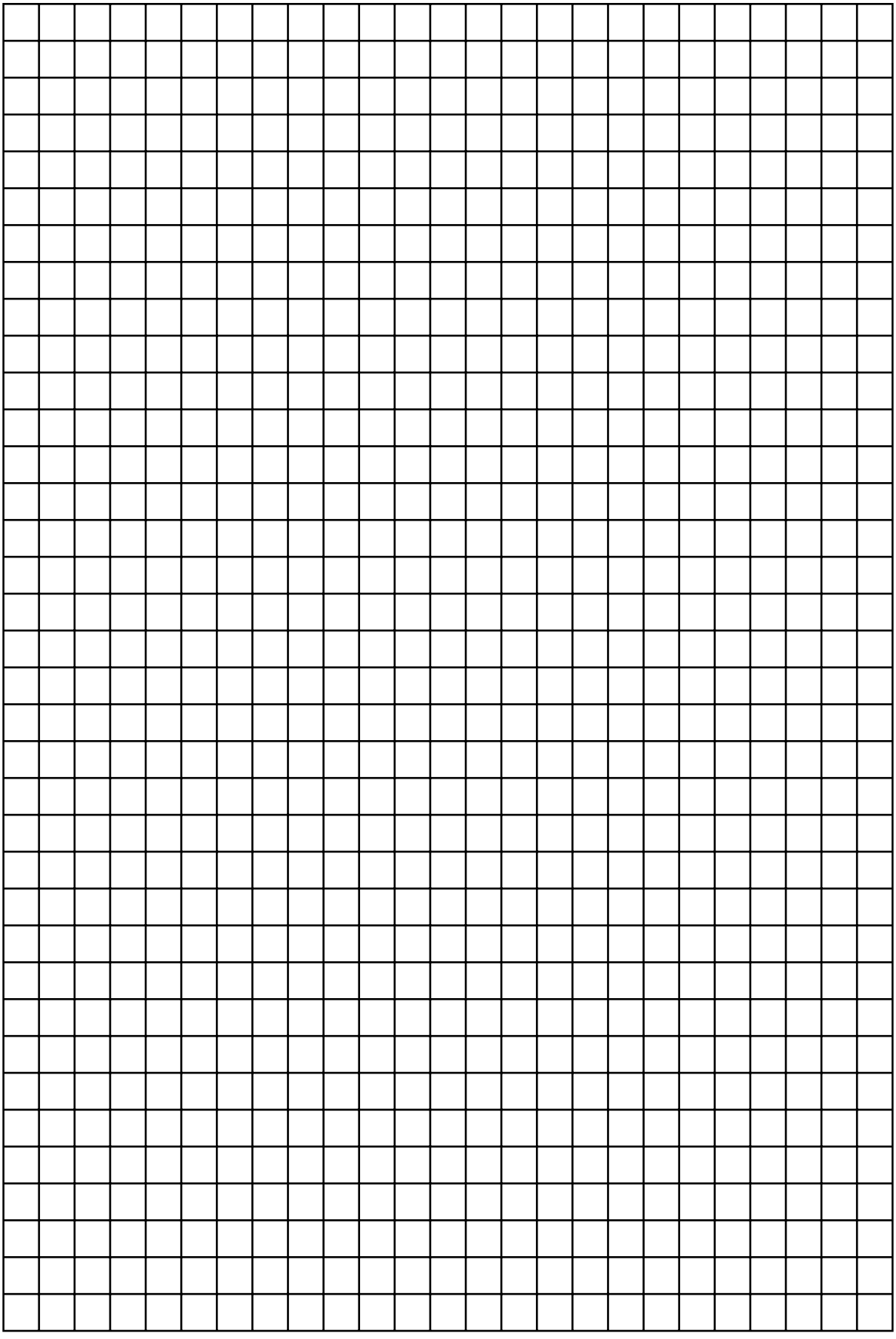
Quarter-Inch Grid



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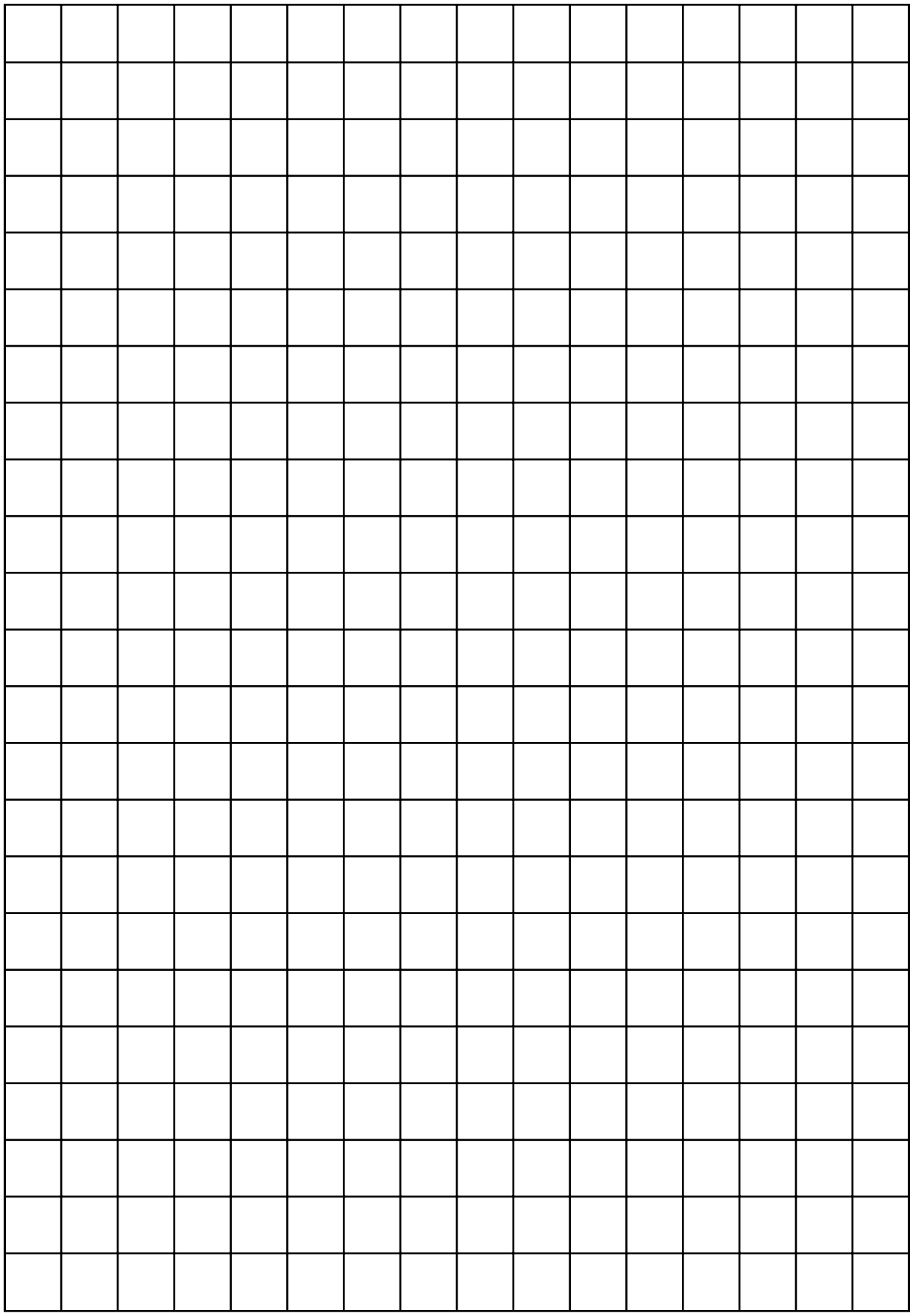
Centimeter Grid



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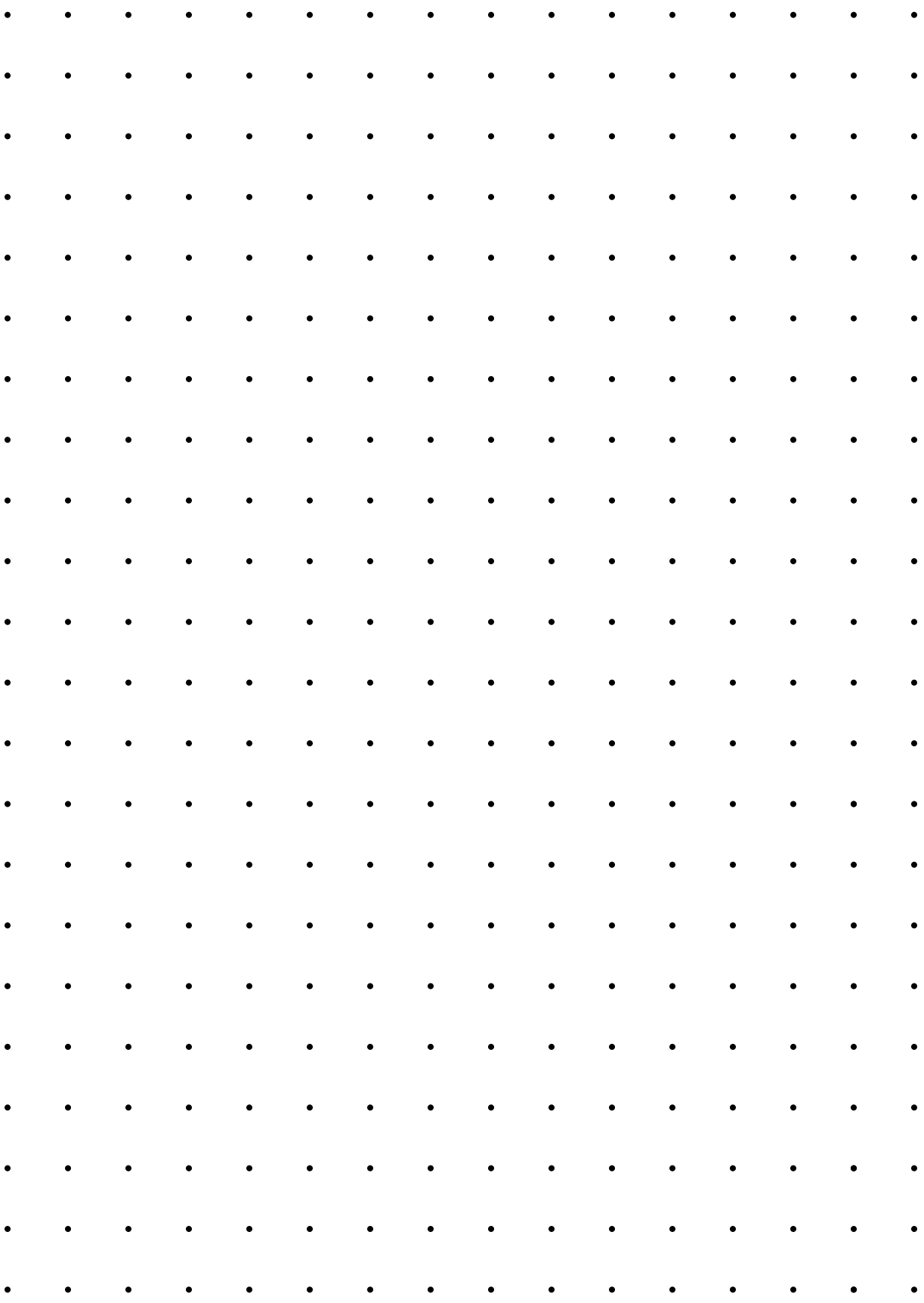
Square Dot Paper



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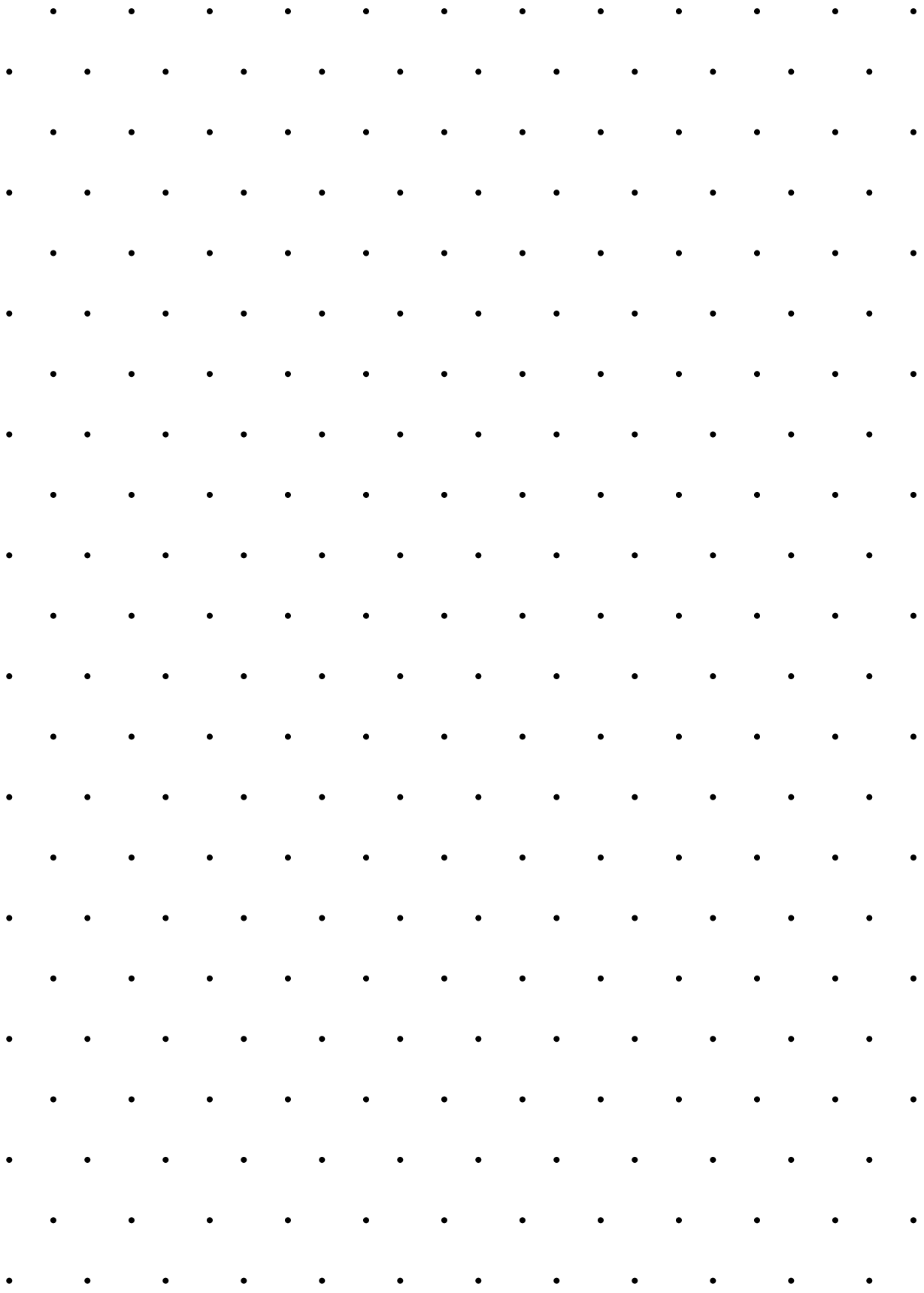
Isometric Dot Paper



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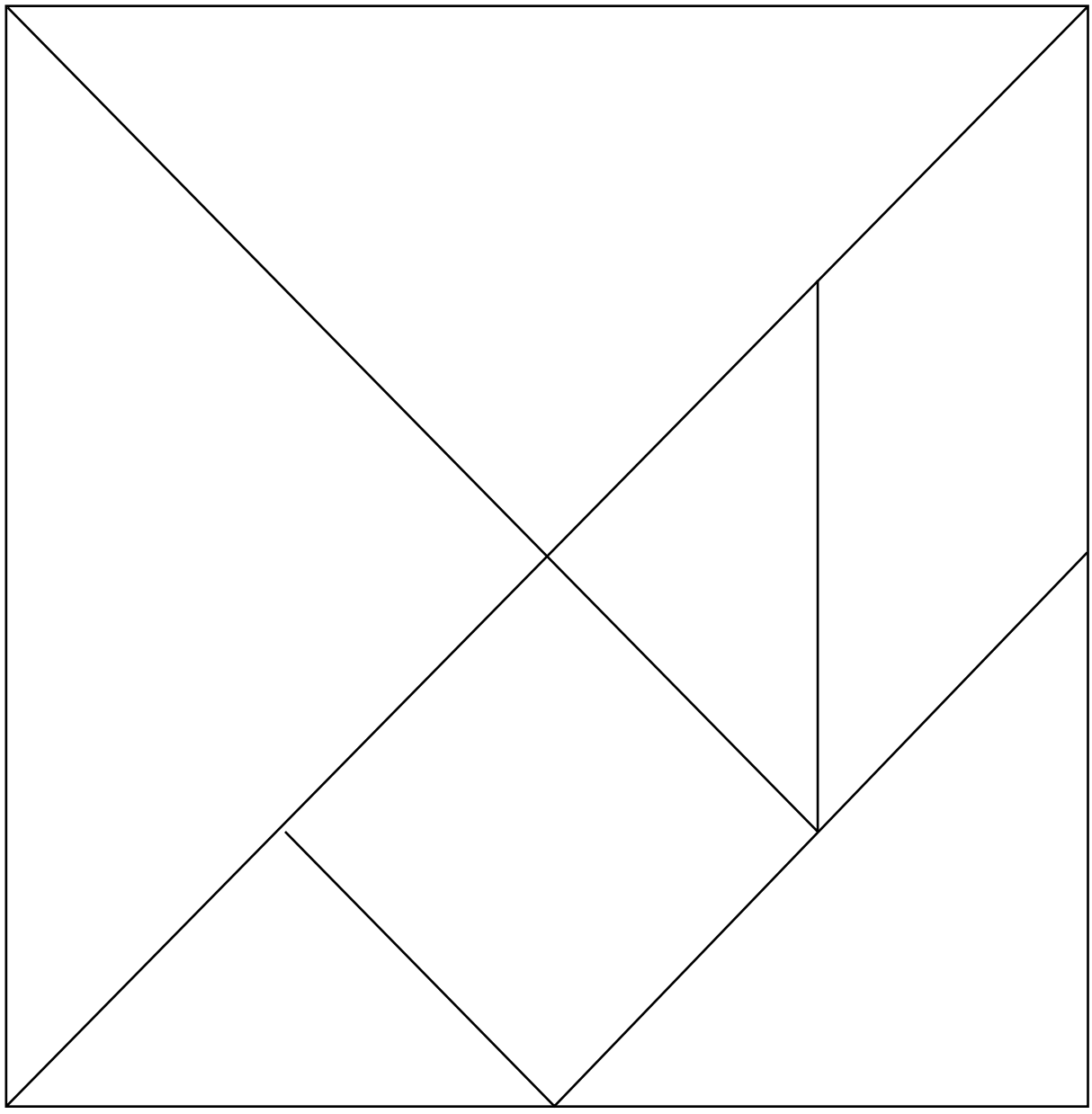
Tangram



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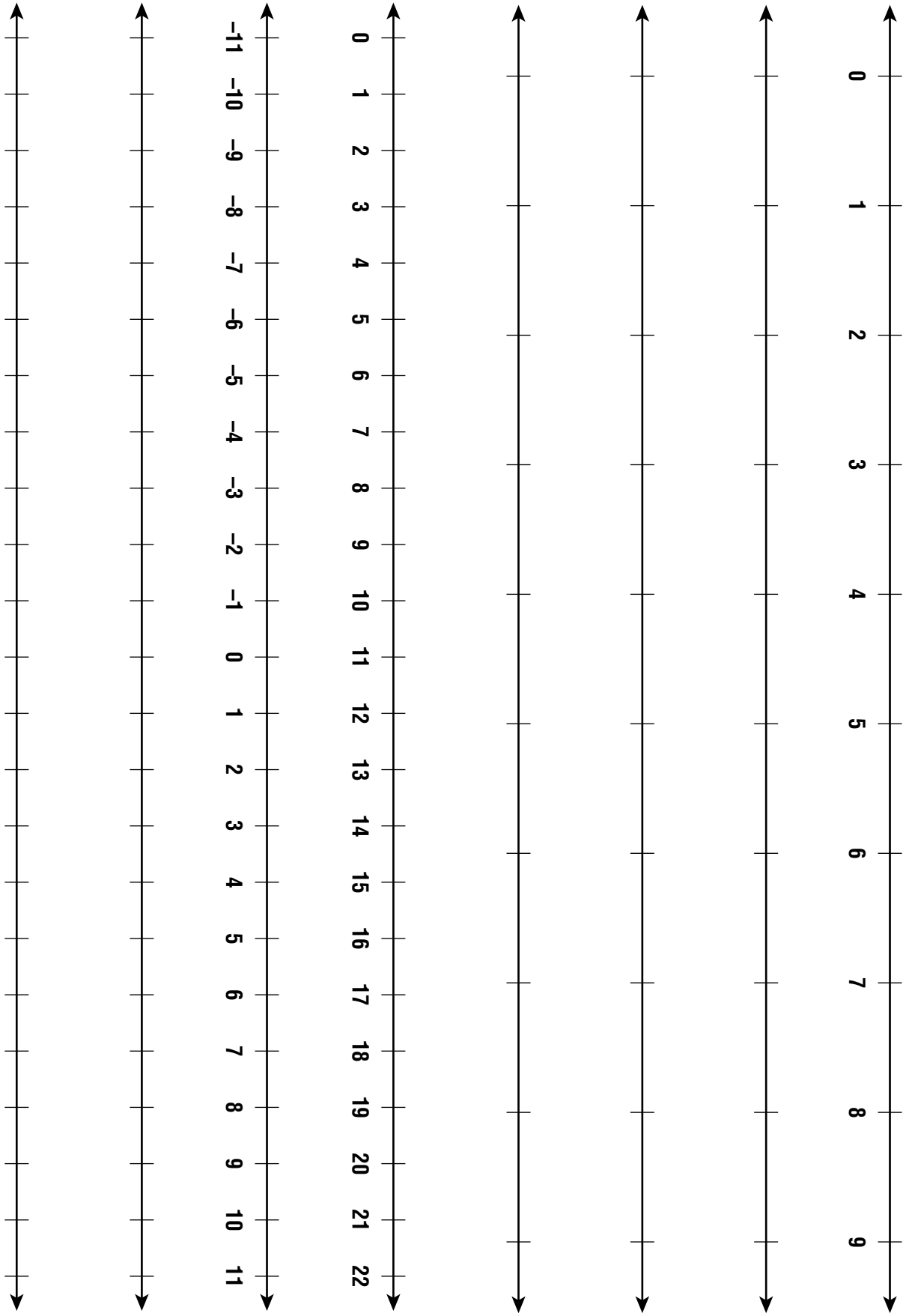
Number Lines



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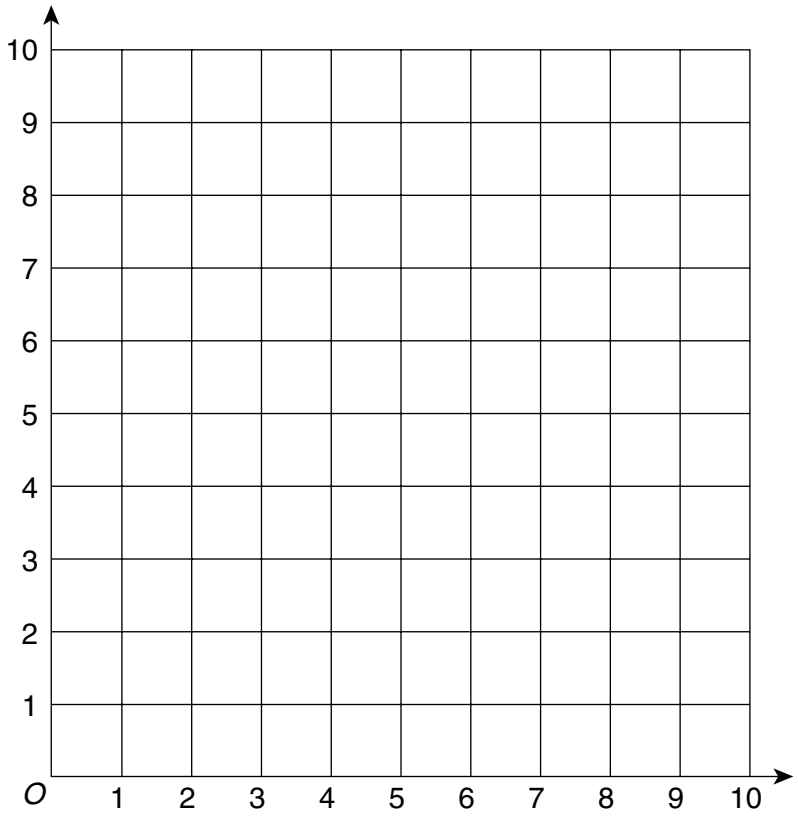
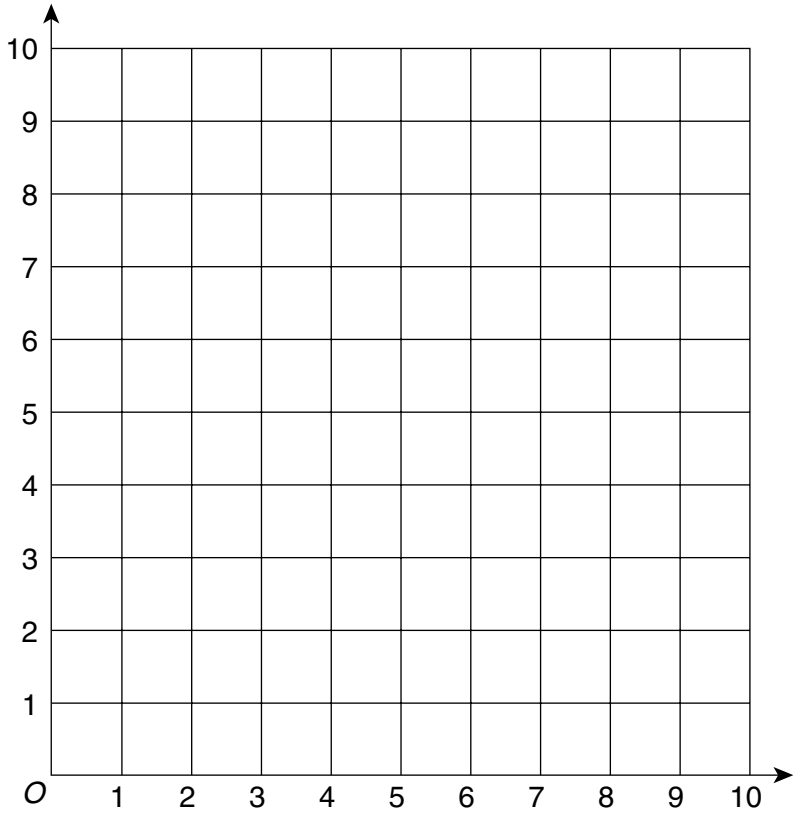
First-Quadrant Grids





