

Ratios

Examples \rightarrow 3 to 4, 3:4, $\frac{3}{4}$
10 to 7, 10:7, $\frac{10}{7}$
a to b, a:b, $\frac{a}{b}$

Write each ratio as a fraction in simplest form.

1) 20 boys: 12 girls 2) 35 pencils to 50 pens

$$\frac{20 \div 4}{12 \div 4} = \frac{5}{3}$$

$$\frac{35 \div 5}{50 \div 5} = \frac{7}{10}$$

Determine whether the ratios are equivalent.

3) 48 questions to 20 multiple choice
36 questions to 15 multiple choice

$$\frac{48 \div 4}{20 \div 4} = \frac{12}{5}$$

$$\frac{36 \div 3}{15 \div 3} = \frac{12}{5}$$

} Yes, they are equivalent.

Rates

Examples \rightarrow 55 mph, \$25/4hr, 25.5 students/class

Find the unit rate. Round to the nearest hundredth if necessary.

1) 100 miles on 14 gallons

$$\frac{100 \text{ mi} \div 14}{14 \text{ gal} \div 14} = \boxed{\frac{7.14 \text{ mi}}{1 \text{ gal}}}$$

2) \$3.36 for 16.4 ounces

$$\frac{\$3.36 \div 16.4}{16.4 \text{ oz} \div 16.4} = \boxed{\frac{\$.20}{1 \text{ oz}}}$$

3) \$5.99 for 12-Pack of Mt. Dew

$$\frac{\$5.99 \div 12}{12 \text{ cans} \div 12} = \boxed{\frac{\$.50}{1 \text{ can}}}$$

Proportions

Determine if the quantities in each pair of