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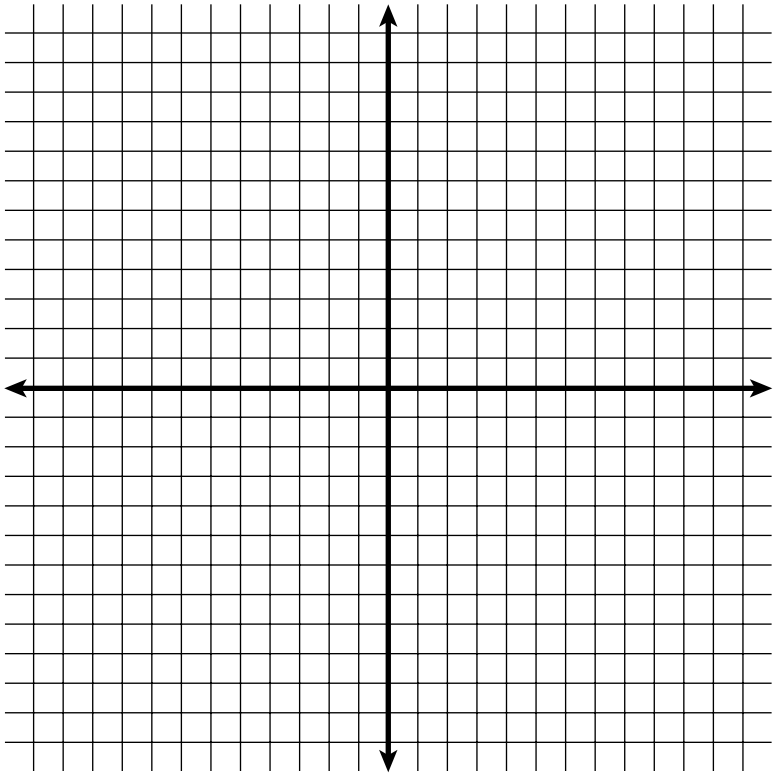
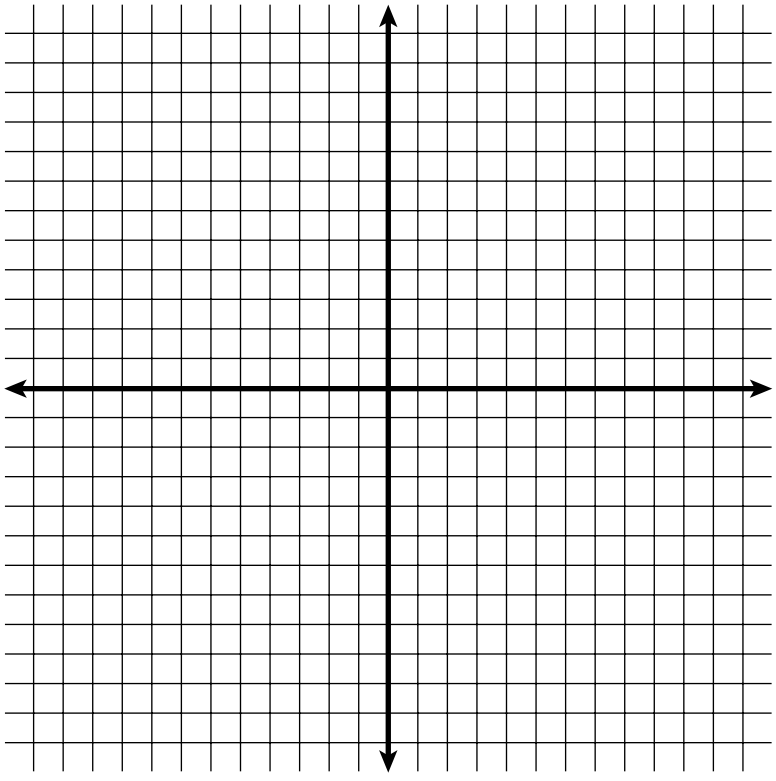
Coordinate Planes





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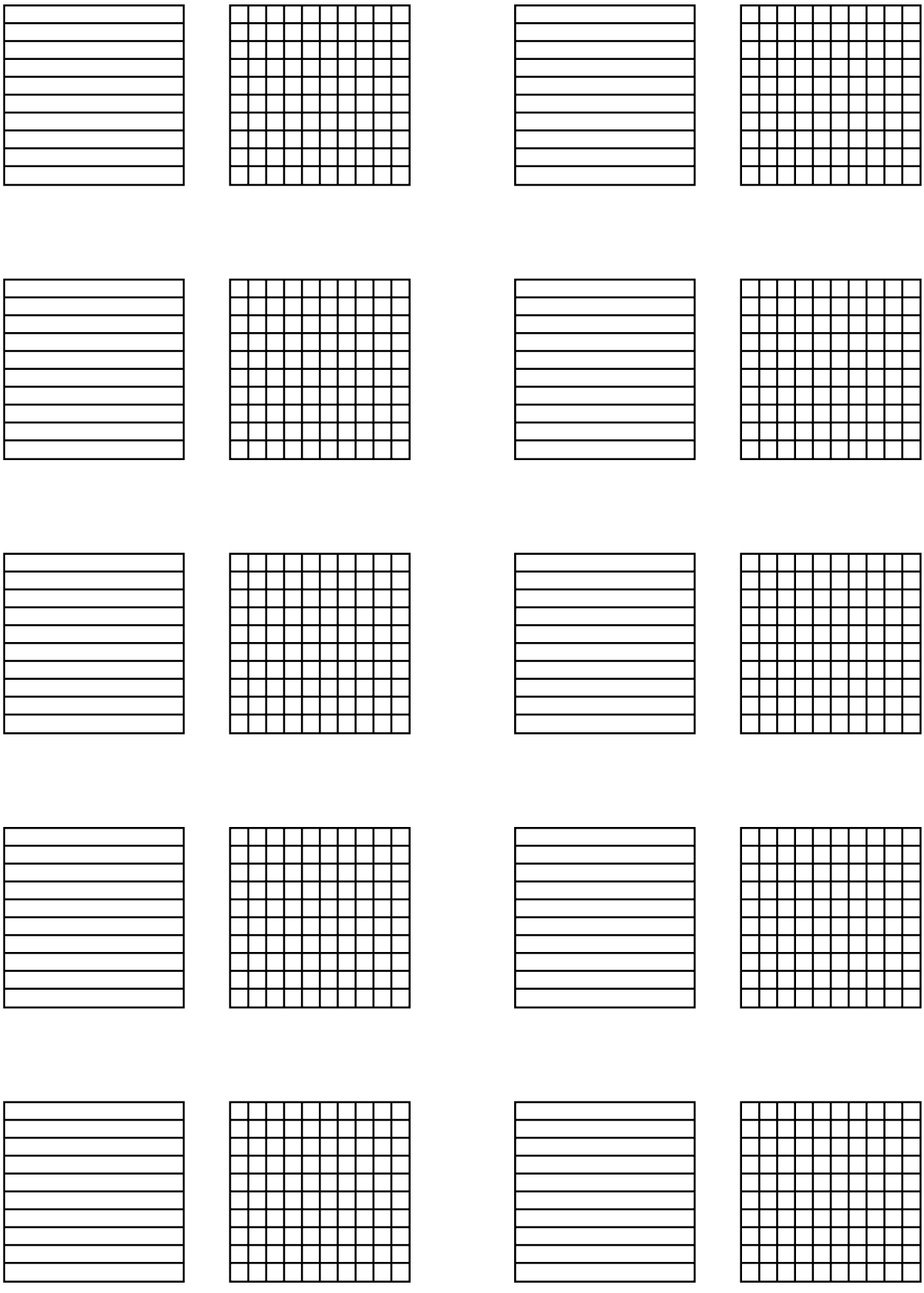
Percent Models



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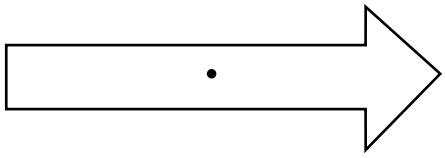
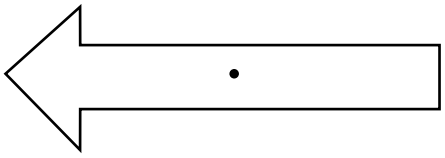
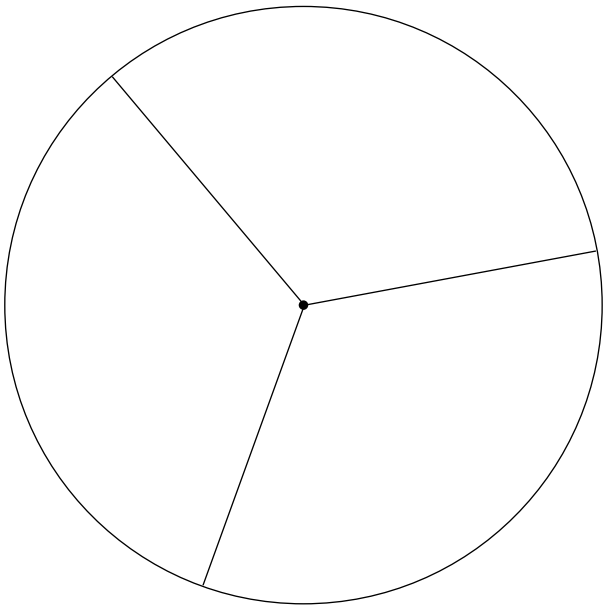
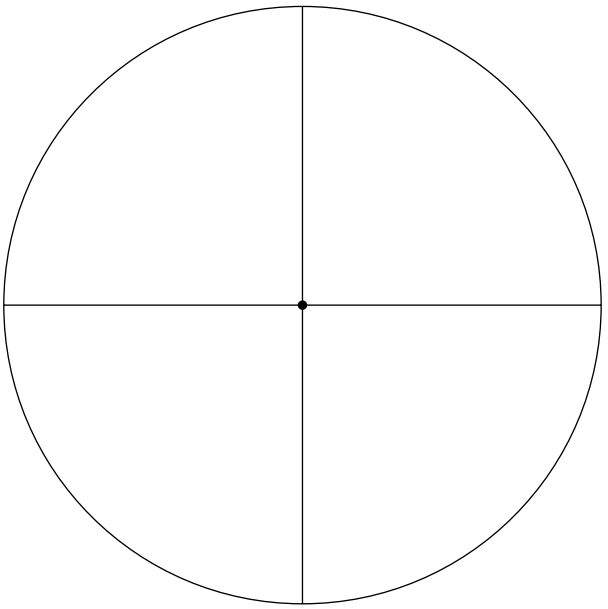
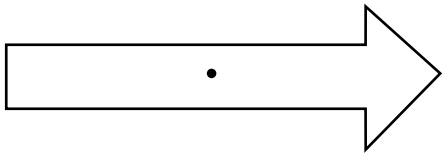
Spinners

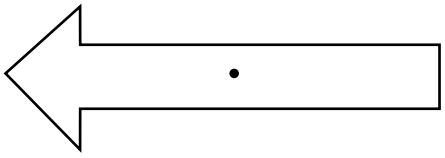


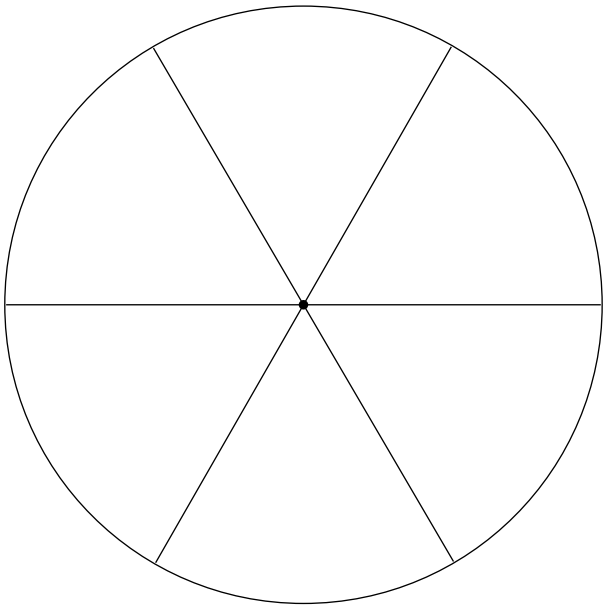


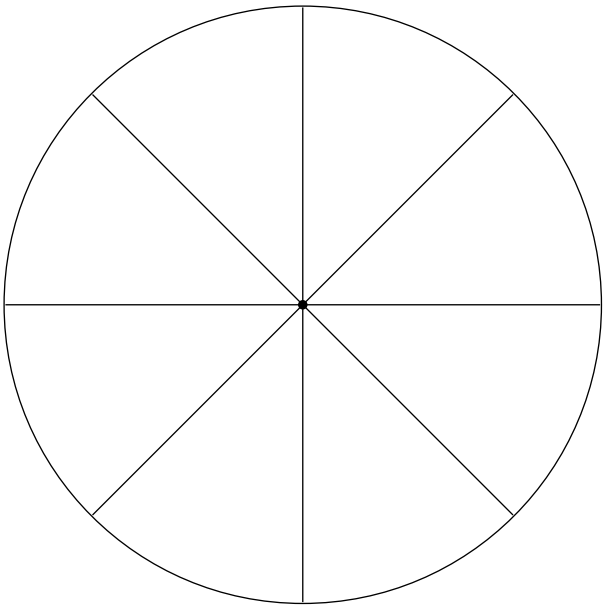












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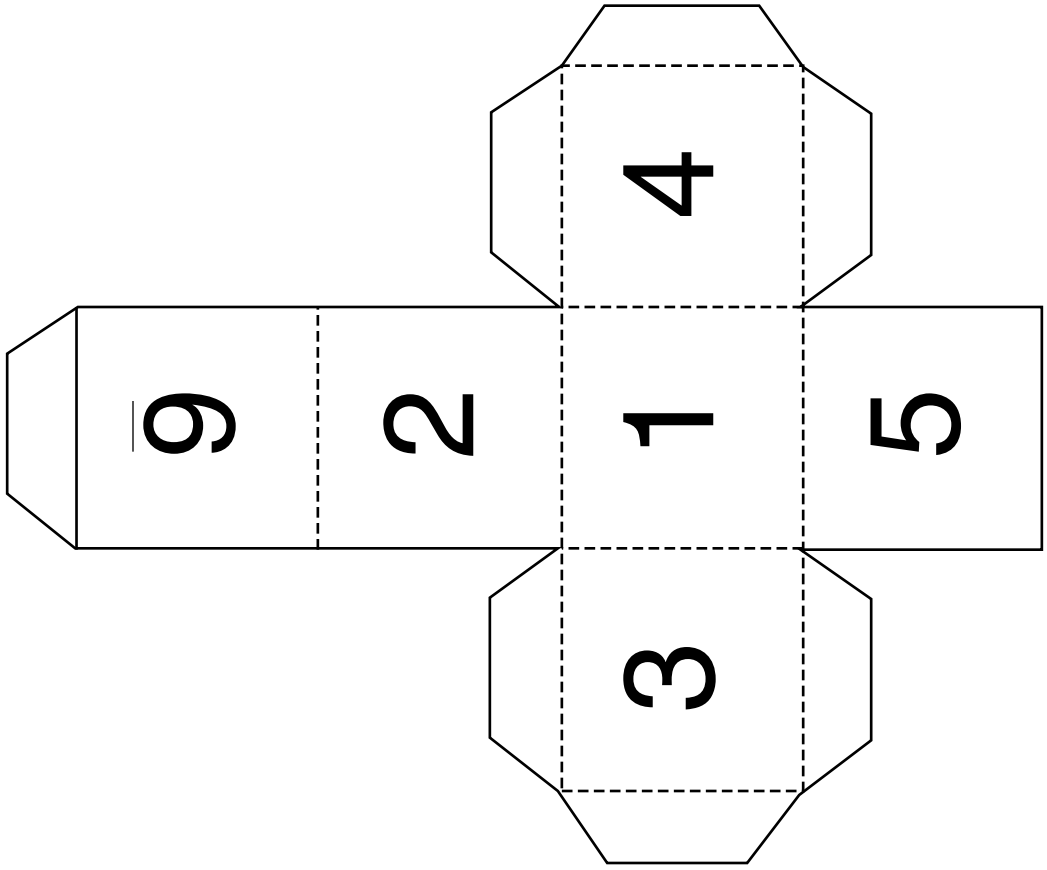
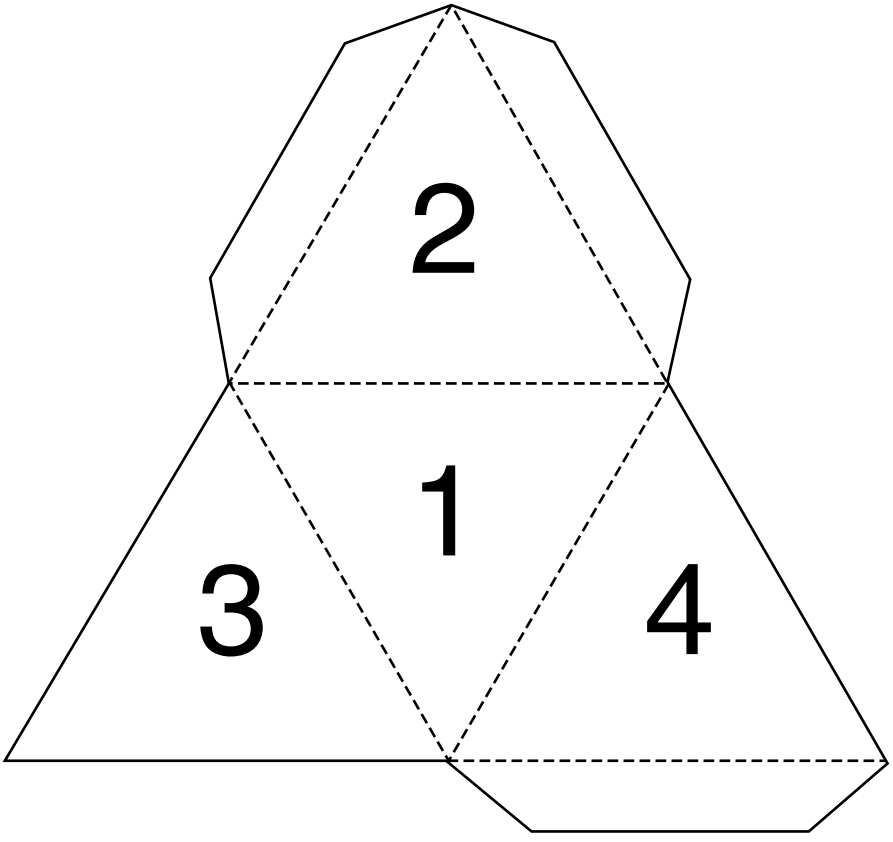
Number Cube Patterns





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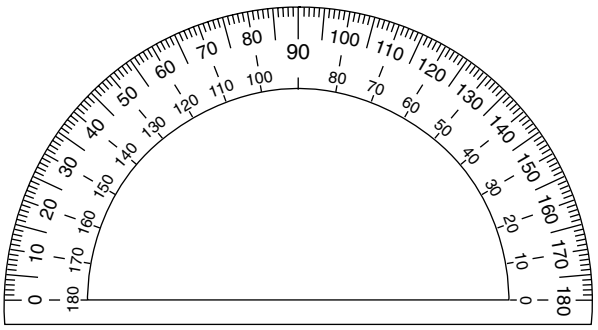
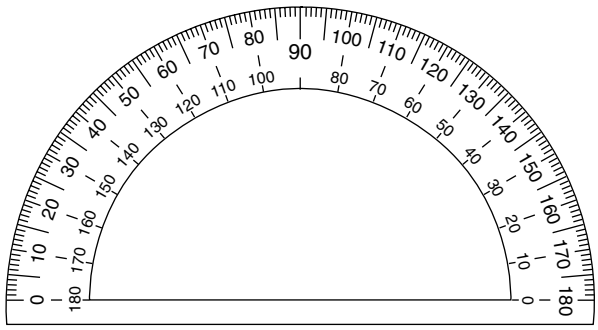
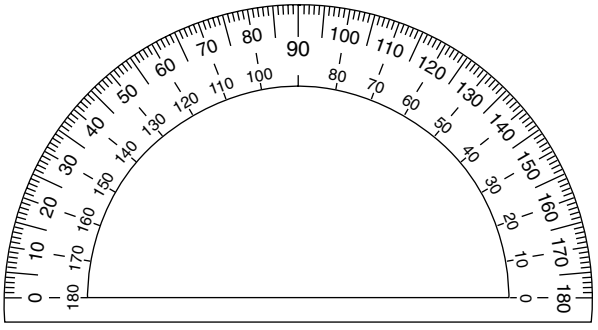
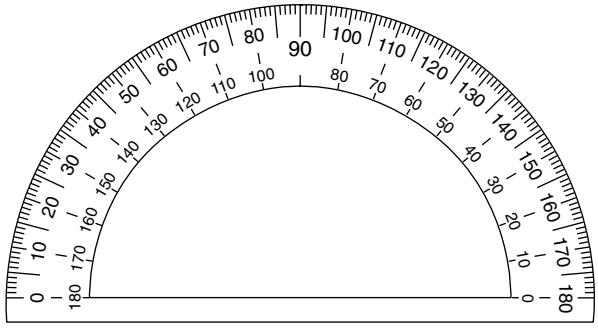
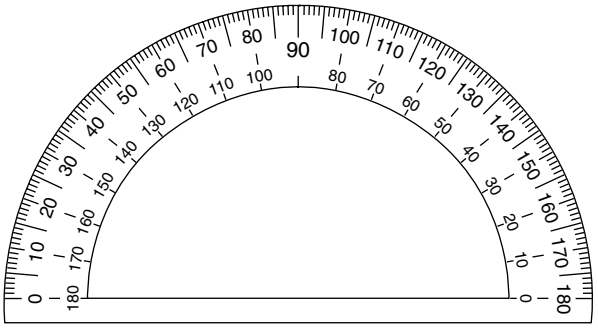
Protractors

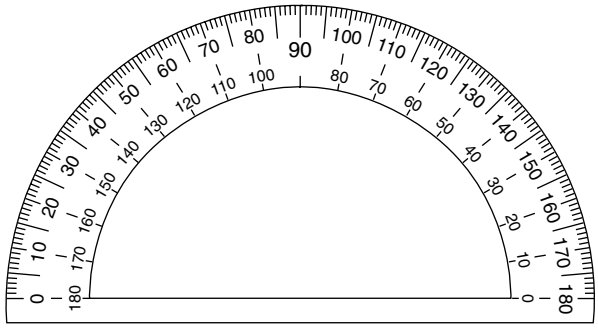












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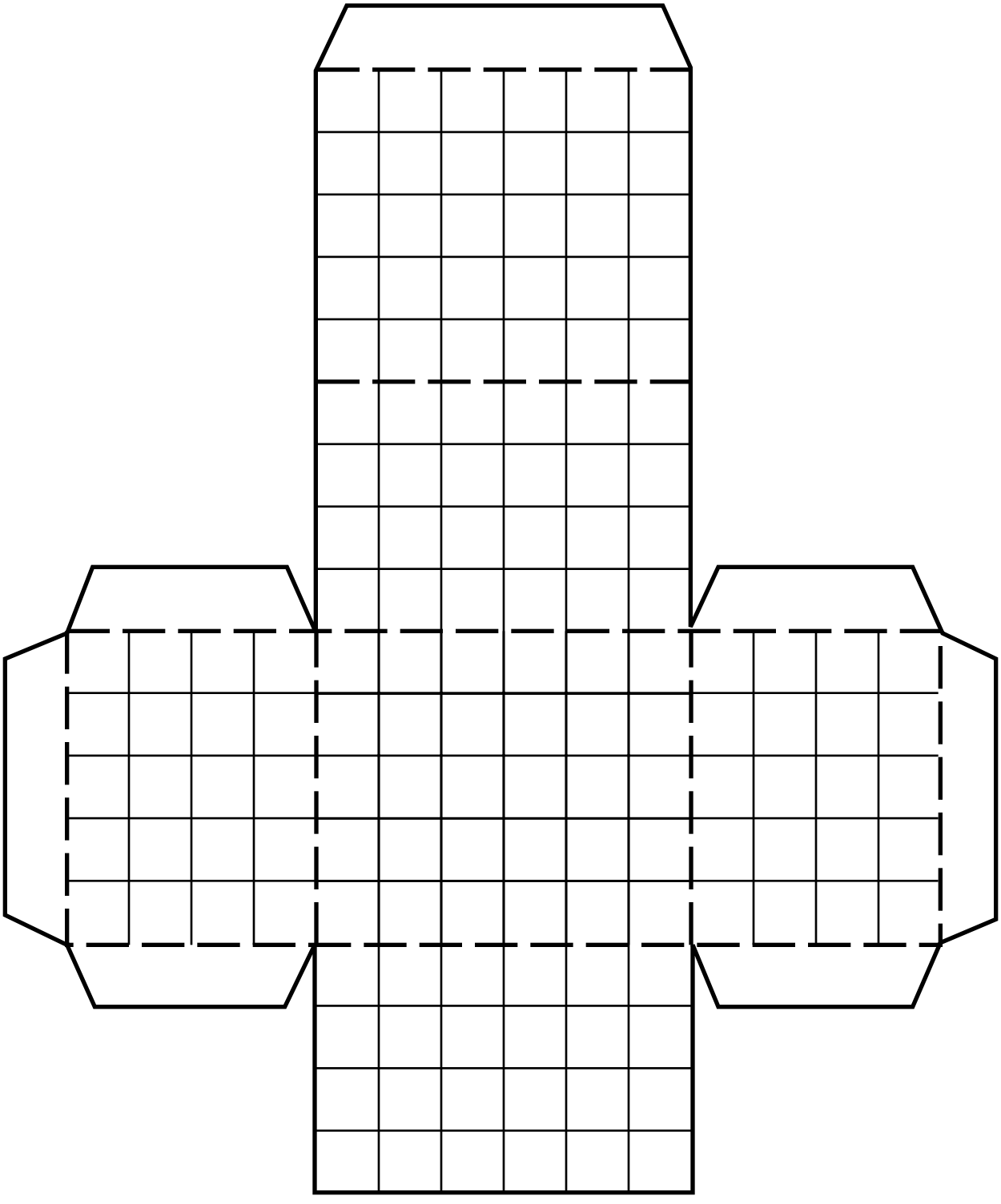
Rectangular Prism Pattern



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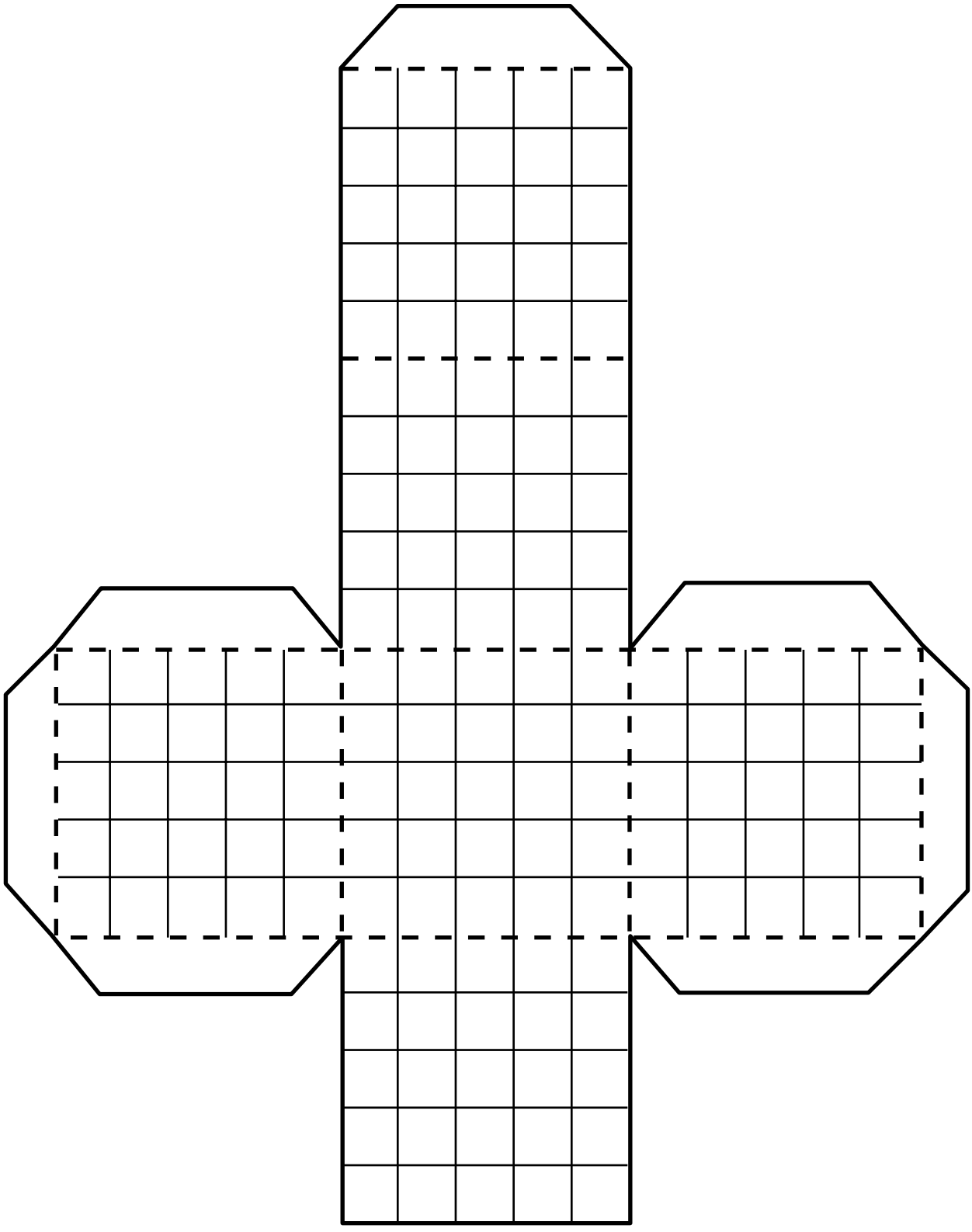
Cube Pattern



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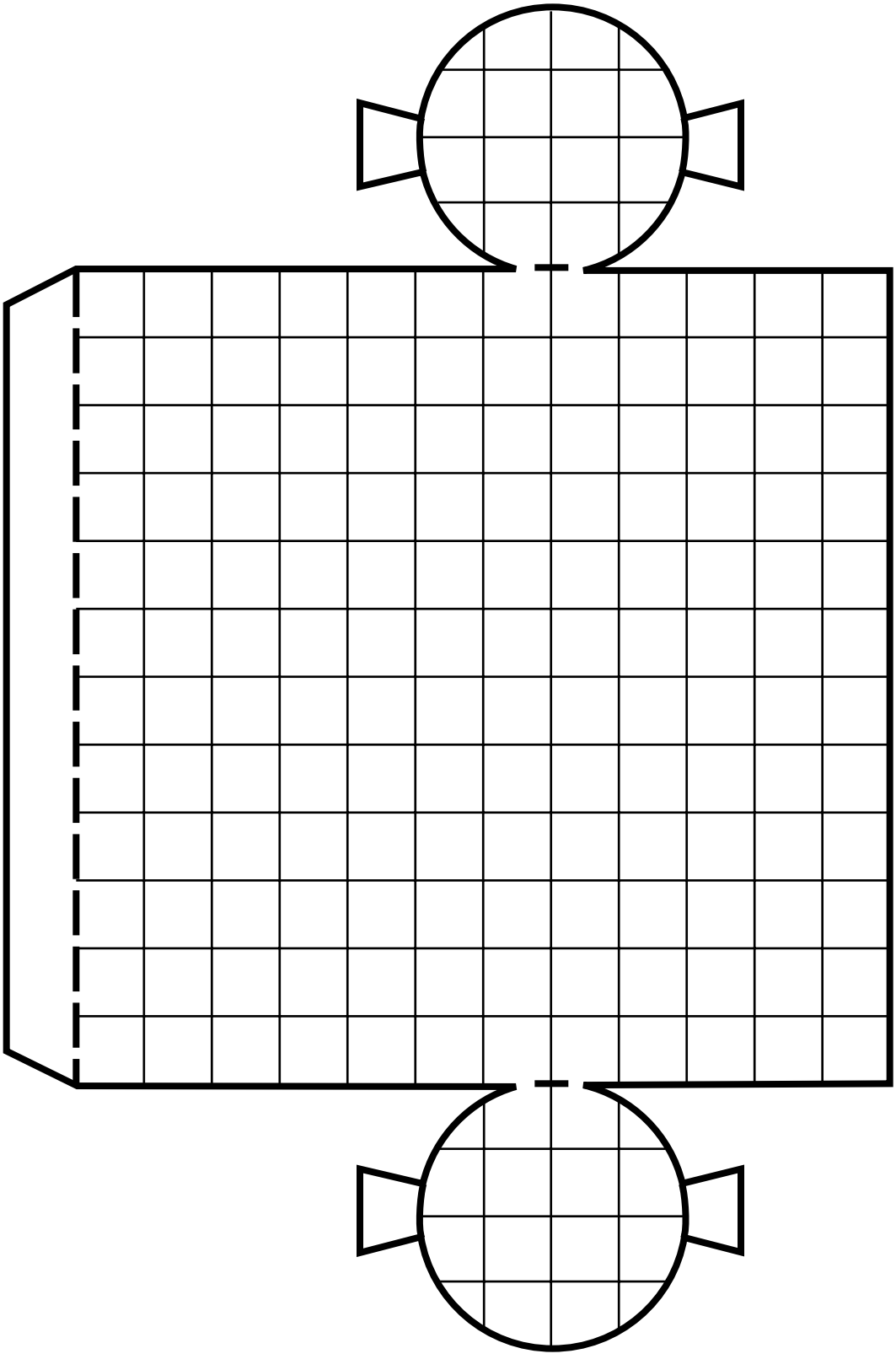
Cylinder Pattern



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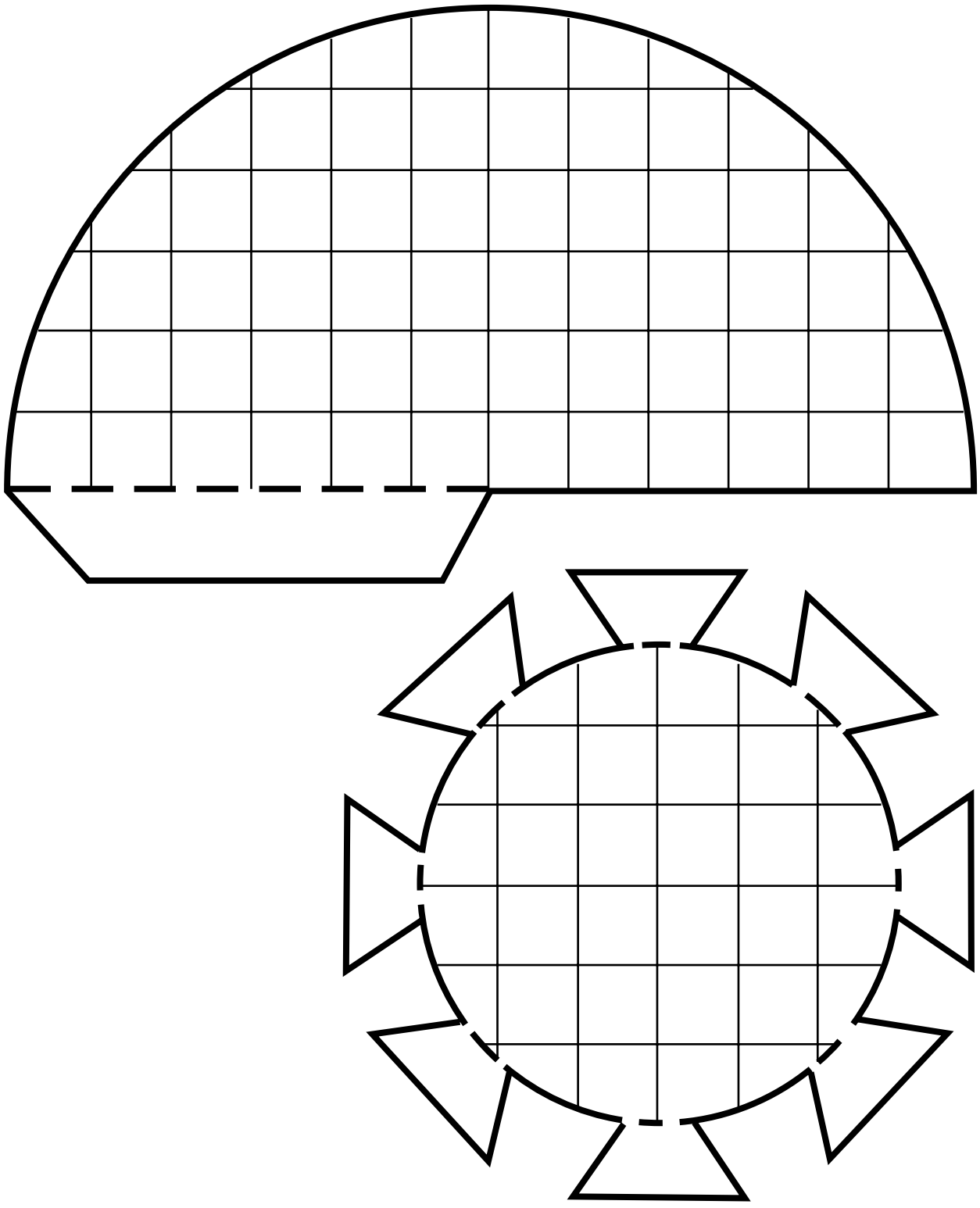
Cone Pattern



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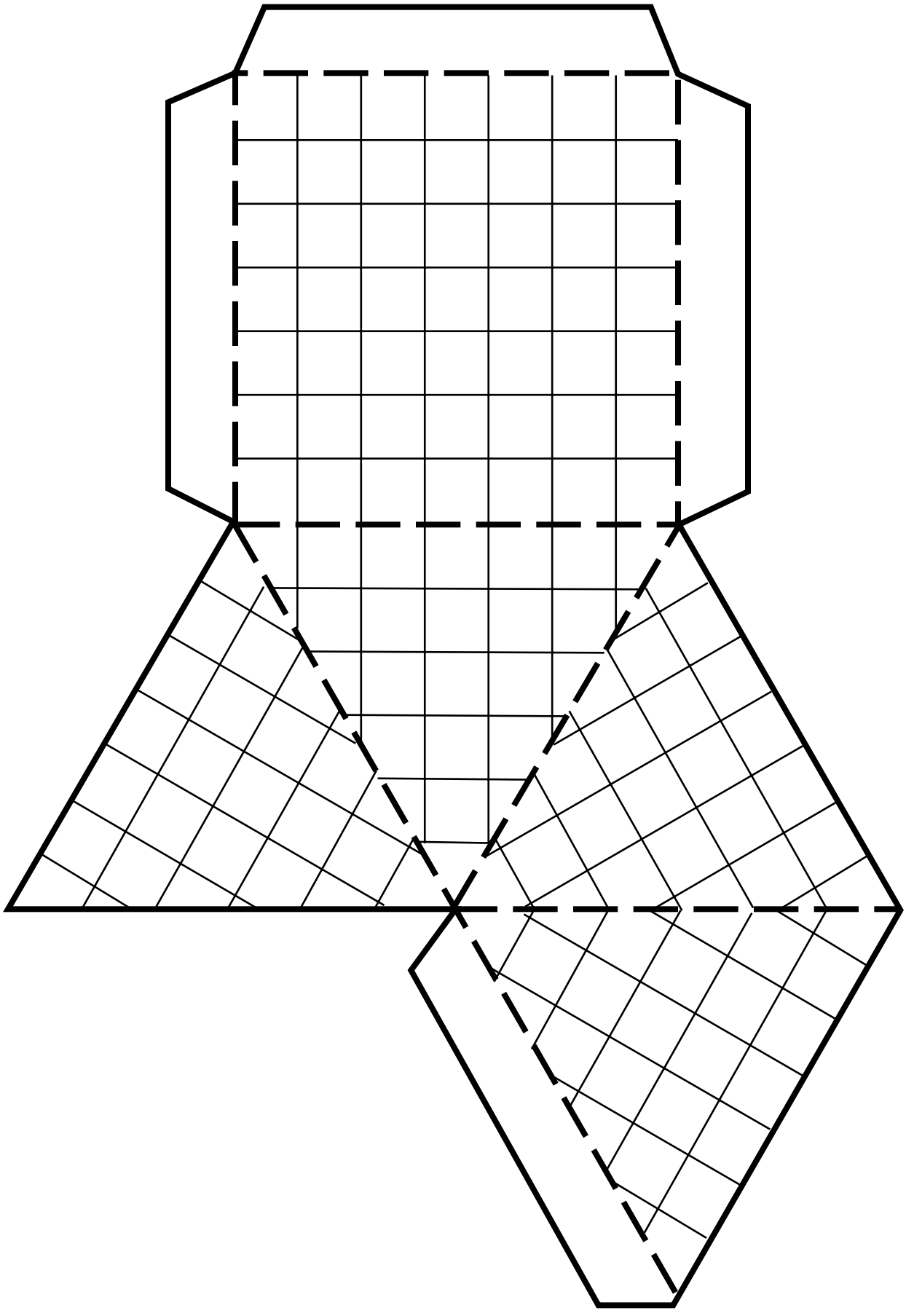
Pyramid Pattern



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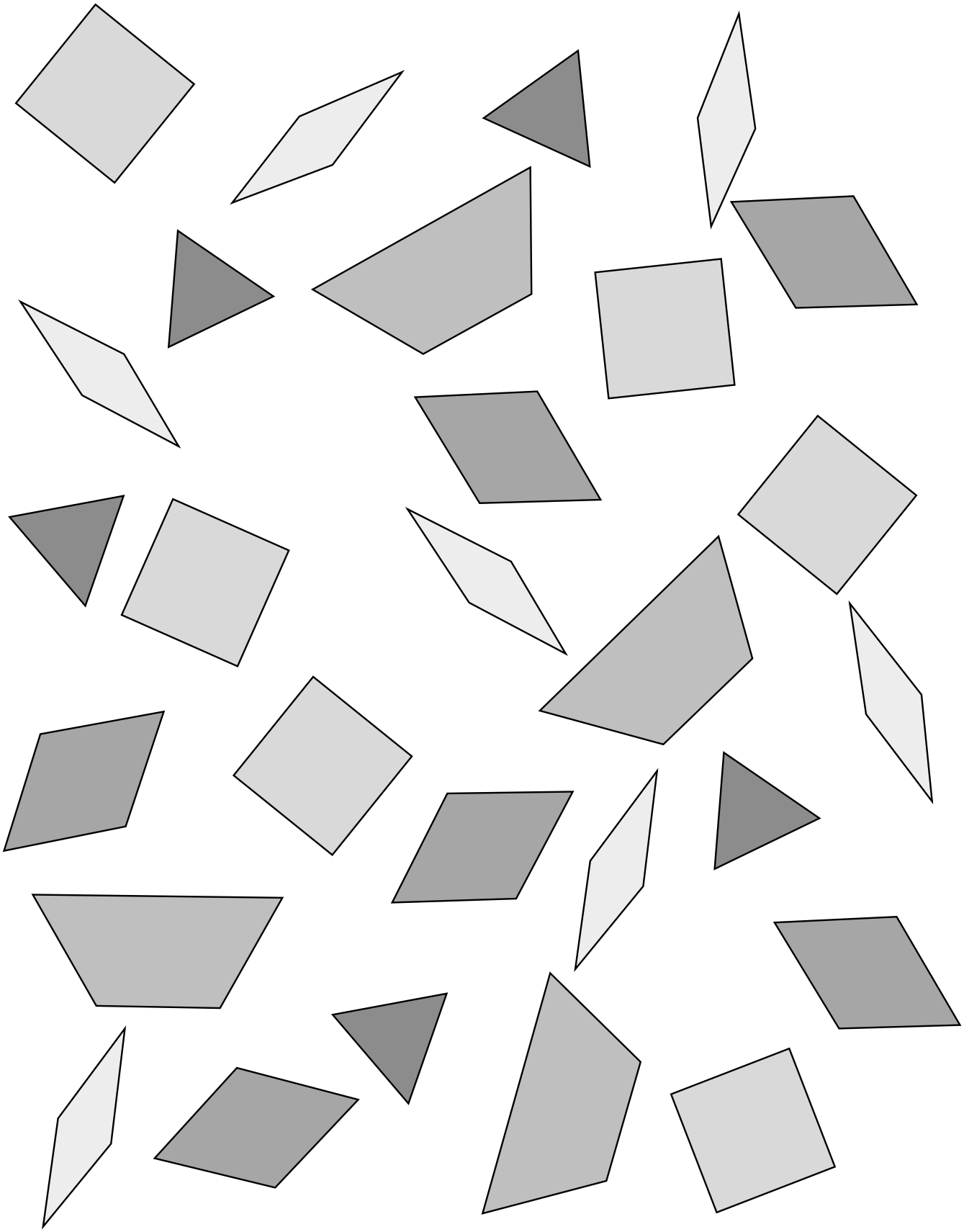
Pattern Blocks



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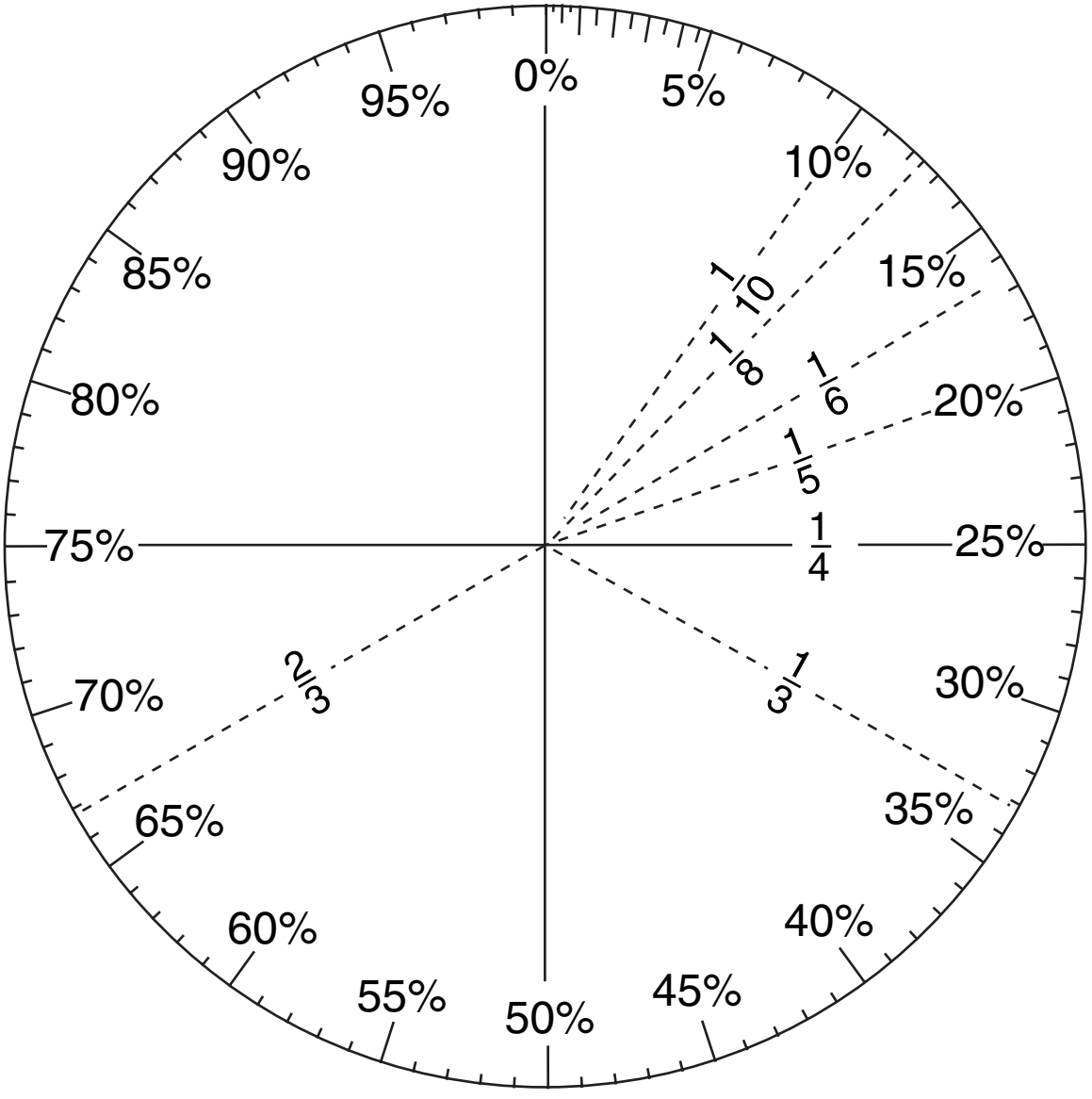
Circle Graph Template



HO-M-CGT

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Problem-Solving Guide

Problem:

Explore

Plan

Solve

Examine

These steps can help you solve problems.

CHAPTER 1

Number Patterns and Algebra
Teaching Notes and Overview

Mini-Project
Divisibility Patterns
(p. 33 of this booklet)

Use With Lesson 1-2.

Objective Apply the rules for divisibility to identify patterns.

Materials
beans
jar

Students use the rules for divisibility to identify divisibility patterns. Students will also count and divide beans to determine all the numbers that will divide a given number.

Answers

1. 1, 2, 3, 6, 9, 18

2. See students' work.

3. 6; $18 \div 1 = 18$; $18 \div 2 = 9$; $18 \div 3 = 6$; $18 \div 6 = 3$; $18 \div 9 = 2$; $18 \div 18 = 1$

4. Sample answer: 15

5. Sample answer: 1, 3, 5, 15

6. See students' work.

7. Sample answers: 4; $15 \div 1 = 15$; $15 \div 3 = 5$; $15 \div 5 = 3$; $15 \div 15 = 1$

Using Overhead Manipulatives
Rectangular Arrays
(pp. 34–35 of this booklet)

Use With Lesson 1-3.

Objective Identify prime and composite numbers by using rectangular arrays.

Materials
algebra tiles*
blank transparency
transparency pens*
* = available in Overhead Manipulative Resources Kit

This demonstration contains two activities.
- Demonstration 1 shows how to identify prime and composite numbers by using rectangular arrays.
- Demonstration 2 asks students how to build a rectangular array and determine whether a given number is prime or composite.
- Students will build rectangular arrays for several numbers independently and tell whether each number is prime or composite.

Answers
Answers appear on the teacher demonstration instructions on pages 34–35.

Using Overhead Manipulatives
Variables and Expressions
(pp. 36–37 of this booklet)

Use With Lesson 1-6.

Objective Model algebraic expressions.

Materials
about 20 counters*
three paper cups
blank transparencies
transparency pen*
* = available in Overhead Manipulative Resources Kit

This demonstration contains four activities.
- Demonstrations 1 and 2 show how to model and find the values of addition expressions.
- Demonstrations 3 and 4 show how to model and find the values of multiplication expressions.

Answers

Answers appear on the teacher demonstration instructions on pages 36–37.

Mini-Project

Solving Equations

(p. 38 of this booklet)

Use With Lesson 1-7.

Objective Model and solve algebraic equations.

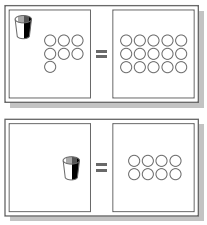
Materials

cups
counters

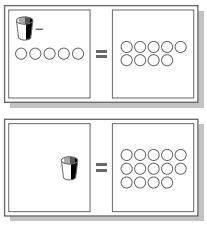
Students use cups and counters to model and solve addition, subtraction, and multiplication equations.

Answers

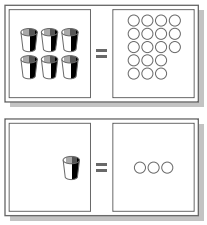
1. 8



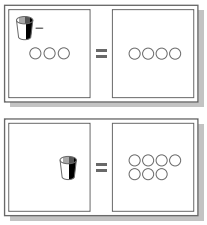
2. 14



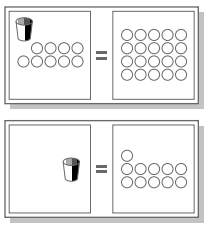
3. 3



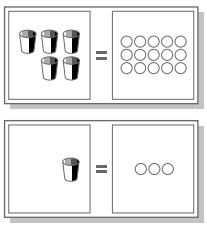
4. 7



5. 11



6. 3



NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 1-2)

Divisibility Patterns

1. List the numbers that divide 18 evenly. Use the rules for divisibility.

_____

2. Take 18 beans. Separate them into groups evenly. The numbers you found in Exercise 1 will work as group numbers. Try to find all of the possible arrangements. Sketch each arrangement you find.

3. How many different numbers divide 18 evenly? _____ List each way.

_____

_____

4. Put your 18 beans into a jar with everyone else's beans. Grab some more beans without counting. When you have the beans you want, count them. How many beans do you have? _____

5. What numbers will divide the number of beans you have evenly? Use your rules of divisibility. _____

6. Separate your beans into groups evenly. Find all of the arrangements you can. Sketch each one.

7. How many different ways can you divide your beans? _____

List the numbers represented by each grouping. _____

_____

Using Overhead Manipulatives

(Use with Lesson 1-3)

Rectangular Arrays

Objective Identify prime and composite numbers by using rectangular arrays.

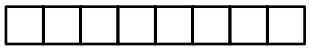
Materials

- algebra tiles*
- blank transparency
- transparency pens*

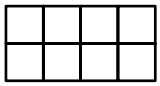
* = available in Overhead Manipulative Resources Kit

Teacher Demonstration for Activity 1

- Tell students that a *composite* number has more than two factors. For example, 4 is a composite number because it has factors of 1, 2, and 4. Place 4 small algebra tiles on the screen. Show that there is more than one way to make a rectangle (2-by-2 and 1-by-4 or 4-by-1).
- Tell students that a *prime* number has exactly two factors. Point out that these factors are the number and 1. Place 3 tiles on the screen. Show that there is only one way to make a rectangle (3-by-1 or 1-by-3). Tell them that this shows that 3 is prime; its only factors are 1 and 3.
- Tell students that you are going to find out whether 8 is a prime or a composite number. Place 8 tiles on the screen to form a 1-by-8 rectangle. Record the arrangement on the blank transparency. Ask students to suggest any other arrangements. **(2-by-4 rectangle)** If students suggest 8-by-1, point out that it is the same shape as 1-by-8, only rotated.



1×8



2×4

- Tell students that these arrangements show that the factors of 8 are 1, 2, 4, and 8. Point out that since 8 has more than two factors, it is a composite number.

Teacher Demonstration for Activity 2

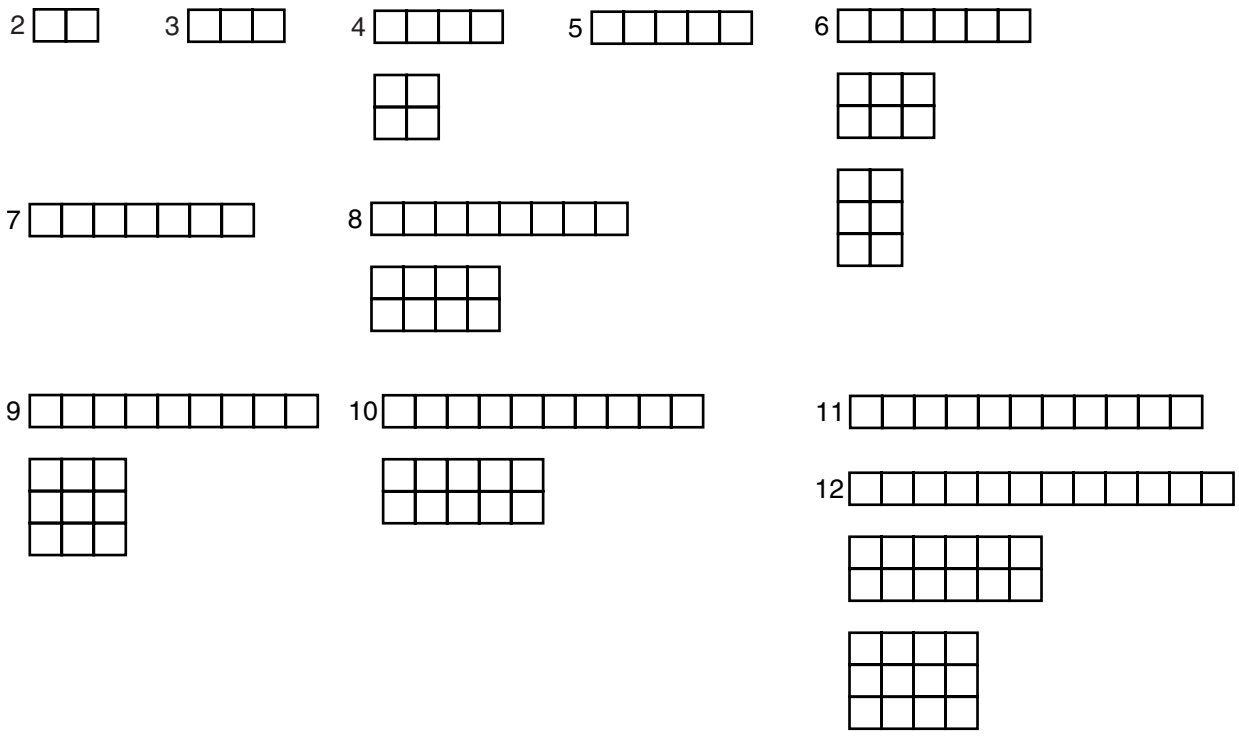
- Tell students that you want to determine whether 13 is a prime number or a composite number.
- Place 13 squares on the screen. Ask students how you can arrange them to make a rectangle. **(1-by-13)** Build the rectangle. Record the arrangement on the transparency.

1×13

- Point out that the drawing shows that the factors of 13 are 1 and 13. Since 13 has exactly two factors, it is a prime number.

Have students complete Exercises 1–6 below. If tiles are not available for all students, they can draw squares on their paper to make the arrangements.

1. Repeat the process with areas of 2 square units through 12 square units.



2. Which numbers of square tiles had only one arrangement? **(2, 3, 5, 7, 11)**

3. Which numbers of square tiles had more than one arrangement? **(4, 6, 8, 9, 10, 12)**

4. Is there a relationship between a multiplication table and the number of arrangements you have? If so, describe it. **(The greater the number of arrangements, the more times the number appears on the multiplication table.)**

5. Make a guess about which numbers between 12 and 25 square units can have more than one rectangular shape. Explain why you selected those numbers. **(12, 14, 15, 16, 18, 20, 21, 22, 24; these numbers have several factors.)**

6. Write a sentence describing the characteristics of prime and composite numbers. **(Sample answer: A prime number has exactly 2 factors, 1 and the number itself. A composite number has more than 2 factors.)**

Using Overhead Manipulatives

(Use with Lesson 1-6)

Variables and Expressions

Objective Model algebraic expressions.

Materials

- about 20 counters*
- three paper cups
- blank transparencies
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration for Activities 1 and 2

- Before turning on the projector, place 4 counters on the left side of the screen. Place 5 counters inside a paper cup and place it on the right side. Then turn on the projector.

- Tell students that this model represents the phrase *the sum of four and some number*. Write the phrase on a blank transparency. Tell them that the *some number* is an unknown value, and that when you assign a value to "some number," then you can find the value of the expression.
- Ask students how many counters they can see. **(4)** Tell them that 4 represents the known value and that the cup can contain any number of counters.
- Tell students that you have assigned a value to "some number" by placing some counters in the cup.
- Empty the cup onto the screen and count the counters that were in the cup. Tell students that by adding this to the 4 other counters, you can find the value of the expression.
- Ask the following questions. In the expression, what is the "some number?" **(the unknown value)** In the expression *the sum of four and some number*, when we replace "some number" with its known value, what is the value of the expression? **(9)**
- Repeat using different numbers of counters in the cup.

- Clear the screen. Ask students how to model the expression *some number plus six*. **(Place a cup and 6 counters on the screen.)** Write the expression on a transparency. Ask, "What is the value of this expression if "some number" is 9?" **(15)** You may want to place 9 counters in the cup, empty the cup, and find the total number of counters.
- Write $a + 8$ on a transparency. Ask, "What is the value of $a + 8$ if a equals 14?" (**22)**

Teacher Demonstration for Activities 3 and 4

- Place 4 counters in each of 3 cups. Place the cups on the screen.

- Tell students that each cup has the same number of counters in it and that this model represents the phrase *3 times some number*.
- Ask students how many cups they see. **(3)** Tell them that 3 represents the known value and that each cup contains an unknown number of counters. Empty the cups to show all the counters. Ask students how many counters there are in all. **(12)**
- Ask the following questions. What is the unknown value? **(4)** What is the product of 3 and some number in this case? **(12)**
- Repeat using a different number of counters under the cups.
- Ask, "How many cups would you need to model the expression *six times a number*?" **(6 cups)**
- Write $5 \times x$ on a blank transparency. Ask, "What is the value of $5 \times x$ if x equals 4?" **(20)**

NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 1-7)

Solving Equations

Materials

cups, counters

Model each equation using cups and counters. Let a cup represent the variable and represent the numbers. Use the guess-and-check strategy to solve each equation.

1. $c + 7 = 15$

2. $n - 5 = 9$

3. $6s = 18$

4. $d - 3 = 4$

5. $9 + e = 20$

6. $5y = 15$

Statistics and Graphs
Teaching Notes and Overview

Mini-Project
Bar Graphs and Line Graphs
(p. 40 of this booklet)

Use With Lesson 2-2.

Objective Collect data to create bar graphs and line graphs.

Materials
paper

Students survey their classmates to find the number of people living in each person's household. They create a bar graph to display their data. Students keep track of the amount of time they spend doing a certain activity each day. They create a line graph to display this data.

Answers
Answers will vary. See students' data and graphs.

Using Overhead Manipulatives
Box-and-Whisker Plots
(pp. 41–42 of this booklet)

Use With Lesson 2-7b.

Objective Display and summarize data in a box-and-whisker plot.

Materials
yellow counters
string
tape
blank transparencies
transparency pens*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows students how to display a set of data in a box-and-whisker plot.
- Students will learn and identify all parts of the box-and-whisker plot.
- Students will then create a box-and-whisker plot independently and use it to summarize the data.

Answers
Answers appear on the teacher demonstration instructions on pages 41–42.

NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 2-2)

Bar Graphs and Line Graphs

Survey your classmates. Ask: "Including yourself, how many persons live in your household?"

Number in Household	Tally	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		

Make a bar graph of the frequency table on a separate sheet of paper.

Keep track of the number of minutes you spend doing some activity each day for one week. Then make a line graph of the data on a separate sheet of paper.

Some examples that you may want to choose include the following: doing homework, reading for pleasure, watching TV, practicing music, exercising, talking on the phone, playing video games, working on a hobby.

Day of the Week	Time Spent (minutes)
Sun.	_____
Mon.	_____
Tues.	_____
Wed.	_____
Thurs.	_____
Fri.	_____
Sat.	_____

Using Overhead Manipulatives

(Use with Lesson 2-7b)

Box-and-Whisker Plots

Objective Display and summarize data in a box-and-whisker plot.

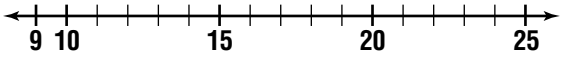
Materials

- yellow counters
- string
- tape
- blank transparencies
- transparency pens*

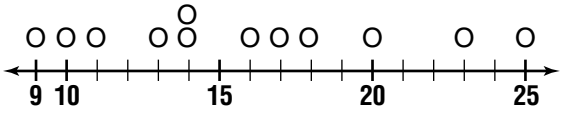
* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

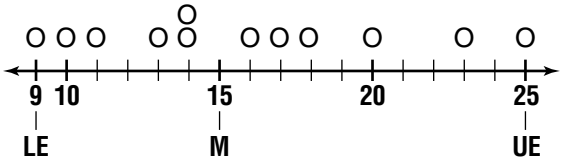
- On a blank transparency, write the data set 16, 9, 14, 20, 13, 17, 11, 23, 18, 25, 10, and 14. Show students the data set. Tell them that you are going to display these data by drawing a *box-and-whisker plot*. Tell them that a box-and-whisker plot displays and summarizes a set of data.
- Place another blank transparency horizontally on the screen. Draw a number line near the bottom of the page. Ask students what the least and greatest data are. **(9 and 25)** Tell them that you will begin the number line with 9 and end with 25. Mark the scale on the number line.



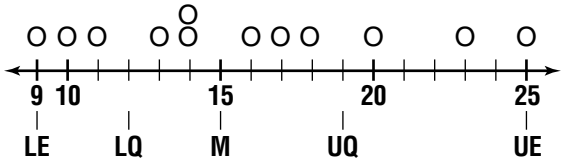
- Have students use the transparency pens to write each of the numbers from the data set on a yellow counter. Then have them place each counter in its appropriate place above the number line.



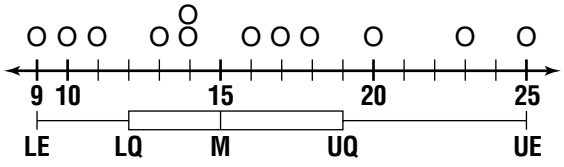
- Tell students that the *extreme values* are the least and greatest values. Mark the extreme values with a vertical line and label them. Remind them that the *median* value is the value in the middle of the data. Then ask what the median value is. **(15)** Mark the median with a vertical line and label it. Point out that the data is now separated into two halves.



- Ask students where the median of the lower half of the data is. **(between the 3rd and 4th data, or between 11 and 13)** Mark this value, 12, with a vertical line. Ask students where the median of the upper half of the data is. **(between the 9th and 10th data, or between 18 and 20)** Mark this value, 19, with a vertical line. Tell students that the median of the lower half of the data is called the *lower quartile* and the median of the upper half of the data is called the *upper quartile*. Label the plot. Point out that the data is now separated into fourths or *quartiles*, each having the same number of data.



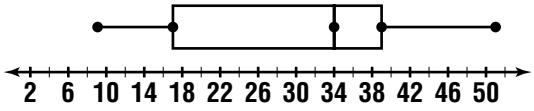
- Tell students that you are going to draw a box using the quartiles as the left and right edges. Draw the box. Then tell them you will draw a line segment through the median. Point out that this is the box of the box-and-whisker plot.
- Tape a piece of string between the lower quartile, 12, and the least number, 9. Tell students that this is one of the whiskers of the box-and-whisker plot. Then tape another piece of string between the upper quartile, 19, and the greatest number, 25. Tell students that this is the other whisker, and that the box-and-whisker plot is now complete.



- Tell students that the plot shows that the bottom fourth of this data has a small range while the top fourth is fairly spread out.

Have students complete Exercises 1–3 below.

1. Draw a box-and-whisker plot for 37, 12, 25, 9, 33, 17, 51, 45, 35, and 39.



2. If the whiskers in a box-and-whisker plot are longer than the box, what does this tell you about the data? **(Sample answer: The middle half of the data is tightly clustered and the lower and upper fourths of the data are widely dispersed.)**

CHAPTER 3

Adding and Subtracting Decimals
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Modeling Decimals
(p. 45 of this booklet)

Use With Lesson 3-1a. This corresponds to the activity on pages 100–101 in the Student Edition.

Objective Use models to represent, compare, order, add, and subtract decimals.

Materials
centimeter grid paper

Using models, students identify, compare, add, and subtract decimals. Space is provided for students to explain how to compute with decimals both with and without models.

Answers
See Teacher Wraparound Edition pp. 100–101.

Using Overhead Manipulatives

Decimals Through Hundredths
(pp. 46–47 of this booklet)

Use With Lesson 3-1a.

Objective Model decimals through hundredths.

Materials
decimal models*
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

This demonstration contains two activities.
- Demonstration 1 shows how to use base-ten models to represent decimals, and how to trade tenths for hundredths.
- Demonstration 2 asks students to identify the decimal modeled with both base-ten models and hundredths blocks.
- Students will perform the same kind of activities independently.
- An Extension activity shows how to trade ones for tenths when modeling a decimal.

Answers
Answers appear on the teacher demonstration instructions on pages 46–47.

Hands-On Lab Recording Sheet

Other Number Systems
(p. 48 of this booklet)

Use With Lesson 3-1b. This corresponds to the activity on pages 106–107 in the Student Edition.

Objective Write numbers using Roman and Egyptian numerals.

Materials
none

Students will write numbers using Roman numerals and identify numbers written in Roman numerals. They will do the same with Egyptian numerals. Students will also compare these number systems to our decimal system and identify disadvantages of using the other number systems.

Answers
See Teacher Wraparound Edition pp. 106–107.

Mini-Project

Comparing and Ordering Decimals

(p. 49 of this booklet)

Use With Lesson 3-2.

Objective Compare decimals using models.

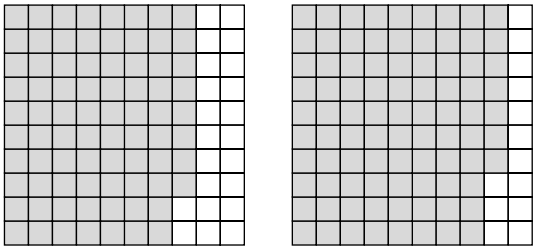
Materials

none

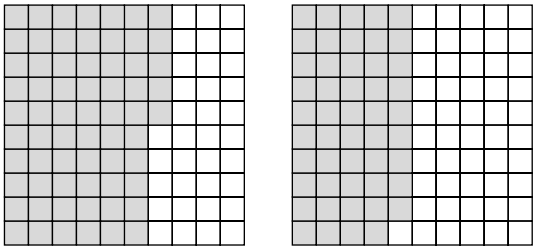
Students draw decimal models for given decimals in order to determine which of two decimals is greater.

Answers

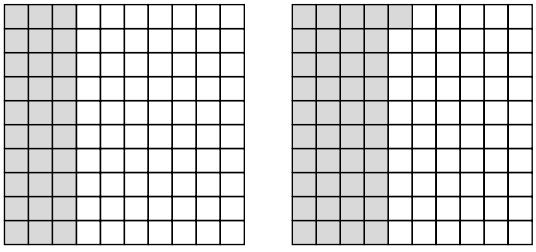
1. 0.87



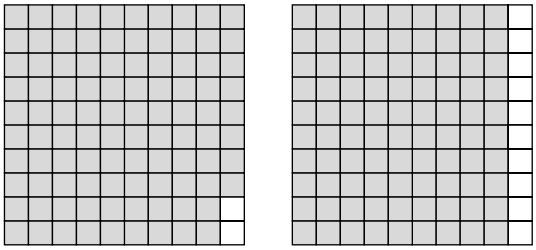
2. 0.65



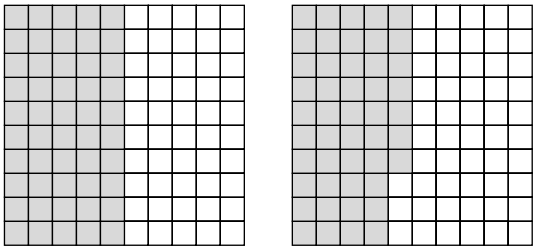
3. 0.41



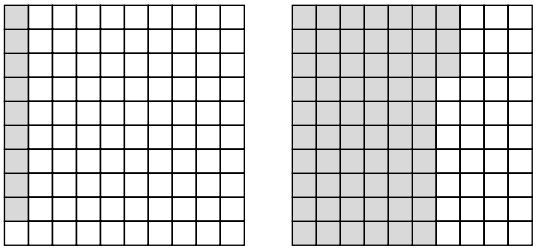
4. 0.98



5. 0.5



6. 0.63



NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 100–101 in Lesson 3-1a of the Student Edition)

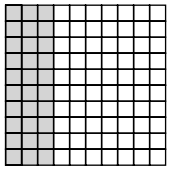
Modeling Decimals

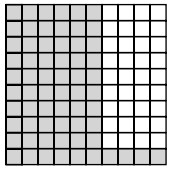
Materials

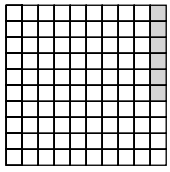
centimeter grid paper

Your Turn

Write the decimal shown by each model.

a. 

b. 

c. 

Compare each pair of decimals using models. Use a separate sheet of paper for your models.

d. 0.68 and 0.65 **e.** 0.2 and 0.28 **f.** 0.35 and 0.4

Your Turn

Find each sum using decimal models.

g. 0.14 + 0.67 **h.** 0.35 + 0.42 **i.** 0.03 + 0.07

Find each difference using decimal models.

j. 0.75 − 0.36 **k.** 0.68 − 0.27 **l.** 0.88 − 0.49

Writing Math

1. **Explain** why 0.3 is equal to 0.30. Use a model in your explanation.

2. **MAKE A CONJECTURE** Explain how you could compare decimals without using models.

3. **Explain** how you can use grid paper to model the following.

   a. 0.25 + 0.3 **b.** 0.8 − 0.37

4. **MAKE A CONJECTURE** Write a rule you can use to add or subtract decimals without using models.

Using Overhead Manipulatives

(Use with Lesson 3-1a)

Decimals Through Hundredths

Objective Model decimals through hundredths.

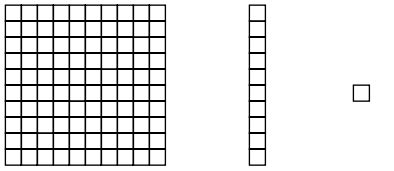
Materials

- decimal models*
- blank transparency
- transparency pen*

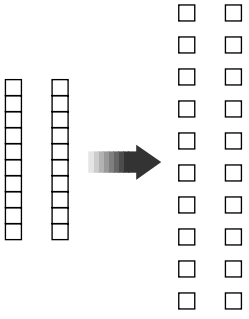
* = available in Overhead Manipulative Resources Kit

Teacher Demonstration for Activity 1

- Tell students that you can use base-ten models to represent decimals. At the top of a blank transparency, label the models as shown below.



- Place two tenths strips on the screen. Point out that this is a model for two tenths.
- Tell students that you can also model two tenths by trading the tenths for hundredths. Trade each tenth for 10 hundredths. You will need to draw additional hundredths on the transparency. Then ask how many hundredths there are. **(20)**



Teacher Demonstration for Activity 2

- Clear the screen. Show three tenths and five hundredths with base-ten models on a blank transparency. Ask students what decimal you have modeled. **(0.35)**
- Show students as you trade the tenths for hundredths. (You will need to draw additional tenths on the transparency.) Ask students how many hundredths blocks there are. **(35)**
- Point out that since there are 35 hundredth blocks, the decimal modeled is 35 hundredths.

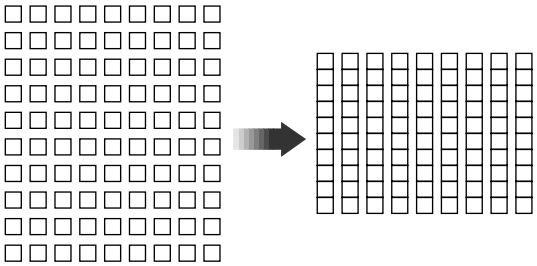
Have students complete Exercises 1–4 below.

If enough base-ten blocks are not available for modeling, have students sketch the blocks on their paper. Tell them to erase or cross out the blocks they trade.

1. Three tenths is the same as how many hundredths? **(thirty hundredths)**

2. Show six tenths and four hundredths with base-ten blocks. Trade the tenths for hundredths. How many hundredths do you have now? **(See students' work; 64 hundredths)**

3. How many tenths are the same as ninety hundredths? Model using base-ten blocks. **(9 tenths)**



4. If you separated a hundredth block into 100 equal parts, what decimal would be modeled by seventeen of the new parts? **(17 ten-thousandths)**

Extension

Show 2 ones and 4 tenths. Trade the ones for tenths. Ask students how many tenths you now have. **(24)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 106–107 in Lesson 3-1b of the Student Edition)

Other Number Systems

Materials

none

Writing Math

1. MAKE A CONJECTURE Study the patterns in the table on page 106. **Write** a sentence or two explaining the rule for forming Roman numerals.

Write each number using Roman numerals.

2. 6 **3.** 40 **4.** 23 **5.** 15 **6.** 55

Write the number for each Roman numeral.

7. XLIX **8.** C **9.** XCVIII **10.** XXIV **11.** XVIII

12. The page numbers at the front of your math book are written using Roman numerals. Write the number for the greatest Roman numeral you find there.

13. Describe a disadvantage of using Roman numerals.

14. *True* or *False*? The Roman numeral system is a place-value system. Explain.

15. Compare and contrast our decimal number system with the ancient Egyptian numbering system.

Write each number using Egyptian numerals.

16. 4 **17.** 20 **18.** 112 **19.** 1,203

Write the number for each Egyptian numeral.

20.  **21.**  **22.**  **23.** 

24. Describe a disadvantage of using Egyptian numerals.

25. Identify any similarities between the Roman numeral system and the ancient Egyptian numbering system.

26. MAKE A CONJECTURE How do you think you would add numbers written with Egyptian numerals? How is it similar to adding in a place-value system?

NAME _____ DATE _____ PERIOD ____

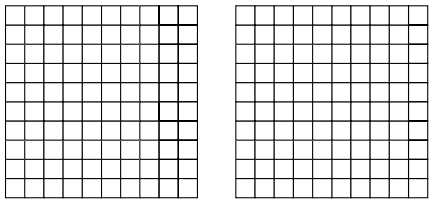
Mini-Project

(Use with Lesson 3-2)

Comparing and Ordering Decimals

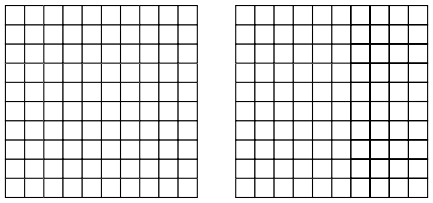
Shade a hundred grid to show each decimal. Which decimal is greater?

1. 0.78 0.87



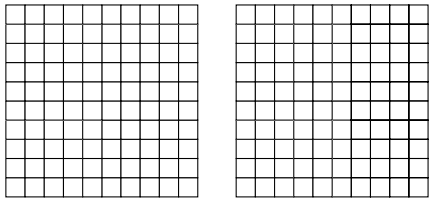
_____ is greater.

2. 0.65 0.49



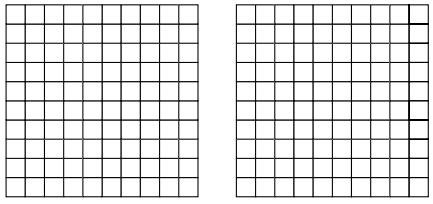
_____ is greater.

3. 0.3 0.41



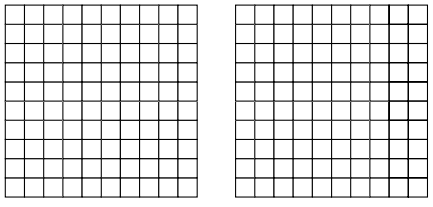
_____ is greater.

4. 0.98 0.9



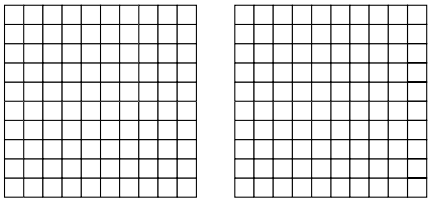
_____ is greater.

5. 0.5 0.47



_____ is greater.

6. 0.09 0.63



_____ is greater.

CHAPTER 4

Multiplying and Dividing Decimals
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Multiplying Decimals by Whole Numbers
(p. 52 of this booklet)

Use With Lesson 4-1a. This corresponds to the activity on page 134 in the Student Edition.

Objective Use models to multiply a decimal by a whole number.

Materials
grid paper
colored pencils
scissors

Students use decimal models to find the product of a decimal and a whole number. Space is also given for students to make and test a conjecture about the product of a whole number and a decimal.

Answers
See Teacher Wraparound Edition p. 134.

Hands-On Lab Recording Sheet

Multiplying Decimals
(p. 53 of this booklet)

Use With Lesson 4-2a. This corresponds to the activity on pages 139–140 in the Student Edition.

Objective Use decimal models to multiply decimals.

Materials
grid paper
colored pencils
scissors

Students use decimal models to find the product of two decimals. Students will identify and explain a pattern in the number of decimal places in each factor and in each product.

Answers
See Teacher Wraparound Edition pp. 139–140.

Hands-On Lab Recording Sheet

Dividing by Decimals
(p. 54 of this booklet)

Use With Lesson 4-4a. This corresponds to the activity on pages 150–151 in the Student Edition.

Objective Use models to divide a decimal by a decimal.

Materials
base-ten blocks

Using base-ten blocks, students will find the quotient of two decimals. Space is also provided for students to explain their work and make conclusions.

Answers
See Teacher Wraparound Edition pp. 150–151.

Mini-Project

Circumference
(p. 55 of this booklet)

Use With Lesson 4-6.

Objective Find the circumference of a circle.

Materials
string
ruler

Using a piece of string and a ruler, students find the circumferences of several circles. They will also measure the diameter of each circle and find its circumference using the formula for the circumference of a circle. Students then compare the two methods for finding the circumference of a circle.

Answers
Sample answers are given.

1. 4 in.; $1\frac{1}{4}$ in.; $3\frac{13}{14}$ in.

2. $4\frac{3}{4}$ in.; $1\frac{1}{2}$ in.; $4\frac{5}{7}$ in.

3. $1\frac{3}{8}$ in.; $\frac{1}{2}$ in.; $1\frac{4}{7}$ in.

4. $2\frac{7}{8}$ in.; $\frac{7}{8}$ in.; $2\frac{3}{4}$ in.

5. The formula method; it is easier to measure the diameter than the circumference.

Using Overhead Manipulatives

Diameter and Circumference
(p. 56 of this booklet)

Use With Lesson 4-6.

Objective Discover the relationship between the diameter and circumference of a circle.

Materials
compass*
string
scissors
ruler*
calculator
blank transparencies
transparency pen*
* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to find the circumference of a circle using a piece of string.
- Students will measure the circumference and diameter of several circles, then divide the circumference by the diameter to discover the relationship between the two measurements.

Answers
Answers appear on the teacher demonstration instructions on page 56.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 134 in Lesson 4-1a of the Student Edition)

Multiplying Decimals by Whole Numbers

Materials

grid paper, colored pencils, scissors

Your Turn

Use decimal models to show each product.

a. 3×0.5 **b.** 2×0.7 **c.** 0.8×4

Writing Math

1. MAKE A CONJECTURE Is the product of a whole number and a decimal greater than the whole number or less than the whole number? Explain your reasoning.

2. Test your conjecture on 7×0.3. Check your answer by making a model or with a calculator.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 139–140 in Lesson 4-2a of the Student Edition)

Multiplying Decimals

Materials

grid paper, colored pencils, scissors

Your Turn

Use decimal models to show each product on grid paper.

a. 0.3×0.3 **b.** 0.4×0.9 **c.** 0.9×0.5

Writing Math

1. Tell how many decimal places are in each factor and in each product Exercises a–c above.

2. MAKE A CONJECTURE Use the pattern you discovered in Exercise 1 to find 0.6×0.2. Check your conjecture with a model or a calculator.

3. Find two decimals whose product is 0.24.

Your Turn

Use decimal models to show each product on grid paper.

d. 1.5×0.7 **e.** 0.8×2.4 **f.** 1.3×0.3

Writing Math

4. MAKE A CONJECTURE How does the number of decimal places in the product relate to the number of decimal places in the factors?

5. Analyze each product.

First Factor		Second Factor		Product
0.9	×	0.6	=	0.54
1.0	×	0.6	=	0.6
1.5	×	0.6	=	0.90

a. Explain why the first product is less than 0.6.

b. Explain why the second product is equal to 0.6.

c. Explain why the third product is greater than 0.6

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 150–151 in Lesson 4-4a of the Student Edition)

Dividing by Decimals

Materials

base-ten blocks

Your Turn

Use base-ten blocks to find each quotient.

a. 2.4×0.6 **b.** 1.2×0.4 **c.** 1.8×0.6

d. 0.9×0.09 **e.** 0.8×0.04 **f.** 0.6×0.05

Writing Math

1. **Explain** why the base-ten blocks representing the dividend must be replaced or separated into the smallest place value of the divisor.

2. **Tell** why the quotient $0.4 \div 0.05$ is a whole number. What does the quotient represent?

3. **Determine** the missing divisor in the sentence $0.8 \div \_\_?\_\_ = 20$. Explain.

4. **Tell** whether $1.2 \div 0.03$ is *less than, equal to, or greater than* 1.2. Explain your reasoning.

NAME _____ DATE _____ PERIOD ____

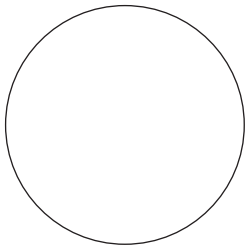
Mini-Project

(Use with Lesson 4-6)

Circumference

Place a piece of string around the circumference of each circle. Measure the string to the nearest eighth of an inch. Record the measurement. Then draw the diameter and measure it to the nearest eighth of an inch. Use the formula C = πd to calculate the circumference. Use $\frac{22}{7}$ as an approximation of π.

1.

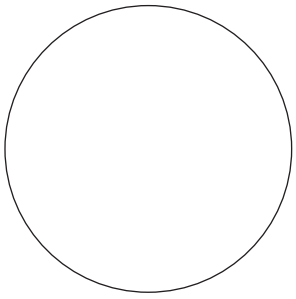


Circumference = _____

Diameter = _____

πd = _____

2.

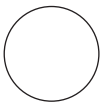


Circumference = _____

Diameter = _____

πd = _____

3.

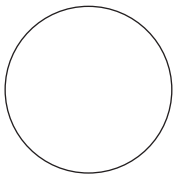


Circumference = _____

Diameter = _____

πd = _____

4.



Circumference = _____

Diameter = _____

πd = _____

Compare the two circumference measures for each circle.

5. Which method is better for determining the circumference? _____

_____

Why? _____

_____

Using Overhead Manipulatives

(Use with Lesson 4-6)

Diameter and Circumference

Objective Discover the relationship between the diameter and circumference of a circle.

Materials

- compass*
- string
- scissors
- ruler*
- calculator
- blank transparencies
- transparency pen*

* = available in Overhead Manipulative Resources Kit.

Teacher Demonstration

- With the compass, draw a circle with a radius of 2 cm on a blank transparency. Show students the circle. Remind them that the *circumference* of the circle is the distance around the circle. Tell students that you can use string to help find the measure of the circumference of this circle. Place the end of the string on a point of the circle. (You may want to tape it in place.) Show students as you trace the circumference of the circle with the string. Cut the string at the point where it meets the other end.
- Place the string on the screen and use the ruler to find the length of the string. **(about 12.6 cm)** Ask students what measurement of the circle this length represents. **(the circumference)**
- Use the ruler to measure the diameter of the circle. **(4 cm)**
- Have students use their calculators to divide the circumference, 12.6, by the diameter, 4, of the circle. **(3.15)** Record the result on the transparency.
- Repeat the steps for circles with other diameters.
- Ask students what they notice about the quotients you obtained when you divided each circle's circumference by its diameter. **(The quotients are close to 3.)**
- Ask students how the circumference and diameter of a circle appear to be related. **(The circumference of a circle is always a little more than 3 times the diameter.)**

CHAPTER 5

Fractions and Decimals
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Simplifying Fractions
(p. 58 of this booklet)

Use With Lesson 5-2a. This corresponds to the activity on page 181 in the Student Edition.

Objective Use models to simplify fractions.

Materials
ruler
paper
scissors
colored pencils

Students use models to simplify fractions. Space is also provided for students to explain a given model and make a conjecture about a rule for writing fractions in simplest form.

Answers
See Teacher Wraparound Edition p. 181.

Using Overhead Manipulatives

Modeling Improper Fractions
(p. 59 of this booklet)

Use With Lesson 5-3.

Objective Use models to express mixed numbers as improper fractions.

Materials
ruler*
transparency pens*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to use models to express mixed numbers as improper fractions.
- Students will also learn to use models to find equivalent fractions.

Answers
Answers appear on the teacher demonstration instructions on page 59.

Using Overhead Manipulatives

Least Common Multiple
(p. 60 of this booklet)

Use With Lesson 5-4.

Objective Use models to find the least common multiple.

Materials
rectangular dot paper transparency*
transparency pens*
centimeter ruler*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to use models to find the least common multiple.
- Students will learn how to make rectangles on rectangular dot paper for their models.

Answers
Answers appear on the teacher demonstration instructions on page 60.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 181 in Lesson 5-2a of the Student Edition.)

Simplifying Fractions

Materials

ruler, paper, scissors, colored pencils

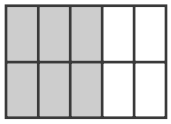
Your Turn

Use a model to simplify each fraction.

a. $\frac{2}{8}$ **b.** $\frac{4}{6}$ **c.** $\frac{3}{9}$

Writing Math

1. Explain how the model at the right shows the simplified form of $\frac{3}{5}$.



2. MAKE A CONJECTURE Write a rule that you could use to write a fraction in simplest form.

Using Overhead Manipulatives

(Use with Lesson 5-3)

Modeling Improper Fractions

Objective Use models to express mixed numbers as improper fractions.

Materials

- ruler*
- transparency pens*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Tell students that you are going to draw a model for a mixed number.
- Draw two rectangles. Shade the rectangles. Tell students that this represents 2.
- Draw an identical rectangle beside the first two. Ask students how to show $\frac{2}{3}$. **(Separate the rectangle into 3 equal parts and shade 2 parts.)** Ask students to name the number shown by the rectangles. $\left(\mathbf{2\frac{2}{3}}\right)$

- Separate the whole number rectangles into thirds. Ask students how many thirds are shaded in all three rectangles. **(8)** Ask students to name the fraction that is equivalent to 223. $\left(\mathbf{\frac{8}{3}}\right)$

Using Overhead Manipulatives

(Use with Lesson 5-4)

Least Common Multiple

Objective Use models to find the least common multiple.

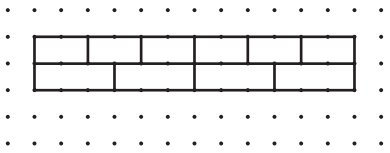
Materials

- rectangular dot paper transparency*
- transparency pens*
- centimeter ruler*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Place the dot paper transparency horizontally on the screen. Tell students that you are going to model multiples of 2 and 3. As you draw 2-by-1 rectangles end-to-end on the dot paper, tell them that each is 2 centimeters long and that you are making a 2-centimeter rectangle train.
- Next, draw 3-by-1 rectangles end-to-end to make a train below the 2-by-1 rectangles as shown. Ask students to look for places where the ends of the rectangles line up.



- Place the ruler below the trains and ask students at what lengths the ends of the rectangles line up. **(0 cm, 6 cm, and 12 cm)** Ask if 0, 6, and 12 are multiples of both 2 and 3. **(yes)**
- Ask students, "At what measurement, other than zero, do the rectangles line up for the first time?" **(6 cm)** Point out that 6 is called the *least common multiple* of 2 and 3.

CHAPTER 6

Adding and Subtracting Fractions
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Rounding Fractions
(p. 63 of this booklet)

Use With Lesson 6-1a. This corresponds to the activity on page 218 in the Student Edition.

Objective Round fractions to 0, $\frac{1}{2}$, and 1.

Materials
grid paper
colored paper

Students will round several fractions to the nearest half. Then they will sort the fractions into three groups: those that round to 0, those that round to $\frac{1}{2}$, and those that round to 1. By comparing the numerators and denominators, students make a conjecture about how to round any fraction to the nearest half.

Answers
See Teacher Wraparound Edition p. 218.

Mini-Project

Adding and Subtracting Fractions
(p. 64 of this booklet)

Use With Lesson 6-3.

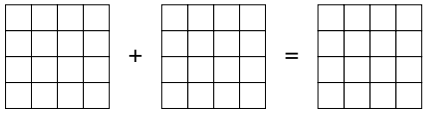
Objective Use models to add and subtract fractions.

Materials
none

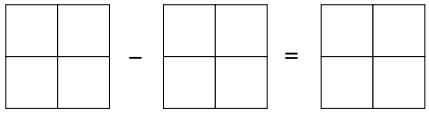
Students shade grids to model fractions. Using these models, students find sums and differences of fractions.

Answers

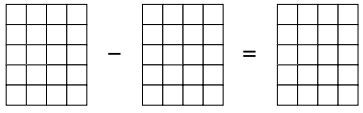
1. $\frac{9}{16}$



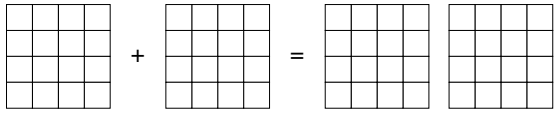
2. $\frac{2}{4}$



3. $\frac{8}{20}$



4. $\frac{22}{16}$ or $1\frac{6}{16}$



Chapter 6

Hands-On Lab Recording Sheet

Common Denominators
(p. 65 of this booklet)

Use With Lesson 6-4a. This corresponds to the activity on page 234 in the Student Edition.

Objective Add and subtract fractions with unlike denominators.

Materials

paper squares
ruler
markers

Students use fraction models to add and subtract fractions with unlike denominators. They will explain why they need a common denominator to add or subtract fractions with unlike denominators. Students will also make a conjecture about the relationship between common multiples and adding and subtracting unlike fractions.

Answers

See Teacher Wraparound Edition p. 234.

Using Overhead Manipulatives

Renaming Sums
(p. 66 of this booklet)

Use With Lesson 6-4.

Objective Find common unit names for adding different objects.

Materials

3 congruent squares (2 inches-by-2 inches)
5 congruent trapezoids (2 inches-by-1 inch)
4 pencils
2 pens
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- Using a group of squares and trapezoids and a group of pens and pencils, students will find common unit names for adding these different objects.
- Students will choose the best common name and explain their reasoning.
- Students will find common unit names for groups of objects that they list themselves.
- An Extension activity asks students to look for objects in their homes that can be described using a common unit name.

Answers

Answers appear on the teacher demonstration instructions on page 66.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 218 in Lesson 6-1a of the Student Edition.)

Rounding Fractions

Materials

grid paper, colored paper

Your Turn

Round each fraction to the nearest half.

a. $\frac{13}{20}$ **b.** $\frac{98}{100}$ **c.** $\frac{9}{10}$ **d.** $\frac{1}{5}$ **e.** $\frac{37}{50}$

f. $\frac{2}{25}$ **g.** $\frac{6}{10}$ **h.** $\frac{17}{20}$ **i.** $\frac{1}{8}$ **j.** $\frac{28}{50}$

Writing Math

1. Sort the fractions in Exercises a–j into three groups: those that round to 0, those that round to $\frac{1}{2}$, and those that round to 1.

2. Compare the numerators and denominators of the fractions in each group. **Make a conjecture** about how to round any fraction to the nearest half.

3. Test your conjecture by repeating Exercise 1 using the fractions $\frac{3}{5}$, $\frac{3}{17}$, $\frac{16}{20}$, $\frac{2}{13}$, $\frac{6}{95}$, $\frac{7}{15}$, $\frac{7}{9}$, and $\frac{9}{11}$.

NAME _____ DATE _____ PERIOD ____

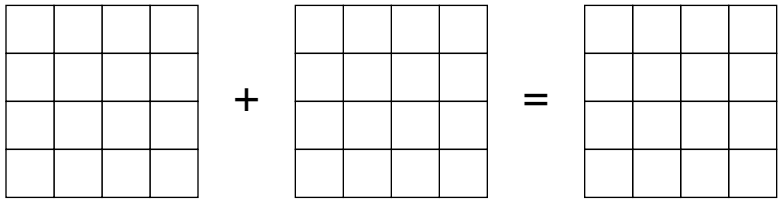
Mini-Project

(Use with Lesson 6-3)

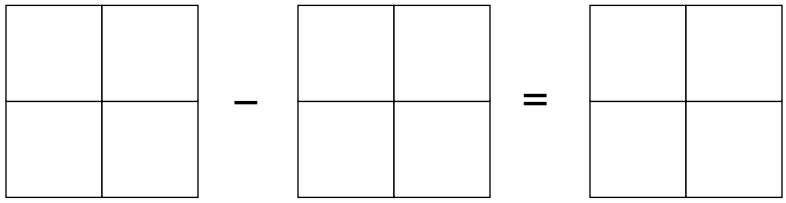
Adding and Subtracting Fractions

Shade each figure to model each fraction. Use the models to find the sum.

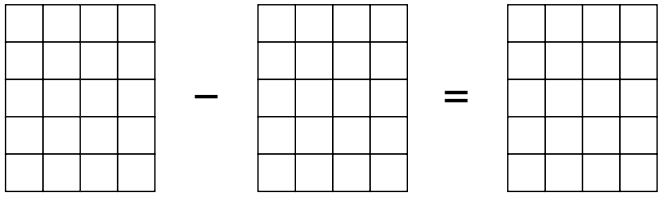
1. $\frac{5}{16} + \frac{4}{16} =$



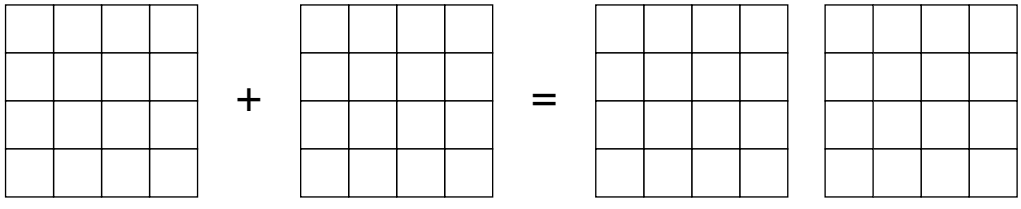
2. $\frac{3}{4} - \frac{1}{4} =$



3. $\frac{11}{20} - \frac{3}{20} =$



4. $\frac{9}{16} + \frac{13}{16} =$



NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 234 in Lesson 6-4a of the Student Edition)

Common Denominators

Materials

paper squares, ruler, markers

Your Turn

Use fraction models to add or subtract.

a. $\frac{1}{4} + \frac{2}{3}$ **b.** $\frac{4}{5} + \frac{1}{2}$ **c.** $\frac{5}{6} - \frac{1}{2}$ **d.** $\frac{4}{5} - \frac{1}{3}$

Writing Math

1. **Explain** why you need a common denominator to add or subtract fractions with unlike denominators.

2. **MAKE A CONJECTURE** What is the relationship between common multiples and adding and subtracting unlike fractions?

Using Overhead Manipulatives

(Use with Lesson 6-4)

Renaming Sums

Objective Find common unit names for adding different objects.

Materials

- 3 congruent squares (about 2 inches-by-2 inches)
- 5 congruent trapezoids (about 2 inches-by-1 inch)
- 4 pencils
- 2 pens
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Place the squares and trapezoids on the screen. Ask students to think of unit names to describe the sum of the squares and trapezoids. Record their responses. **[Sample answers: 8 things, 8 shapes, 8 four-sided shapes (or quadrilaterals)]**
- Have students look at the list and choose the best unit name for the squares and trapezoids. **(four-sided shapes, quadrilaterals)**
- Hold up the pencils and pens. Ask students to think of unit names to describe their sum. Record their responses. **(6 things; 6 objects; 6 long, thin objects; 6 writing tools)**
- Have students choose the best unit name for the pens and pencils. **(writing tools)**
- Ask students why a common unit name is needed to find the sum. **(The addends and sum must have the same unit name.)**
- Ask students what helped them select the "best" unit name for the objects. **(Sample answer: The name "writing tools" is more representative; it describes the objects in the group better.)**

Have students complete Exercises 1–2 below.

1. Make a list of different objects that could have a common unit name. **(Sample answer: paper clips, staples, and rubber bands—paper holders)**

2. What do you think you need to do to find the sum of $\frac{1}{2}$ and $\frac{3}{4}$? **(Sample answer: Find a common denominator.)**

Extension

Ask students to look for groups of objects in their homes that can be described using a common unit name. For example, sheets and pillowcases can be called bed linens.

Multiplying and Dividing Fractions
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Multiplying Fractions
(p. 68 of this booklet)

Use With Lesson 7-2a. This corresponds to the activity on pages 259–260 in the Student Edition.

Objective Multiply fractions using models.

Materials
paper
markers

Students will use models to find the products of fractions. Then they will explain the relationship between the numerators of the problem and the numerator of the product. They will also compare the denominators of the problem and the denominator of the product. Finally, students will write a rule they can use to multiply fractions.

Answers
See Teacher Wraparound Edition pp. 259–260.

Hands-On Lab Recording Sheet

Dividing Fractions
(p. 69 of this booklet)

Use With Lesson 7-4a. This corresponds to the activity on pages 270–271 in the Student Edition.

Objective Divide fractions using models.

Materials
paper
colored pencils
scissors

Students will divide fractions using models. They will also determine when the quotient of two fractions is greater than, less than, or equal to 1 and give examples to support their reasoning. Finally, students will explain whether division is commutative and give examples to support their answer.

Answers
See Teacher Wraparound Edition pp. 270–271.

Using Overhead Manipulatives

The Fibonacci Sequence
(pp. 70–71 of this booklet)

Use With Lesson 7-6.

Objective Discover the numbers that make up the Fibonacci sequence.

Materials
2 blank transparencies
transparency pens*
* = available in Overhead Manipulative Resources Kit

- By tracing paths on a diagram, students discover the numbers that make up the Fibonacci sequence.
- An Extension activity asks students to compare the terms in two Fibonacci sequences that each begin at a different number.

Answers
Answers appear on the teacher demonstration instructions on pages 70–71.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 259–260 in Lesson 7-2a of the Student Edition)

Multiplying Fractions

Materials

paper, markers

Your Turn

Find each product using a model.

a. $\frac{1}{4} \times \frac{1}{2}$ **b.** $\frac{1}{3} \times \frac{1}{4}$ **c.** $\frac{1}{2} \times \frac{1}{5}$

Your Turn

Find each product using a model. Then write in simplest form.

d. $\frac{3}{4} \times \frac{2}{3}$ **e.** $\frac{2}{5} \times \frac{5}{6}$ **f.** $\frac{4}{5} \times \frac{3}{8}$

Writing Math

1. Describe how you would change the model in Activity 1 to find $\frac{1}{2} \times \frac{1}{3}$. Is the product the same as $\frac{1}{3} \times \frac{1}{2}$? Explain.

2. Draw a model to show that $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18}$. Then explain how the model shows that $\frac{10}{18}$ simplifies to $\frac{5}{9}$.

3. Explain the relationship between the numerators of the problem and the numerator of the product. What do you notice about the denominators of the problem and the denominator of the product?

4. MAKE A CONJECTURE Write a rule you can use to multiply fractions.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 270–271 in Lesson 7-4a of the Student Edition)

Dividing Fractions

Materials

paper, colored pencils, scissors

There are 8 pieces of candy that are given away 2 at a time. How many people will get candy?

1. How many 2s are in 8? Write as a division expression.

Suppose there are two granola bars divided equally among 8 people. What part of a granola bar will each person get?

2. What part of 8 is in 2? Write as a division expression.

Your Turn

Find each quotient using a model.

a. $2 \div \frac{1}{5}$ **b.** $3 \div \frac{1}{3}$ **c.** $3 \div \frac{2}{3}$ **d.** $2 \div \frac{3}{4}$

e. $\frac{4}{10} \div \frac{1}{5}$ **f.** $\frac{3}{4} \div \frac{1}{2}$ **g.** $\frac{4}{5} \div \frac{1}{5}$ **h.** $\frac{1}{6} \div \frac{1}{3}$

Writing Math

Use *greater than*, *less than*, or *equal to* to complete each sentence. Then give an example to support your answer.

1. When the dividend is equal to the divisor, the quotient is _____?_____ 1.

2. When the dividend is greater than the divisor, the quotient is _____?_____ 1.

3. When the dividend is less than the divisor, the quotient is _____?_____ 1.

4. You know that multiplication is commutative because the product of 3×4 is the same as 4×3. Is division commutative? Give examples to explain your answer.

Using Overhead Manipulatives

(Use with Lesson 7-6)

The Fibonacci Sequence

Objective Discover the numbers that make up the Fibonacci sequence.

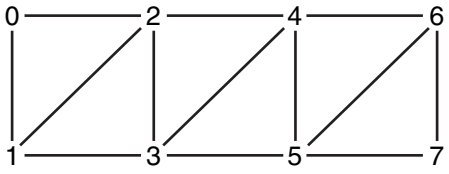
Materials

- 2 blank transparencies
- transparency pens*

* = available in Overhead Manipulative Resources Kit

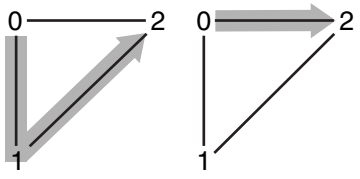
Teacher Demonstration

- Prepare one transparency by copying the diagram and chart shown below. Do not copy the numbers shown in parentheses.

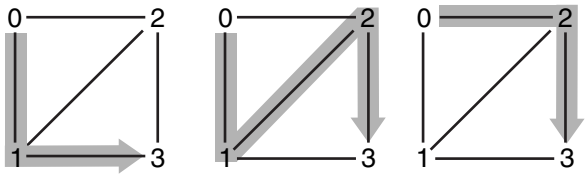


Destination	Number of Paths
1	**(1)**
2	**(2)**
3	**(3)**
4	**(5)**
5	**(8)**
6	**(13)**
7	**(21)**

- Show students the diagram. Tell them that you can find paths from 0 to other numbers under the following rules. You must move along the lines and the path must have the numbers in numerical order.
- Place a blank transparency on top of the diagram. Use a colored transparency pen to show students the one path from 0 to 1. Record the number of paths in the chart.
- Move the top transparency so the space on top of the diagram is clear. Show the 2 paths possible from 0 to 2. Record in the chart.



- Move the top transparency again. Ask students to describe possible paths from 0 to 3. Draw them with the colored transparency pen. Record the number of possible paths from 0 to 3 in the chart.



- Repeat for paths from 0 to 4, 5, and 6. (7 will be done in the next step.)
- Ask students to look for a pattern in the number of paths. **(Each number is the sum of the previous two numbers.)** Then ask them to predict the number of paths to 7. **(21)**
- Verify by drawing the paths.

Have students complete Exercises 1–3 below.

1. Look for a pattern in the numbers in the second row. How is each number related to the previous numbers? **(Each number is the sum of the previous two numbers.)**

2. Without making a drawing, how many ways are there to build a road that is 7 units long? **(21 ways)**

3. The numbers in the second row of the chart are the first numbers in the Fibonacci sequence. Write the ratios of consecutive terms of the Fibonacci sequence, $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots, \frac{89}{55}$, as decimals rounded to the nearest thousandth. Describe the pattern. **(Sample answer: All the decimals are about 1.62.)**

Extension

Tell students that a Fibonacci sequence can begin at any number. Write the numbers 2, 2, 4 on a blank transparency. Have students use mental math and/or a calculator to tell what the next 10 terms would be. **(6, 10, 16, 26, 42, 68, 110, 178, 288, 466)** Write the original sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233) on the transparency. Ask students to compare the terms in the two sequences. **(Each term in the sequence beginning with 2 is twice the corresponding term in the sequence beginning with 1.)**

CHAPTER 8

Algebra: Integers
Teaching Notes and Overview

Using Overhead Manipulatives

Ratios and Probability
(pp. 73–74 of this booklet)

Use With Lesson 8-1.

Objective Explore ratios and probability.

Materials
transparent spinner with 4 sections*
blank transparency
transparency pens*
*= available in Overhead Manipulative Resources Kit

- Demonstration 1 shows how to find the probability that a player makes a free throw by spinning a spinner.
- Demonstration 2 repeats the activity using a spinner with three sections instead of four.
- Students then compare the results and probabilities from each activity.

Answers
Answers appear on the teacher demonstration instructions on pages 73–74.

Hands-On Lab Recording Sheet

Zero Pairs
(p. 75 of this booklet)

Use With Lesson 8-2a. This corresponds to the activity on page 299 in the Student Edition.

Objective Model zero pairs.

Materials
counters
integer mat

Students use counters to model pairs of integers, then form zero pairs to find the sum of the integers.

Answers
See Teacher Wraparound Edition p. 299.

Mini-Project

The Coordinate System
(p. 76 of this booklet)

Use With Lesson 8-6.

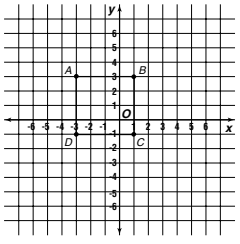
Objective Graph and label points.

Materials
none

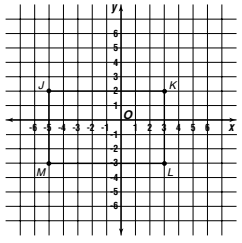
Students graph and label sets of points. They connect the points and identify each figure.

Answers

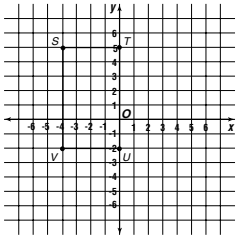
1. square *ABCD*



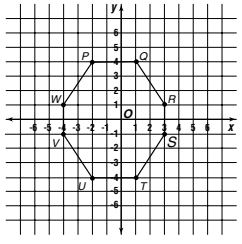
2. rectangle *JKLM*



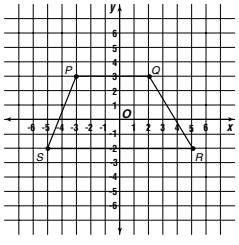
3. trapezoid *PQRS*



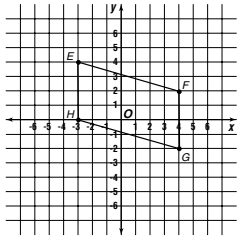
4. parallelogram *EFGH*



5. rectangle *STUV*



6. octagon *PQRSTUVW*



Using Overhead Manipulatives

(Use with Lesson 8-1)

Ratios and Probability

Objective Explore ratios and probability.

Materials

- transparent spinner with 4 sections*
- blank transparency
- transparency pens*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration for Activity 1

- Prepare the transparent spinner with four equal sections by marking one section "make" and the other sections "miss." Tell students that each section represents a basketball player making or missing a basket from the free throw line.
- Tell them that you are going to spin the spinner 100 times to represent 100 free throws made by a player.
- On a blank transparency, sketch the spinner. Below it, copy the following chart.

Result	Tally	Frequency
Make		
Miss		

- Spin the spinner 100 times. Record the results in the tally column of the chart. When you have finished 100 spins, record the frequency of "make" and "miss" results.
- On the transparency, write the ratio $\frac{\text{baskets made}}{100}$. Have students find the ratio for the data in the chart. Then have them express the ratio as a decimal. **(Answers will vary.)** Point out that this ratio represents the probability that the player makes a basket from the free throw line.

Teacher Demonstration for Activity 2

- Change the spinner so that three sections are labeled "make" and one section is labeled "miss." Again, tell students that each section in this spinner represents a player making or missing a basket.
- Repeat the steps in Activity 1 for this spinner.

Have students complete Exercises 1–5 below.

1. Write a ratio that compares the area of each region marked "make" to the total area of the circle for each spinner. $\left(\frac{1}{4}; \frac{3}{4}\right)$
2. What is the probability of making a basket on each spinner? Express the probability as a decimal. **(0.25; 0.75)**
3. Compare your results from the activity to each probability in Exercise 2. **(Sample answer: The results are about the same.)**
4. Describe how ratio and probability are related. **(Sample answer: Probability is the ratio of the number of times an event occurs to the total number of possible outcomes.)**
5. A player makes a basket every 50 out of 80 times. What is the probability that he will make a basket on his next attempt? Express the probability as a decimal. **(0.625)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 299 in Lesson 8-2a of the Student Edition)

Zero Pairs

Materials: counters, integer mat

Your Turn

Use counters to model each pair of integers. Then form zero pairs to find the sum of the integers.

a. $+3, -3$ **b.** $+5, -5$ **c.** $-7, +7$

Writing Math

1. What is the value of a zero pair? Explain your reasoning.

2. Suppose there are 5 zero pairs on an integer mat. What is the value of these zero pairs? Explain.

3. **Explain** the effect of removing a zero pair from the mat. What effect does this have on the remaining counters?

4. Integers like $+4$ and -4 are called *opposites*. What is the sum of any pair of opposites?

5. **Write** a sentence describing how zero pairs are used to find the sum of any pair of opposites.

6. **MAKE A CONJECTURE** How do you think you could find $+5 + (-2)$ using counters?

NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 8-6)

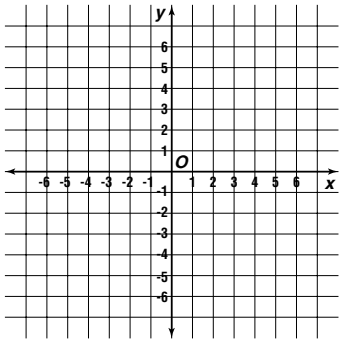
The Coordinate System

a. Graph and label each point.

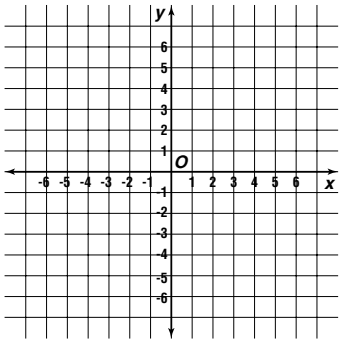
b. Connect the points in order, including the last and the first points.

c. Name the figure.

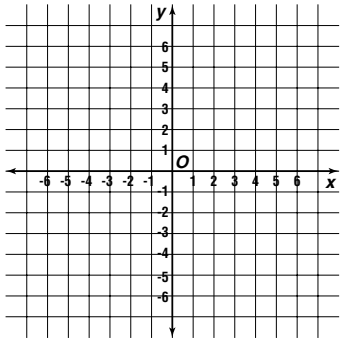
1. $A(-3, 3)$, $B(1, 3)$, $C(1, -1)$, $D(-3, -1)$



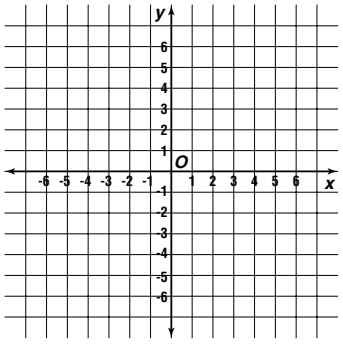
2. $J(-5, 2)$, $K(3, 2)$, $L(3, -3)$, $M(-5, -3)$



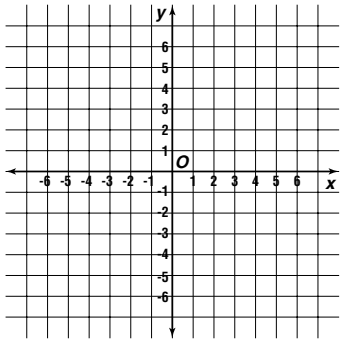
3. $P(-3, 3)$, $Q(2, 3)$, $R(5, -2)$, $S(-5, -2)$



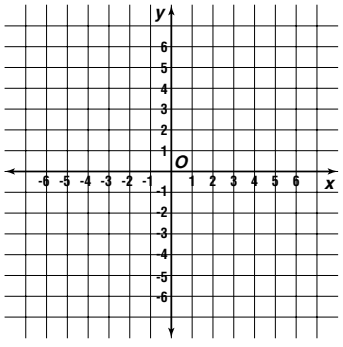
4. $E(-3, 4)$, $F(4, 2)$, $G(4, -2)$, $H(-3, 0)$



5. $S(-4, 5)$, $T(0, 5)$, $U(0, -2)$, $V(-4, -2)$



6. $P(-2, 4)$, $Q(1, 4)$, $R(3, 1)$, $S(3, -1)$, $T(1, -4)$, $U(-2, -4)$, $V(-4, -1)$, $W(-4, 1)$



CHAPTER 9

Algebra: Solving Equations
Teaching Notes and Overview

Hands-On Lab Recording Sheet
The Distributive Property
(p. 80 of this booklet)

Use With Lesson 9-1a. This corresponds to the activity on page 332 in the Student Edition.

Objective Solve equations using the Distributive Property.

Materials
none

Students will use and draw models to illustrate the Distributive Property. They will use these models and the Distributive Property to solve equations.

Answers
See Teacher Wraparound Edition p. 332.

Hands-On Lab Recording Sheet
Solving Addition Equations Using Models
(p. 81 of this booklet)

Use With Lesson 9-2a. This corresponds to the activity on pages 337–338 in the Student Edition.

Objective Solve addition equations using cups and counters.

Materials
cups
counters
equation mat

Using cups and counters, students will solve addition equations using models. Students will explain how to solve equations using models and discover a rule they can use to solve addition equations without using models.

Answers
See Teacher Wraparound Edition pp. 337–338.

Mini-Project
Solving Addition Equations
(p. 82 of this booklet)

Use With Lesson 9-2.

Objective Solve addition equations using models.

Materials
cups
counters
equation mat

Students write equations represented by models. Using cups and counters, they solve addition equations with models. Space is provided for students to sketch their models.

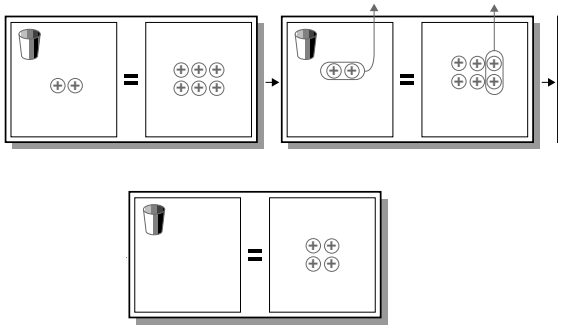
Answers
1. $4 + x = 6$

2. $x - 3 = 9$ or $x + (-3) = 9$

3. $x + 2 = 6$

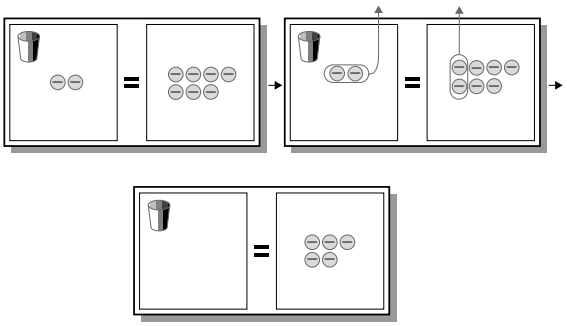
4. $x + (-3) = 4$ or $x - 3 = 4$

5. $x = 4$

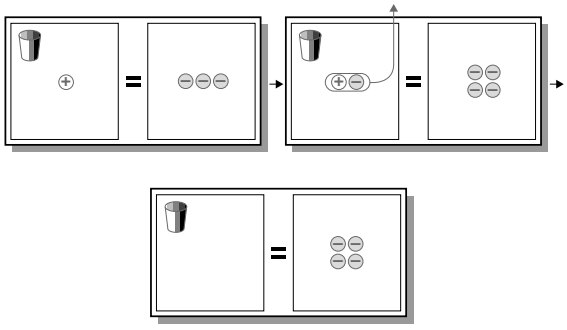


Chapter 9

6. $x = -5$



7. $x = -4$



8. -7

Hands-On Lab Recording Sheet

Solving Subtraction Equations Using Models
(p. 83 of this booklet)

Use With Lesson 9-3a. This corresponds to the activity on page 343 in the Student Edition.

Objective Solve subtraction equations with models.

Materials
cups
counters
equation mat

Using cups and counters, students will solve subtraction equations using models. Students will explain how to solve equations using models and discover a rule they can use to solve subtraction equations without using models.

Answers
See Teacher Wraparound Edition p. 343.

Using Overhead Manipulatives

Solving Multiplication Equations
(pp. 84–85 of this booklet)

Use With Lesson 9-4.

Objective Solve multiplication equations by using models.

Materials
cups*
counters*
equation mat transparency*
transparency pens*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to solve multiplication equations by using models.
- Students then solve equations independently using models.
- Students explain how they would solve multiplication equations without using models.

Answers
Answers appear on the teacher demonstration instructions on pages 84–85.

Hands-On Lab Recording Sheet

Solving Inequalities Using Models

(p. 86 of this booklet)

Use With Lesson 9-4b. This corresponds to the activity on page 354 in the Student Edition.

Objective Solve inequalities using models.

Materials
cups
counters
equation mat

Students use models to solve inequalities. They compare and contrast solving addition and subtraction inequalities with solving addition and subtraction equations. Students discover a rule they can use for solving inequalities without using models.

Answers
See Teacher Wraparound Edition p. 354.

Hands-On Lab Recording Sheet

Function Machines

(p. 87 of this booklet)

Use With Lesson 9-6a. This corresponds to the activity on pages 360–361 in the Student Edition.

Objective Solve equations using function machines.

Materials
scissors
tape

Students make function machines for given rules and input values, then record the input and output values in function tables. Students will also create their own function machines, write inputs and outputs, and have other students determine the rule. Finally, students will explain why using a function machine is like finding a pattern.

Answers
See Teacher Wraparound Edition pp. 360–361.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 332 in Lesson 9-1a of the Student Edition)

The Distributive Property

Materials

none

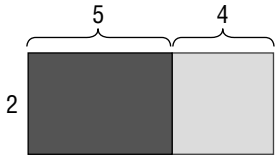
Your Turn

Draw a model showing that each equation is true.

a. $2(4 + 6) = (2 \times 4) + (2 \times 6)$ **b.** $4(3 + 2) = (4 \times 3) + (4 \times 2)$

Writing Math

1. Write two expressions for the total area of the rectangle at the right.



2. OPEN ENDED Draw any two rectangles that have the same width. Find the total area in two ways.

3. MAKE A CONJECTURE Write an expression that has the same value as $2(4 + 3)$. Explain your reasoning.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 337–338 in Lesson 9-2a of the Student Edition)

Solving Addition Equations Using Models

Materials

cups, counters, equation mat

Your Turn

Solve each equation using models.

a. $1 + x = 8$ **b.** $x + 2 = 7$ **c.** $9 = x + 3$

d. $x + 3 = -7$ **e.** $2 + x = -5$ **f.** $-3 = x + 3$

Writing Math

1. **Explain** how you decide how many counters to add or subtract from each side.

2. **Write** an equation in which you need to remove zero pairs in order to solve it.

3. **Model** the equation *some number plus 5 is equal to* -2. Then solve the equation.

4. **MAKE A CONJECTURE** Write a rule that you can use to solve an equation like $x + 3 = 6$ without using models.

NAME _____ DATE _____ PERIOD ____

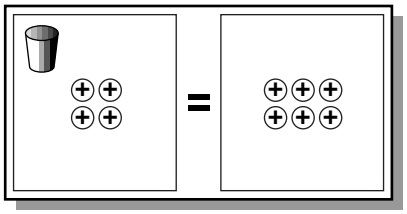
Mini-Project

(Use with Lesson 9-2)

Solving Addition Equations

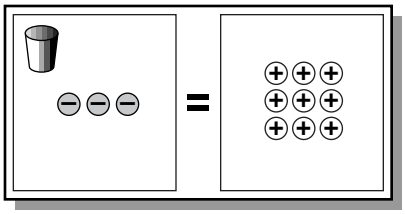
Write the equation that is represented by each model.

1.



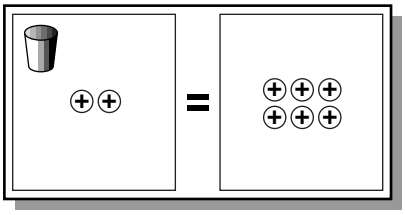
_____

2.



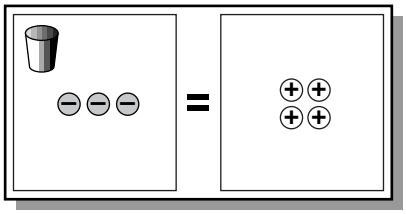
_____

3.



_____

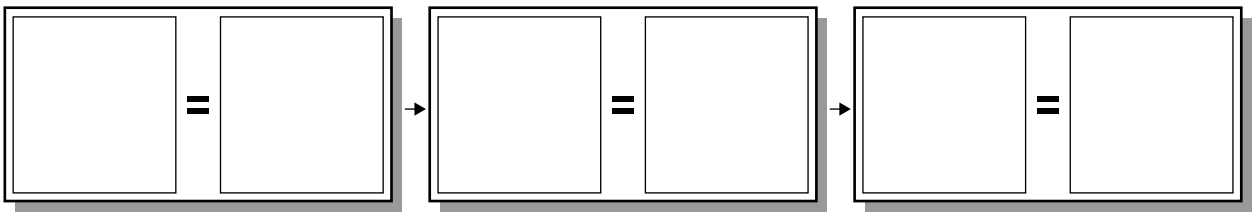
4.



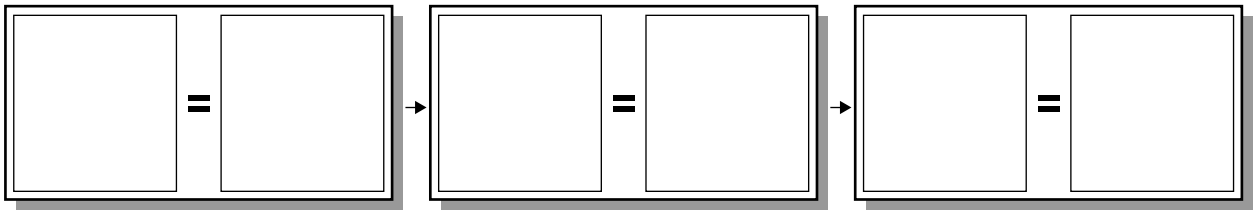
_____

Solve each equation using cups and counters. Sketch the arrangement in the boxes.

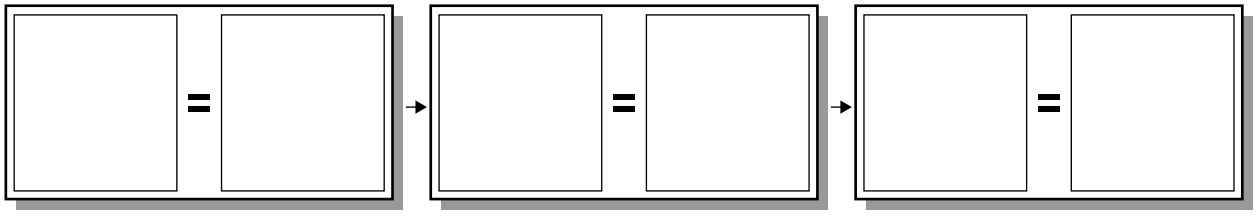
5. $x + 2 = 6$



6. $x + (-2) = -7$



7. $x + 1 = -3$



8. Solve $x + 4 = -3$ without using models. $x =$ _____

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 343 in Lesson 9-3a of the Student Edition)

Solving Subtraction Equations Using Models

Materials

cups, counters, equation mat

Your Turn

Solve each equation using models.

a. $x - 4 = 2$ **b.** $-3 = x - 1$ **c.** $x - 5 = -1$

Writing Math

1. Explain why it is helpful to rewrite a subtraction problem as an addition problem when solving equations using models.

2. MAKE A CONJECTURE Write a rule for solving equations like $x - 7 = -5$ without using models.

Using Overhead Manipulatives

(Use with Lesson 9-4)

Solving Multiplication Equations

Objective Solve multiplication equations by using models.

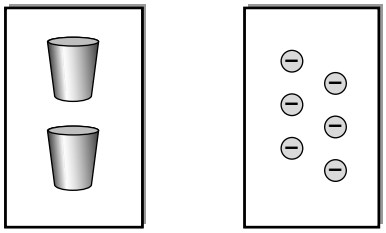
Materials

- cups*
- counters*
- equation mat transparency*
- transparency pens*

* = available in Overhead Manipulative Resources Kit

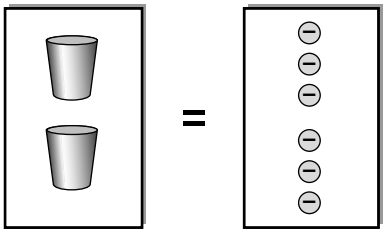
Teacher Demonstration

- Place 2 cups on the left side of the equation mat. Place 6 red counteres on the right side of the mat. Remind students that each cup represents an unknown value, x. Ask them to state the equation represented by the models. **($2x = -6$)** Write the equation at the bottom of the mat.



$2x = -6$

- Tell students that each cup must contain the same number of counters. Arrange the counters into two equal groups to correspond to the two cups.



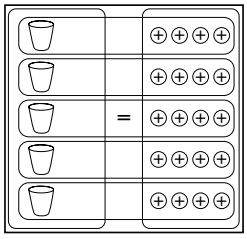
$2x = -6$

- Ask students what the solution of the equation is. **(-3)**

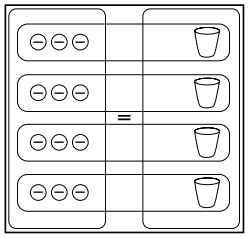
Have students complete Exercises 1–4 below.

1. Solve each equation using models.

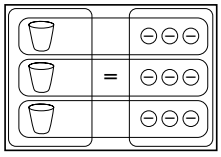
a. $5x = 20$ **(4)**



b. $-12 = 4x$ **(−3)**



c. $3x = 3$ **(1)**



2. What operation did you use to find each solution? **(division)**

3. The coefficient of an expression like $3x$ is the numerical part, 3. How can you use the coefficient to solve the equation $3x = 12$? **(Divide each side by the coefficient, 3.)**

4. How would you solve $2x = 5$ without using models? **(Divide 5 by 2.)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 354 in Lesson 9-4b of the Student Edition)

Solving Inequalities Using Models

Materials

cups, counters, equation mat

Your Turn

Solve each inequality using models.

a. $x + 5 > 9$ **b.** $x - 7 < 3$ **c.** $6 < x + 1$

Writing Math

1. **Compare and contrast** solving addition and subtraction inequalities with solving addition and subtraction equations.

2. **Examine** the inequality $x + 7 > 12$. Can the solution be $x = 8.5$? Explain your reasoning.

3. **MAKE A CONJECTURE** Write a rule for solving inequalities like $x - 3 > 8$ without using models.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 360–361 in Lesson 9-6a of the Student Edition)

Function Machines

Materials

scissors, tape

Your Turn

Make a function machine for each rule. Use the input values 0, 1, 2, and 3 for *n*. Record the input and output in a function table.

a. $n + 3$ **b.** $n + 5$ **c.** $n - 2$

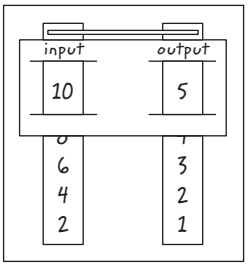
d. $n - 3$ **e.** $n \times 2$ **f.** $n \times 3$

Writing Math

Work in small groups.

1. **Explain** what a function machine would do for the rule $n \times 4$.

2. Use the function machine at the right. Complete the function table. Then write the function rule for the table.



Input	Output
10	5
8	
6	
4	
2	

3. **Explain** how a function machine would evaluate the rule $n \times 3 + 4$.

4. **Make** a function machine using the rule $n \times 3 + 4$. Use the numbers 1–5 as the input values. Record the input and output values in a function table.

5. **Create** your own function machine on a separate sheet of paper. Write pairs of inputs and outputs and have the other members of your group determine the rule.

6. Refer to page 361. **Tell** what the function rule is for each set of input and output values.

   a. _____ **b.** _____

7. **Explain** why using a function machine is like finding a pattern.

CHAPTER 10

Ratio, Proportion, and Percent
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Ratios and Tangrams
(p. 90 of this booklet)

Use With Lesson 10-1b. This corresponds to the activity on pp. 384–385 in the Student Edition.

Objective Explore ratios and the relationship between ratio and area.

Materials
2 sheets of patty paper
scissors

Students use a tangram to explore ratios and the relationship between ratio and area. They will compare the areas of triangles using ratios to discover the relationship between ratio and area.

Answers
See Teacher Wraparound Edition pp. 384–385.

Using Overhead Manipulatives

Ratios
(pp. 91–92 of this booklet)

Use With Lesson 10-2.

Objective Explore ratios that are equivalent and ratios that are not equivalent.

Materials
paper or colored transparency, if available
scissors
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- Using a triangle as the unit of measure, students create shapes and use them to show ratios that are equivalent and ratios that are not equivalent.
- Students then explain how to find equivalent ratios and how to determine when two ratios are not equivalent.

Answers
Answers appear on the teacher demonstration instructions on pp. 91–92.

Hands-On Lab Recording Sheet

Construct Scale Drawings
(p. 93 of this booklet)

Use With Lesson 10-3b. This corresponds to the activity on p. 394 in the Student Edition.

Objective Construct scale drawings.

Materials
grid paper

Students will create scale drawings given actual measurements and a scale. They will explain how the scale is used to determine the dimensions of an object on a scale drawing.

Answers
See Teacher Wraparound Edition p. 394.

Using Overhead Manipulatives

Fractions and Percents
(p. 94 of this booklet)

Use With Lesson 10-5.

Objective Express fractions as percents.

Materials
compass*
paper clip
12-inch ruler
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- This demonstration illustrates dropping a paper clip onto a piece of paper and recording the number of times it lands within a circle drawn on the paper.
- Students will write a ratio of the number of times the clip lands within the circle to the number of drops that are made.
- Using this ratio, students will find the percent of drops that land within the circle.

Answers
Answers appear on the teacher demonstration instructions on p. 94.

Hands-On Lab Recording Sheet

Percent of a Number
(p. 95 of this booklet)

Use With Lesson 10-7a. This corresponds to the activity on pages 407–408 in the Student Edition.

Objective Use a model to find the percent of a number.

Materials
grid paper

Students will draw models to find the percent of a number. They will explain how to find the percent of a number and how knowing 10% of a number can help them find the percent of a number.

Answers
See Teacher Wraparound Edition pp. 407–408.

Mini-Project

Percent and Estimation
(p. 96 of this booklet)

Use With Lesson 10-8.

Objective Estimate the percent of a figure that is shaded.

Materials
none

Students estimate the percent of each figure that is shaded by just looking. Then they count the grid squares to find the actual percent that is shaded. Students compare their estimates to the actual percents.

Answers
All estimates are sample answers.

1. 75%; 60%

2. 50%; 50%

3. 35%; 35%

4. 50%; 51%

5. Sample answer: Estimates are close to the actual percents, but not exact.

6. Answers will vary. See students' work.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 384–385 in Lesson 10-1b of the Student Edition)

Ratios and Tangrams

Materials

2 sheets of patty paper, scissors

Writing Math

Work with a partner. Refer to pp. 384–385.

1. Suppose the area of triangle B is 1 square unit. Find the area of each triangle below.

a. triangle C **b.** triangle F

2. Explain how the area of each of these triangles compares to the area of triangle B.

3. Explain why the ratio of the area of triangle C to the original large square is 1 to 8.

4. Tell why the area of square E is equal to the area of figure G.

5. Find the ratio of the area of triangle F to the original large square. Explain your reasoning.

6. Complete the table. Write the fraction that compares the area of each figure to the original square. What do you notice about the denominators?

Figure	A	B	C	D	E	F	G
Fractional Part of the Large Square							

Using Overhead Manipulatives

(Use with Lesson 10-2)

Ratios

Objective Explore ratios that are equivalent and ratios that are not equivalent.

Materials

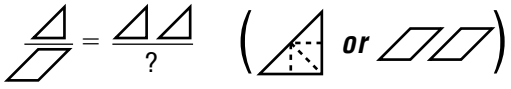
- paper or colored transparency, if available
- scissors
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

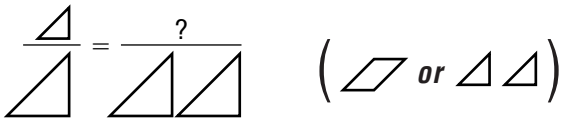
- Create the shapes shown below. Use a 2-inch square cut along a diagonal as the triangular unit of measure. The dashed lines on each shape are to be used only as a help in preparing the shapes, and should not appear on the finished pieces. You will need a minimum of 10 small triangles, 3 parallelograms, 2 trapezoids, 4 large triangles, and 2 pentagons.

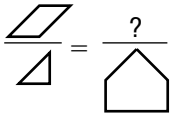
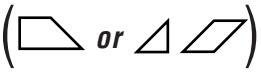
- Show students the small triangular shape. Say, "This triangle will be our unit of measure for this activity."
- Copy the following ratio using the shapes and a transparency pen. Ask students what shape or shapes can be used to make an equal ratio.

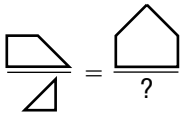
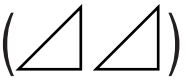


- Repeat the previous step for the following ratios. Be prepared to build any shape with the small triangles if needed to show students how the shapes compare.

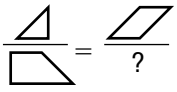
a. 

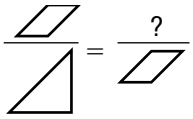
b. 

c.  ()

d.  ()

- Ask students to suggest a shape that has a ratio not equal to each ratio shown below.

e.  (*not* )

f.  (*not* )

- Ask, "How did you find the equivalent ratios?" **(Sample answer: Compare the known parts; keep the same comparison to find the unknown shape.)**
- Ask, "How did you know when two ratios were not equivalent?" **(Sample answer: The comparison between corresponding parts was not the same.)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 394 in Lesson 10-3b of the Student Edition)

Construct Scale Drawings

Materials

grid paper

Your Turn

a. A rectangular flower bed is 4 feet wide and 14 feet long. Make a scale drawing of the flower bed that has a scale of $\frac{1}{4}$ inch $=$ 2 feet.

b. A playground has dimensions 150 feet wide and 75 feet long. Make a scale drawing of the playground that has a scale of $\frac{1}{4}$ inch $=$ 10 feet.

Writing Math

1. **Explain** how the scale is used to determine the dimensions of the object on the scale drawing.

2. **Describe** $\frac{1}{2}$-inch grid paper.

3. Suppose you were making a scale drawing of a football field. What size grid paper would you use? What would be an appropriate scale?

Using Overhead Manipulatives

(Use with Lesson 10-5)

Fractions and Percents

Objective Express fractions as percents.

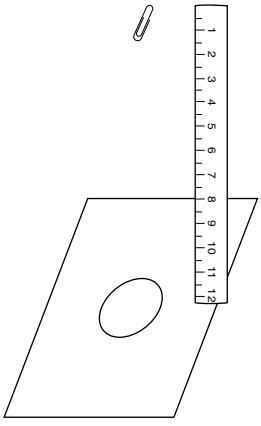
Materials

- compass*
- paper clip
- 12-inch ruler
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Draw a circle with a 3-inch diameter in the center of a blank transparency. Place this transparency on the screen.
- Hold the ruler on its end so that it shows a height of 12 inches. Drop the paper clip 20 times from a height of 12 inches. Record the number of times the clip lands completely within the circle. **(Sample answer: 9 times)**



- Ask students to name the ratio of the number of times the clip landed within the circle to the number of drops made. **(Sample answer: $\frac{9}{20}$)**
- Write $\frac{9}{20} = \frac{?}{100}$ at the bottom of the transparency. Ask students how to calculate the percent of drops that landed within the circle. **(Set up a proportion. Find the cross products. Solve for the unknown value.)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 407–408 in Lesson 10-7a of the Student Edition)

Percent of a Number

Materials

grid paper

Your Turn

Draw a model on grid paper to find the percent of each number.

a. 20% of 120 **b.** 60% of 70 **c.** 90% of 400

Draw a model on grid paper to find the percent of each number. If it is not possible to find an exact answer from the model, estimate.

d. 25% of 140 **e.** 7% of 50 **f.** 0.5% of 20

Writing Math

1. **Explain** how to determine the units that get labeled on the left side of the percent model.

2. **Write** a sentence explaining how you can find 7% of 50.

3. **Explain** how knowing 10% of a number will help you find the percent of a number when the percent is a multiple of 10%.

4. **Explain** how knowing 10% of a number can help you determine whether a percent of a number is a reasonable amount.

NAME _____ DATE _____ PERIOD ____

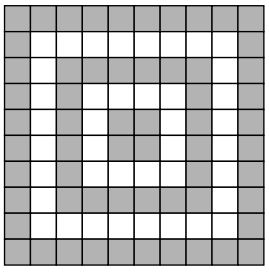
Mini-Project

(Use with Lesson 10-8)

Percent and Estimation

Estimate the percent of each figure that is shaded by just looking. Then count the grid squares to find the actual percent shaded.

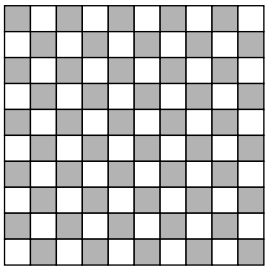
1.



Estimate: _____

Actual: _____

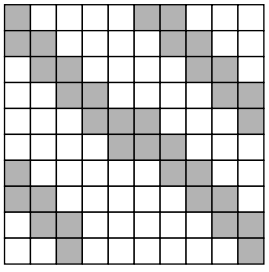
2.



Estimate: _____

Actual: _____

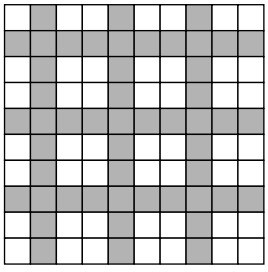
3.



Estimate: _____

Actual: _____

4.



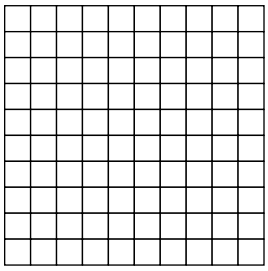
Estimate: _____

Actual: _____

5. How did your estimates compare with the actual percents?

_____

6. Shade your own grid. Estimate the percent shaded and count to find the exact percent.



Estimate: _____

Actual: _____

CHAPTER 11

Probability
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Simulations
(p. 99 of this booklet)

Use With Lesson 11-1a. This corresponds to the activity on pages 426–427 in the Student Edition.

Objective Explore experimental probability by conducting a simulation.

Materials
3 two-colored counters
cups
spinner

Students conduct simulations to find the experimental probabilities of given events. Space is provided for them to explain what experimental probability is and how a simulation is used to find the experimental probability of an event.

Answers
See Teacher Wraparound Edition pp. 426–427.

Using Overhead Manipulatives

Fair and Unfair Games
(pp. 100–101 of this booklet)

Use With Lesson 11-1.

Objective Explore fair and unfair games.

Materials
2 spinners*
blank transparency, prepared as described
transparency pens*

* = available in Overhead Manipulative Resources Kit

This demonstration contains two activities.

- Demonstration 1 shows students how to play an addition game using spinners and determine whether the game is fair.
- An Extension activity asks students to compare what results should occur most often in the game to the actual results of the game.
- Demonstration 2 shows students how to play a multiplication game using the spinners and determine whether the game is fair.

Answers
Answers appear on the teacher demonstration instructions on pages 100–101.

Hands-On Lab Recording Sheet

Experimental and Theoretical Probability
(p. 102 of this booklet)

Use With Lesson 11-1b. This corresponds to the activity on page 432 in the Student Edition.

Objective Compare experimental probability with theoretical probability.

Materials
2 number cubes

Students will conduct a simulation by rolling two number cubes. They will use the results from several trials to compare the experimental and theoretical probabilities of given events. Students will examine the results within their group, as well as the results of the other groups in the class.

Answers
See Teacher Wraparound Edition p. 432.

Using Overhead Manipulatives

Experimental Probability
(p. 103 of this booklet)

Use With Lesson 11-1b.

Objective Determine the experimental probability for a given set of data.

Materials

6-section spinner*
blank transparency
transparency pens*

* = available in Overhead Manipulative Resources Kit

- Students conduct a simulation by spinning a spinner 60 times to find the experimental probability of given events.
- Students will also use a given set of results from a different experiment to find the experimental probability of given events.

Answers

Answers appear on the teacher demonstration instructions on page 103.

Hands-On Lab Recording Sheet

Bias
(p. 104 of this booklet)

Use With Lesson 11-3a. This corresponds to the activity on page 437 in the Student Edition.

Objective Determine whether a group is biased.

Materials

none

Students answer given questions, record their responses, then compare results to determine whether the questions result in bias. They explain why and suggest ways to rewrite the questions so they do not result in answers that are biased.

Answers

See Teacher Wraparound Edition p. 437.

Mini-Project

Probability and Area
(p. 105 of this booklet)

Use With Lesson 11-4.

Objective Explore probability and area.

Materials

rice

Using a shaded grid, students find the area of various regions. Then they find the ratio of the area of each region to the total area of the grid. Using the same shaded grid, students drop grains of rice onto the grid and record where they land. Then they find the ratio of the number of grains in each region to the total number of grains dropped on the grid. Students compare the two sets of ratios and summarize their results.

Answers

1. 36 squares; 24 squares; 4 squares; 100 squares

2. $\frac{9}{25}$; $\frac{6}{25}$; $\frac{1}{25}$

3. See students' work.

4. See students' work.

5. Sample answer: The area ratios and grain ratios are close, but not exactly the same.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 426–427 in Lesson 11-1a of the Student Edition)

Simulations

Materials

3 two-colored counters, cups, spinner

Your Turn

a. Describe a simulation to explore the experimental probability that two of five children in a family are boys. Then conduct your experiment. What is the experimental probability?

b. The probability of rain on Monday is 0.75, and the probability of rain on Tuesday is 0.4. Describe a simulation you could use to explore the probability of rain on both days. Conduct your simulation to find the experimental probability of rain on both days.

Writing Math

1. Explain experimental probability.

2. How is a simulation used to find the experimental probability of an event?

Using Overhead Manipulatives

(Use with Lesson 11-1)

Fair and Unfair Games

Objective Explore fair and unfair games.

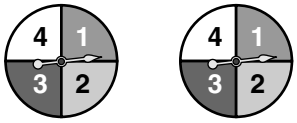
Materials

- 2 spinners*
- blank transparency, prepared as described below
- transparency pens*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration for Activity 1

- Tell students that in this activity you will explore fair and unfair games. Ask students to define a fair game. **(A fair game is a game in which players have an equal chance of winning.)** Then ask them to define an unfair game. **(In an unfair game, players do not have an equal chance of winning.)**
- Prepare two identical spinners with four equal sections. Number the sections on each spinner 1–4. (*Hint:* You could also use just one spinner and spin it twice for each round of the game.)



- On the blank transparency, prepare a chart like the one shown below.

Sum	2	3	4	5	6	7	8

Also prepare the addition table shown in the Extension for Activity 1, and the chart shown in Activity 2. Place the transparency on the screen so only the first table is showing. Use tape to hold it in place if necessary.

- Divide the class into two sections, for example, left side and right side. Call them Team One and Team Two. Tell them the rules for the game as follows. "We will spin the spinners. If the sum of the two numbers is even, Team One gets a point. If the sum of the two numbers is odd, Team Two gets a point."
- Spin both spinners. Keep track of each sum by marking an X above the number in the chart. Repeat until you have recorded 30 sums.
- Have a member from each team count and report the number of points for their team. **(Answers will vary; there will be about 15 of each.)**
- Ask students whether they think the game is fair and why or why not. **(This is a fair game. There are 8 ways to get an odd-numbered sum and**

8 ways to get an even-numbered sum. However, this particular trial might not look fair. Encourage students to express any reasonable answers.)

- The chart should resemble a bar graph. Ask students to describe the shape of the graph. **(Sample answer: The bars are lower on the ends and higher in the middle.)**
- Ask students which sums occurred most often and least often. **(Sample answer: 5; 2 and 8)**

Extension for Activity 1

Show students the addition table at the right. Ask the following questions. Which sum or sums from the addition table occur most often? **(5)** Which sum or sums occur least often? **(2 and 8)** Ask students to compare these answers to the results of the game. **(They should be very similar.)**

	1	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Teacher Demonstration for Activity 2

- Place the chart transparency so that the following chart is showing.

Product	1	2	3	4	6	8	9	12	16

- Tell students that for this activity, Team One gets a point if the product of the two numbers is even, and Team Two gets a point if the product is odd.
- Spin both spinners. Keep track of each product by marking an X above the number in the chart. Repeat until you have recorded 30 products.
- Have a member from each team count and report the number of points for their team. **(Answers will vary; there will be about 3 times as many even products as odd products.)**
- Ask students whether they think the game is fair and why or why not. **(Sample answer: The game in not fair because the even products occur much more often. Some students may suggest that this is because, although there are the same number of even and odd factors, the only products that are odd are the products of two odd numbers, while odd times even, even times odd, and even times even give even products.)**
- Ask them to describe the shape of the graph. **(Sample answer: It goes up and down more than once.)**
- Ask which product(s) occurred most often and least often. **(Sample answer: 4 occurred most often; 1, 9, and 16 occurred least often.)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 432 in Lesson 11-1b of the Student Edition)

Experimental and Theoretical Probability

Materials

2 number cubes

Writing Math

1. **Find** the experimental probability of rolling doubles for the 30 trials. How does the experimental probability compare to the theoretical probability? Explain any differences.

2. **Compare** the results of your experiment with the results of the other groups in your class. Why do you think experimental probabilities usually vary when an experiment is repeated?

3. **Find** the experimental probability for the entire class's trials. How does the experimental probability compare to the theoretical probability?

4. **Explain** why the experimental probability obtained in Exercise 3 may be closer in value to the theoretical probability than the experimental probability in Exercise 1.

Using Overhead Manipulatives

(Use with Lesson 11-1b)

Experimental Probability

Objective Determine the experimental probability for a given set of data.

Materials

- 6-section spinner*
- blank transparency
- transparency pens*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

	1	3	5
estimate			
actual			

- Prepare the spinner by writing the numbers from 1 to 6 in the sections of the spinner. Prepare the blank transparency with a chart like the one shown.
- Show students the chart and the spinner. Point out that the spinner has 6 equal-size numbered sections. Tell them that you are going to spin the spinner. Ask students to estimate the number of times the spinner will show 1, 3, and 5 if you spin it 60 times. **(Estimates may vary; a reasonable estimate is about 10 times for each.)**
- Have students help spin the spinner 60 times. Have another student make a tally of each spin in the chart.
- Compare the estimates to the actual results. **(Answers will vary.)**
- Tell students that experimental probability is a ratio that compares the number of ways a certain outcome occurs to the total number of outcomes. Since you spun the spinner 60 times, there were 60 outcomes in this experiment. Write a fraction comparing the number of times the spinner showed a 3 to 60. Tell students that this ratio represents the experimental probability of spinning a 3. Have students tell you how to use the results in the chart to write ratios representing the experimental probability of spinning a 1 and of spinning a 5. **(Answers will vary.)**

Have students complete Exercises 1–4 below.

Six index cards are labeled L, O, C, K, E, and R. Without looking, Jackie chooses a card, records its letter, and replaces it. She repeats the activity 48 times. The chart shows the results of her experiment.

L	O	C	K	E	R
8	7	10	10	5	8

1. What is the experimental probability of choosing an O?

2. What is the experimental probability of choosing an E?

3. What is the experimental probability of choosing an O or a C?

4. Find the experimental probability of choosing a vowel. Write the answer in simplest form.

(1. $\frac{7}{48}$ **2.** $\frac{5}{48}$ **3.** $\frac{17}{48}$ **4.** $\frac{1}{4}$**)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 437 in Lesson 11-3a of the Student Edition)

Bias

Materials

none

Writing Math

Work with a partner.

1. **Compare** the responses of your group to the responses of the other groups. Which questions may result in bias?

2. **Describe** the ways in which the wording of these questions may have influenced your answers.

3. **Tell** how these questions can be rewritten so they do not result in answers that are biased.

Tell whether each of the following survey locations might result in bias.

	Type of Survey	Survey Location
4.	favorite hobby	model train store
5.	favorite season	public library
6.	favorite TV show	skating rink
7.	favorite food	Mexican restaurant

NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 11-4)

Probability and Area

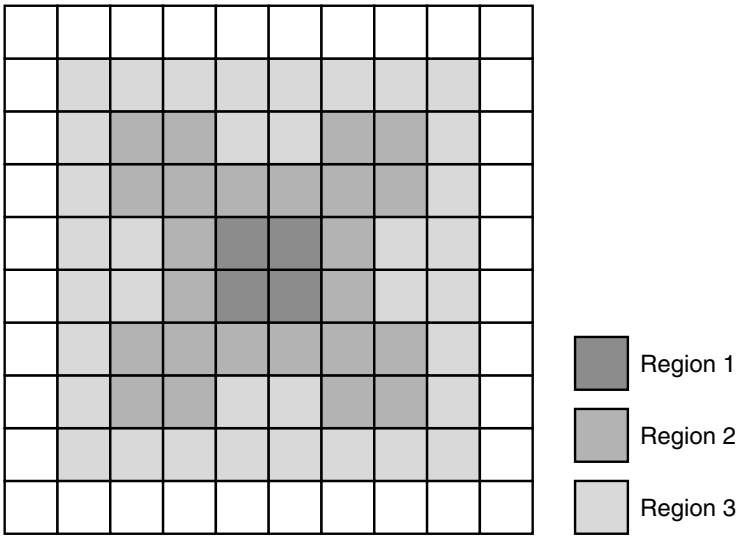
1. Count the number of squares in each region to find the area.

Region 1: _____

Region 2: _____

Region 3: _____

Total Area: _____



2. Find the ratio $\frac{\text{area of region}}{\text{total area}}$ for each region.

Region 1: _____

Region 2: _____

Region 3: _____

Drop 50 grains of rice onto the square target from a height of 2 or 3 inches. Mark an X wherever a grain lands inside the target. If a grain does not fall completely inside of one square, mark the X on the square that contains most of the grain.

3. Count the number of grains that landed in each region.

Region 1: _____

Region 2: _____

Region 3: _____

Total dropped on target: _____

4. For each region, find the ratio $\frac{\text{number of grains in region}}{\text{number of grains dropped on target}}$.

Region 1: _____

Region 2: _____

Region 3: _____

5. Compare the ratios in Exercise 2 to the ratios in Exercise 4 and write a summary of your results. _____

_____

_____

CHAPTER 12

Measurement
Teaching Notes and Overview

Hands-On Lab Recording Sheet

Area and Perimeter
(p. 108 of this booklet)

Use With Lesson 12-1a. This corresponds to the activity on p. 464 in the Student Edition.

Objective Explore changes in area and perimeter of rectangles.

Materials
centimeter grid paper

Students will find the area and perimeter of different rectangles to investigate relationships between the areas and perimeters of original figures and those of the new figures. They will discover how changing the dimensions of a figure affects its area and perimeter.

Answers
See Teacher Wraparound Edition p. 464.

Using Overhead Manipulatives

Measurement
(pp. 109–110 of this booklet)

Use With Lesson 12-2.

Objective Compare weights among the Sun and the planets.

Materials
transparency showing weight relative to Earth
6 measuring cups
10 containers (having a 3-cup capacity)
water
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- Using containers of water, students compare weights among the Sun and the planets.
- Students find the weight of various objects on certain planets.
- An Extension activity asks students to find their relative Earth weight for several planets.

Answers
Answers appear on the teacher demonstration instructions on pp. 109–110.

Hands-On Lab Recording Sheet

The Metric System
(p. 111 of this booklet)

Use With Lesson 12-3a. This corresponds to the activity on pp. 474–475 in the Student Edition.

Objective Measure in metric units.

Materials
tape measure

Students use metric units to measure several items. They explain which metric unit is most appropriate for each item. Then they examine their measurements to identify a relationship between the numbers. Students also select three objects around the classroom that would be best measured in each of three metric units. Finally, students will measure the sides of rectangles and find the area and perimeter of each one.

Answers
See Teacher Wraparound Edition pp. 474–475.

Hands-On Lab Recording Sheet

Significant Digits

(p. 112 of this booklet)

Use With Lesson 12-3b. This corresponds to the activity on pages 480–481 in the Student Edition.

Objective Determine and apply significant digits in a real-life context.

Materials

centimeter ruler
meterstick
yardstick
tape measure

Students measure several items in the classroom using the smallest unit on their measuring tool. Then they find each measure using significant digits. Students will explain how they used significant digits to find the measures. Students will also describe a real-life situation in which measurement precision was used, as well as a real-life situation in which a less precise or estimated measure was sufficient.

Answers

See Teacher Wraparound Edition pp. 480–481.

Mini-Project

Using the Metric System

(p. 113 of this booklet)

Use With Lesson 12-5.

Objective Measure in metric units.

Materials

centimeter ruler

Students measure several items in centimeters. They write the measurement of each item in millimeters, meters, and kilometers.

Answers

1. 5 cm; 50 mm; 0.05 m; 0.005 km

2. 3 cm; 30 mm; 0.03 m; 0.003 km

3. 7.5 cm; 75 mm; 0.075 m; 0.0075 km

4. 1.5 cm; 15 mm; 0.015 m; 0.0015 km

5–8. Answers will vary. See students' work.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 464 in Lesson 12-1a of the Student Edition)

Area and Perimeter

Materials

centimeter grid paper

Record your data in the table below.

Rectangle	Length (cm)	Width (cm)	Area (sq cm)	Perimeter (cm)
original	6	2		
A	12	4		
B	18	6		
C	24	8		

Writing Math

1. Describe how the dimensions of rectangles A, B, and C are different than the original rectangle.

2. Describe how the area of the original rectangle changed when the length and width were both doubled.

3. Describe how the perimeter of the original rectangle changed when the length and width were both doubled.

4. Describe how the area and the perimeter of the original rectangle changed when the length and width were both tripled.

5. **Draw** a rectangle whose length and width are half those of the original rectangle. Describe how the area and perimeter changes.

6. Suppose the perimeter of a rectangle is 15 centimeters. **Make a conjecture** about the perimeter of the rectangle if the length and the width are both doubled.

Using Overhead Manipulatives

(Use with Lesson 12-2)

Measurement

Objective Compare weights among the Sun and the planets.

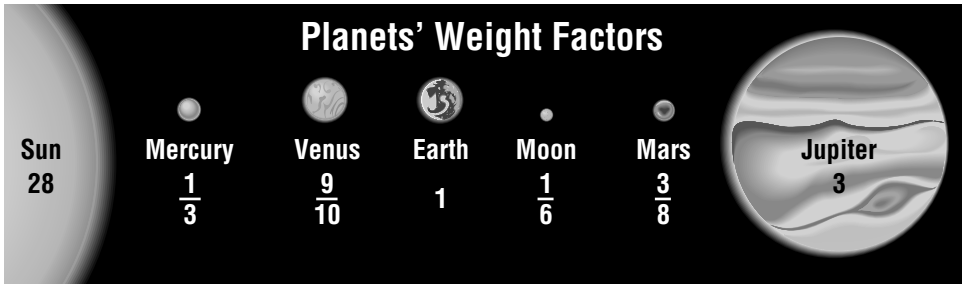
Materials Needed

- transparency showing weight relative to Earth
- 6 measuring cups
- 10 containers (having a 3-cup capacity)
- water
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Prepare a blank transparency with the diagram shown below.



- Fill a measuring cup with one cup of water to represent the weight of water on Earth. Pour this amount into a container. Label the container Earth.
- Show the transparency of Weight relative to Earth. Separate the class into 5 groups and assign each group a planet or the moon. Each group should have two containers, a measuring cup, and water.
- Instruct the groups to pour one cup of water into a container and label it Earth, as you have shown.
- Instruct each group to fill as many measuring cups with water as needed to represent the weight of water of their planet. For example, the moon's weight is $\frac{1}{6}$ that of Earth, so that group would empty $\frac{1}{6}$ cup of water into a container and label it Moon.
- Ask each group which of their two containers weighs more and why. On the blank transparency, record the name of each planet and whether the water weighs more or less than on Earth. **(Mercury, less; Venus, less; Moon, less; Mars, less; Jupiter, more)**

- Then ask whether the water on each planet weighs more or less than the weight of one cup of water on Jupiter. **(less)**
- Have students complete Exercises 2–5 on page 295.

Have students complete Exercises 1–4 below.

1. Which planet's container weighs the most? Explain your reasoning. **(The container labeled *Jupiter*. It has the most water.)**

2. How much would a 22-pound dog weigh on Jupiter? **(66 pounds)**

3. How much would you weigh on the moon? **(Sample answer: A 126-pound student would weigh 21 pounds.)**

4. On Earth, a certain object weighs 1 pound. How many ounces would the same object weigh on Mars? **(6 ounces)**

Extension

Have students do research to find their relative Earth weight for the planets Saturn, Neptune, Uranus, and Pluto.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 474–475 in Lesson 12-3a of the Student Edition)

The Metric System

Materials

tape measure

Record your data in the table at the right.

Object	Measure		
	m	cm	mm
width of math book			
length of pencil			
length of sheet of paper			
length of your hand			
width of your little finger			
length of table or desk			
length of chalkboard eraser			
width of door			
height of door			
distance from doorknob to the floor			
length of classroom			

Writing Math

1. **Tell** which unit of measure is most appropriate for each item. How did you decide which unit is most appropriate?

2. **Examine** the pattern between the numbers in each column. What relationship do the numbers have to each other?

3. **Select** three objects around your classroom that would be best measured in meters, three objects that would be best measured in centimeters, and three objects that would be best measured in millimeters. Explain your choices.

4. **Write** the name of a common object that you think has a length that corresponds to each length. Explain your choices.
   a. 5 centimeters **b.** 3 meters **c.** 1 meter **d.** 75 centimeters

Refer to page 475 for Exercises 5–7. Write your answers below.

5. _____ **6.** _____ **7.** _____

_____ _____ _____

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 480 in Lesson 12-3b of the Student Edition)

Significant Digits

Materials

centimeter ruler, meterstick, yardstick, tape measure

Record your data in the table at the right.

Writing Math

Identify the smallest unit of each measuring tool on page 481.

1.

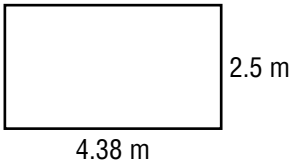
2.

Object	Measure	
	Using Smallest Unit	Using Significant Digits
height of desk		
length of calculator		
length of a pencil		
height of door		
length of classroom		
width of classroom		

3. Explain how you used significant digits to find the measures.

4. Choose the most precise unit of measurement: inches, feet, or yards. Explain.

5. When adding measurements, the sum should have the same precision as the *least precise* measurement. Find the perimeter of the rectangle at the right using significant digits.



6. Describe a real-life situation in which a family member or neighbor used measurement precision. Then describe a real-life situation in which a less precise or estimated measure is sufficient.

7. Find the area of your classroom using the precision unit measures from your table and using the significant digit measures. Which area is more precise? Explain.

NAME _____ DATE _____ PERIOD ____

Mini-Project

(Use with Lesson 12-5)

Using the Metric System

Use a ruler to measure the lines in centimeters. Then write your measurement in millimeters, meters, and kilometers.

1. _____ centimeters

_____ millimeters _____ meters _____ kilometers

2. _____ centimeters

_____ millimeters _____ meters _____ kilometers

3. _____ centimeters

_____ millimeters _____ meters _____ kilometers

4. _____ centimeters

_____ millimeters _____ meters _____ kilometers

Find four small objects in your classroom to measure with your ruler. Record what you are measuring. Record its length in centimeters. Write the measurement in millimeters, meters, and kilometers.

5. Object: _____ _____ centimeters

_____ millimeters _____ meters _____ kilometers

6. Object: _____ _____ centimeters

_____ millimeters _____ meters _____ kilometers

7. Object: _____ _____ centimeters

_____ millimeters _____ meters _____ kilometers

8. Object: _____ _____ centimeters

_____ millimeters _____ meters _____ kilometers

CHAPTER 13

Geometry: Angles and Polygons
Teaching Notes and Overview

Using Overhead Manipulatives

Angles
(pp. 117–118 of this booklet)

Use With Lesson 13-2.

Objective Estimate measures of angles.

Materials
wax paper, or colored transparency
compass*
scissors
protractor*
blank transparencies
transparency pens*

* = available in Overhead Manipulative Resources Kit

- Students estimate the measures of angles using sections cut out of a circle.
- Students discuss how they could estimate the measure of an angle without using a protractor or the sections from the circle.

Answers

Answers appear on the teacher demonstration instructions on pages 117–118.

Hands-On Lab Recording Sheet

Construct Congruent Segments and Angles
(p. 119 of this booklet)

Use With Lesson 13-3a. This corresponds to the activity on pages 513–514 in the Student Edition.

Objective Construct congruent segments and angles.

Materials
straightedge
compass

Students construct segments congruent to given segments and explain the process they use. They also construct angles congruent to given angles and explain the process they use.

Answers

See Teacher Wraparound Edition pp. 513–514.

Using Overhead Manipulatives

Bisecting Segments
(p. 120 of this booklet)

Use With Lesson 13-3.

Objective Use paper-folding to bisect a line segment.

Materials
ruler*
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to use paper-folding to bisect a line segment.
- Students review the definition of *bisect* and discover the midpoint of the line segment.

Answers

Answers appear on the teacher demonstration instructions on page 120.

Using Overhead Manipulatives

Bisecting Angles
(p. 121 of this booklet)

Use With Lesson 13-3.

Objective Use paper-folding to bisect an angle.

Materials
straightedge*
protractor*
blank transparency
transparency pen*

* = available in Overhead Manipulative Resources Kit

- This demonstration shows how to use paper-folding to bisect an angle.
- Students discover that bisecting an angle creates two angles with the same measure.

Answers

Answers appear on the teacher demonstration instructions on page 121.

Hands-On Lab

Recording Sheet
Triangles and Quadrilaterals
(p. 122 of this booklet)

Use With Lesson 13-4b. This corresponds to the activity on pages 526–527 in the Student Edition.

Objective Explore, classify, and draw triangles and quadrilaterals.

Materials
notebook paper
scissors
protractor
dot paper
colored pencils

Students find missing angle measures for triangles and classify them as *acute, right,* or *obtuse.* Students will also draw quadrilaterals and compare them using given characteristics.

Answers

See Teacher Wraparound Edition pp. 526–527.

Mini-Project

Lines of Symmetry
(p. 123 of this booklet)

Use With Lesson 13-5.

Objective Identify lines of symmetry.

Materials
scissors
sheet of paper

Students cut out any shape from a folded sheet of paper. They identify all lines of symmetry and explain their reasoning.

Answers

1. Answers will vary.

2. Sample answers: No; there is no place I can fold the paper and have the same shape on both sides. Yes; I can fold the paper so the shape on each side of the line is the same.

3. no

4. no

5. When you alter one side of the paper and not the other, the fold is no longer a line of symmetry.

Hands-On Lab Recording Sheet

Transformations

(p. 124 of this booklet)

Use With Lesson 13-5b. This corresponds to the activity on pages 532–533 in the Student Edition.

Objective Investigate transformations.

Materials

grid paper
pattern blocks
geomirror
colored pencils

Students perform given transformations of a figure on a coordinate grid. They also describe and identify transformations.

Answers

See Teacher Wraparound Edition pp. 532–533.

Hands-On Lab Recording Sheet

Tessellations

(p. 125 of this booklet)

Use With Lesson 13-6b. This corresponds to the activity on page 537 in the Student Edition.

Objective Create tessellations using pattern blocks.

Materials

pattern blocks

Students create tessellations using pattern blocks shown. Students also identify two figures that cannot be used to create a tessellation and use a drawing to justify their answers.

Answers

See Teacher Wraparound Edition p. 537.

Using Overhead Manipulatives

(Use with Lesson 13-2)

Angles

Objective Estimate measures of angles.

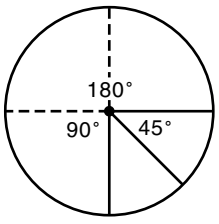
Materials

- wax paper, or colored transparency
- compass*
- scissors
- protractor*
- blank transparencies
- transparency pens*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- Use the compass to draw a circle with a radius of 4 inches on wax paper or a colored transparency. Cut out the circle.
- Place the circle on the screen. Show students as you fold it in half and then in half again. Unfold the circle and tell students that the point where the folds intersect is the center of the circle. Mark the center.
- Draw a line from the center to the edge of the circle along one of the folds. Ask students how you could draw an angle of 180° using this line as one side of the angle. **(Since 180° is a straight angle, you can extend the line to the other side of the circle. Another solution is to position a protractor with the center mark at the center of the circle and the 0° mark on the line. Find 180° on the opposite side. Make a mark. Draw a line from the vertex through the mark.)** Draw the angle and write the measure on the paper.
- On the same circle, draw angles of 90° and 45°, as shown. Label each section. Cut out the labeled angles. Make sure the edges of each section have pen lines showing. (If you are using a colored transparency and do not wish to cut it, you can use the angles without cutting the circle.)



- Ask a student to draw an angle on a blank transparency. Then ask students how they could use the labeled sections to estimate the measure of this angle. **(Choose a section to compare. Place the side of the angle on the side of that section. See if the angle is greater or less than the labeled angle. Compare to the next larger or smaller section.**

Continue until you find the 2 measures that the angle is between. Then estimate.) Measure the angle with a protractor to verify the accuracy of the estimate.

- Repeat for several different angles.
- Ask students to discuss how they could estimate the measure of an angle without using a protractor or the sections above. **[Sample answer: Compare the angle to 0°, 90° (like a square corner), and 180° (straight line). Then estimate how far it is from the known angle and add or subtract that amount. For example, if it is about halfway between 0° and 90°, then the angle is about 45°.]**
- Show students the remaining circle wedge. Ask what the measure of this remaining wedge is. **(45°)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 513–514 in Lesson 13-3a of the Student Edition)

Construct Congruent Segments and Angles

Materials

straightedge, compass

Your Turn

Construct a segment congruent to each segment below.

a. 

b. 

c. 

Writing Math

1. Explain, in your own words, how to construct a line segment that is congruent to a given line segment.

2. Find the measure of $\overline{JK}$. How does this compare to the measure of $\overline{LP}$?

3. Suppose the length of $\overline{JK}$ is 26 centimeters. If $\overline{JK}$ is separated into two congruent parts, what will be the length of each part? Explain.

4. Explain the relationship between $\angle JKM$ and $\angle UST$.

5. Explain how to construct an angle that is congruent to a 65° angle.

Using Overhead Manipulatives

(Use with Lesson 13-3)

Bisecting Segments

Objective Use paper-folding to bisect a line segment.

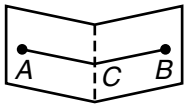
Materials

- ruler*
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- On a blank transparency, show students as you use a straightedge to draw a line segment and label it $\overline{AB}$.
- Fold point A onto point B and pinch the transparency where the fold intersects the segment. Open the transparency and mark the intersection point. Label it point C.



- Review the definition of *bisect*. Point out that the crease bisects $\overline{AB}$. Measure $\overline{AC}$ and $\overline{CB}$. Ask, "What can you say about the location of point C?" **(It is halfway between points *A* and *B*.)**

Using Overhead Manipulatives

(Use with Lesson 13-3)

Bisecting Angles

Objective Use paper-folding to bisect an angle.

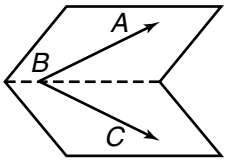
Materials

- straightedge*
- protractor*
- blank transparency
- transparency pen*

* = available in Overhead Manipulative Resources Kit

Teacher Demonstration

- On a blank transparency, show students as you use a straightedge to draw an acute angle and label it $\angle ABC$.



- Fold ray BC onto ray BA and make a crease. Draw a dashed line along the crease. Point out that the crease bisects $\angle ABC$.
- Use a protractor to measure the two angles formed by the crease. Ask, "What can you say about the two angles?" **(They have the same measure.)**

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 526–527 in Lesson 13-4b of the Student Edition)

Triangles and Quadrilaterals

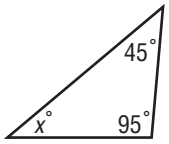
Materials

notebook paper, scissors, protractor, dot paper, colored pencils

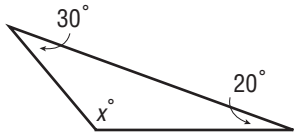
Your Turn

Find the missing angle measure for each triangle shown. Then classify each triangle as *acute, right,* or *obtuse.*

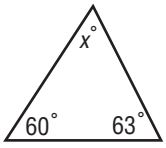
a.



b.



c.



Writing Math

1. If a triangle has angles with measures 45°, 35°, and 100°, what type of triangle is it? Explain.

2. Is the statement *All rectangles are parallelograms, but not all parallelograms are rectangles* true or false? Explain.

3. Tell why a triangle must always have at least two acute angles. Include drawings in your explanation.

4. Two different quadrilaterals each have four congruent sides. However, one has four 90° angles, and the other has no 90° angles. Draw the figures and compare them using the given characteristics.

NAME _____ DATE _____ PERIOD ____

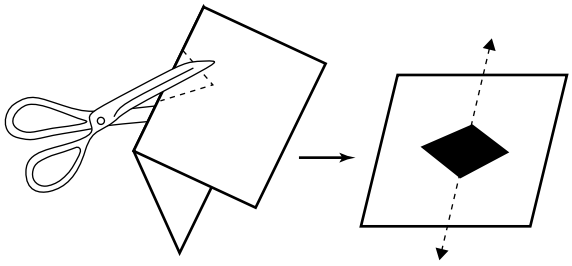
Mini-Project

(Use with Lesson 13-5)

Lines of Symmetry

Materials

scissors
sheet of paper
pencil



Take a sheet of paper and fold it in half. Begin cutting out a pattern at the folded edge. Cut any shape that comes to mind. Unfold what remains of your paper. The folded line is the line of symmetry because the shape on each side of the line is the same. You can trace the fold with a pen or pencil.

1. Are there any other lines of symmetry? _____

2. How do you know? _____

_____

_____

_____

Now take your paper with the line of symmetry. Without folding it, cut one section out of one side. Refold on the same line as before.

3. Do the sides still match? _____

4. Is the fold line still a line of symmetry? _____

5. How do you know? _____

_____

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 532–533 in Lesson 13-5b of the Student Edition)

Transformations

Materials

grid paper, pattern blocks, geomirror, colored pencils

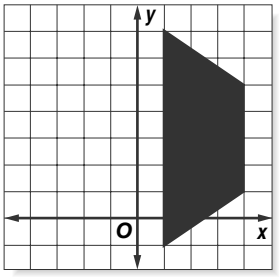
Your Turn

Using the pattern block shown, perform each transformation described on a coordinate grid.

a. a translation 5 units left

b. a reflection across the *y*-axis

c. a 90° rotation counterclockwise



Writing Math

1. A square is transformed across the *y*-axis. How could this transformation be interpreted as a slide, a flip, and a turn?

2. A rectangle has vertices at (3, 2), (7, 2), (3, 8), and (7, 8). What happened to the figure if its transformed image has vertices at (8, 2), (12, 2), (8, 8), and (12, 8)?

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 537 in Lesson 13-6b of the Student Edition)

Tessellations

Materials

pattern blocks

Your Turn

Create a tessellation using the pattern blocks shown.

a. 

b. 

c. 

Writing Math

1. **Tell** if a tessellation can be created using a square and an equilateral triangle. Justify your answer with a drawing.

2. What is the sum of the measures of the angles where the vertices of the figures meet? Is this true for all tessellations?

3. Name two figures that cannot be used to create a tessellation. Use a drawing to justify your answer.

CHAPTER 14

Geometry: Measuring Area and Volume
Teaching Notes and Overview

Using Overhead Manipulatives

Area of Irregular Shapes
(pp. 129–130 of this booklet)

Use With Lesson 14-1.

Objective Find the area of irregular shapes.

Materials
centimeter grid transparency*
transparency pens*
*= available in Overhead Manipulative Resources Kit

This demonstration contains two activities.
- Demonstration 1 shows how to estimate the area of an irregular figure on a centimeter grid.
- Demonstration 2 shows how to estimate the area of an irregular figure by drawing a rectangle around the figure.
- Students explain which estimation method they think is more accurate.
- An Extension activity asks students to choose a method and estimate the area of a circle and a triangle on a centimeter grid.

Answers
Answers appear on the teacher demonstration instructions on pages 129–130.

Hands-On Lab Recording Sheet

Area of Triangles
(p. 131 of this booklet)

Use With Lesson 14-2a. This corresponds to the activity on page 550 in the Student Edition.

Objective Find the area of a triangle using the properties of parallelograms.

Materials
grid paper
colored pencils
scissors

Students use the properties of parallelograms to find the area of triangles. Students discover the formula for the area of a triangle.

Answers
See Teacher Wraparound Edition p. 550.

Mini-Project

Areas of Triangles
(p. 132 of this booklet)

Use With Lesson 14-2.

Objective Find the areas of triangles.

Materials
metric ruler

Students use a metric ruler to measure the base and height of several triangles. Using these measurements, students find the area of each triangle to the nearest square millimeter.

Answers
Sample answers are given.

1. 450 mm^2

2. 672 mm^2

3. 308 mm^2

4. 263 mm^2

5. 463 mm^2

6. 442 mm^2

Hands-On Lab Recording Sheet

Area of Trapezoids

(p. 133 of this booklet)

Use With Lesson 14-2b. This corresponds to the activity on page 555 in the Student Edition.

Objective Find the area of a trapezoid using the properties of triangles.

Materials
grid paper

Students use the properties of triangles to find the area of trapezoids. They explain how the area of a trapezoid is related to the area of a triangle and compare the formulas for the area of each.

Answers
See Teacher Wraparound Edition p. 555.

Hands-On Lab Recording Sheet

Making Circle Graphs

(p. 134 of this booklet)

Use With Lesson 14-3b. This corresponds to the activity on pages 560–561 in the Student Edition.

Objective Make circle graphs.

Materials
colored pencils
ruler
compass
protractor
calculator

Students interpret data displayed in a circle graph. They also create circle graphs for given sets of data. Students compare data displayed in a table versus a circle graph. Students explain how each set of data compares part to whole relationships and how the area of a circle is related to making a circle graph.

Answers
See Teacher Wraparound Edition pp. 560–561.

Hands-On Lab Recording Sheet

Three-Dimensional Figures

(p. 135 of this booklet)

Use With Lesson 14-4b. This corresponds to the activity on page 567 in the Student Edition.

Objective Draw three-dimensional figures.

Materials
isometric dot paper
ruler

Students draw a rectangular prism on isometric dot paper and explain which faces are the bases of the prism. Students draw other three-dimensional figures and explain why they think isometric dot paper is used to draw a three-dimensional object.

Answers
See Teacher Wraparound Edition p. 567.

Hands-On Lab Recording Sheet

Using a Net to Build a Cube

(p. 136 of this booklet)

Use With Lesson 14-6a. This corresponds to the activity on page 574 in the Student Edition.

Objective Build a three-dimensional figure from a net and vice versa.

Materials

cube
scissors
paper

Students identify and draw nets that will form a cube, as well as nets that will not form a cube. They draw a net for a rectangular prism and compare it to the nets that form a cube. Finally, students identify the figures formed by given nets.

Answers

See Teacher Wraparound Edition p. 574.

Using Overhead Manipulatives

(Use with Lesson 14-1)

Area of Irregular Shapes

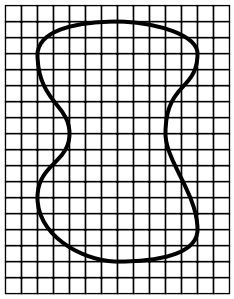
Objective Find the area of irregular shapes.

Materials

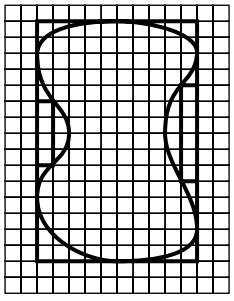
- centimeter grid transparency*
- transparency pens*

*= available in Overhead Manipulative Resources Kit

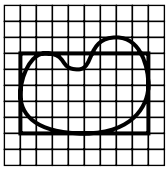
Teacher Demonstration for Activity 1



- On the centimeter grid transparency, draw a figure similar to the one shown at the right.
- Shade the whole squares within the outline of the figure. Have students count the number of shaded squares. Record this number on the grid.
- Using a different colored transparency pen, shade the squares that touch the outline anywhere. Ask students to count the number of these squares. Add this to the number of whole squares within the outline. Record this number on the grid.
- Ask students to find the mean of the two recorded numbers. Ask students how this number represents an estimate of the area within the outline. **(The area is at least the number of whole squares and at most the number of whole and partial squares. The mean is halfway between these two numbers.)**
- Ask students to think of another way to estimate the area of the figure. **(Sample answer: Draw a rectangle around the shape and 2 small rectangles in the area not in the shape. Subtract the area of the small rectangles from the area of the large rectangle.)**



Teacher Demonstration for Activity 2

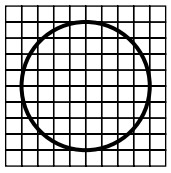


- On the centimeter grid transparency, draw a figure similar to the one shown at the right.
- Draw a rectangle that encloses most of the figure.
- Have students count squares to find the length and the width of the rectangle. Record the numbers on the grid. **(Answers will vary.)**
- Ask students to estimate the area of the figure by finding the area of the rectangle. **(The estimate is the length times the width.)**
- Next, use the method in Activity 1 to estimate the area of the figure. **(Answers will vary.)**
- Ask students which estimation method they think is more accurate. Have them explain their reasoning. **(Answers will vary. In general, the method using the rectangle that encloses most of the figure is more accurate when the parts of the figure inside the rectangle and outside the rectangle are about the same.)**

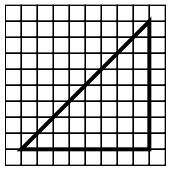
Extension

On the centimeter grid, draw the following figures. Ask students to describe a way to estimate the area of each figure. Then estimate each area using those methods. **(Answers will vary.)**

a. circle



b. triangle



NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 550 in Lesson 14-2a of the Student Edition)

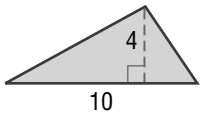
Area of Triangles

Materials

grid paper, colored pencils, scissors

Your Turn

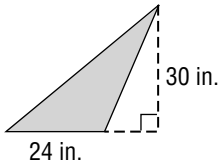
a. Draw the triangle shown on grid paper. Then draw a parallelogram and find the area of the triangle.



Writing Math

1. Suppose a parallelogram has an area of 84 square units with a height of 7 units. **Describe** a triangle related to this parallelogram, and find the triangle's area, base, and height.

2. Draw a parallelogram that is related to the triangle at the right. How could you use the drawing to find the area of the triangle?



3. **Write** a formula for the area of a triangle.

NAME _____ DATE _____ PERIOD ____

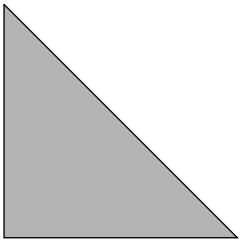
Mini-Project

(Use with Lesson 4-2)

Areas of Triangles

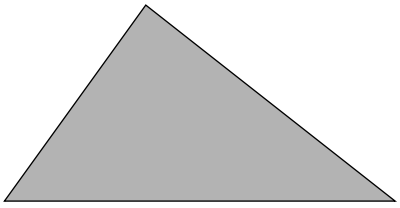
Use a metric ruler to measure the base and height of each triangle. If the height is not shown, sketch it. Label these segments with their measurements to the nearest millimeter. Use your measurements to calculate the area to the nearest square millimeter.

1.



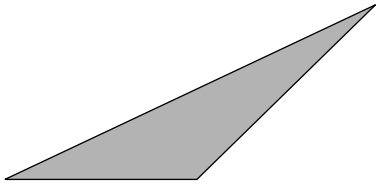
Area = _____

2.



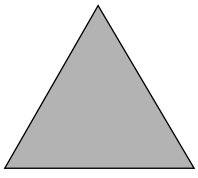
Area = _____

3.



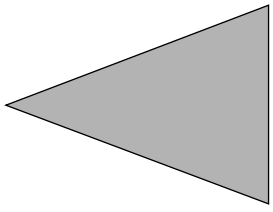
Area = _____

4.



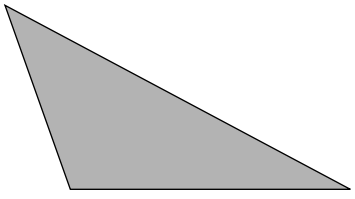
Area = _____

5.



Area = _____

6.



Area = _____

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 555 in Lesson 14-2b of the Student Edition)

Area of Trapezoids

Materials

grid paper

Your Turn

a. Find the area of the trapezoid on page 555.

Writing Math

1. Explain why the area of a trapezoid is related to the area of a triangle.

2. Why can $A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$ be written as $A = \frac{1}{2}h(b_1 + b_2)$?

3. Explain how you would separate any trapezoid into triangles to find its area.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on pages 560–561 in Lesson 14-3b of the Student Edition)

Making Circle Graphs

Materials

colored pencils, ruler, compass, protractor, calculator

"How important is sunny weather in a vacation location?" The circle graph on page 560 shows how people responded to this question.

1. What percent of the people said that having sunshine while on vacation was not at all important?

2. What percent is represented by the whole circle graph? How many degrees are in the circle?

3. Explain when a circle graph is the best choice to display a set of data.

Your Turn

Refer to page 561 for Exercises a and b. Use the space below to display each set of data in a circle graph.

a. **b.**

Writing Math

1. **Compare** each circle graph to its corresponding table. Does the graph or table display the data more clearly? Explain.

2. **Examine** each data set you displayed. Explain how each set of data compares part to whole relationships.

3. **Give an example** of a data set that *cannot* be represented by a circle graph. What type of graph would you use to best represent the data set?

4. **Explain** how the area of a circle is related to making a circle graph.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 567 in Lesson 14-4b of the Student Edition)

Three-Dimensional Figures

Materials

isometric dot paper, ruler

Writing Math

1. Explain which faces are the bases of the prism.

2. Use isometric dot paper to draw each figure.

a. a cube with length, width, and height of 3 units

b. a rectangular prism with length 4 units, width 2 units, and height 2 units

3. How would you draw a prism with a triangular base?

4. Explain why you think isometric dot paper is used to draw a three-dimensional object.

5. Suppose you need to draw a three-dimensional representation of a sphere. Do you think this method would work? Explain.

NAME _____ DATE _____ PERIOD ____

Hands-On Lab Recording Sheet

(Use with the activity on page 574 in Lesson 14-6a of the Student Edition)

Using a Net to Build a Cube

Materials

cube, scissors, paper

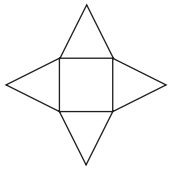
Writing Math

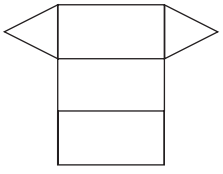
1. **Explain** whether both nets formed a cube. If not, describe why the net or nets did not cover the cube.

2. **Draw** three other nets that will form a cube and three other nets that will not form a cube. Describe a pattern in the nets that do form a cube.

3. **Draw** a net for a rectangular prism. Explain the difference between this net and the nets that formed a cube.

4. **Tell** what figure would be formed by each net. Explain.

a. 

b. 

c.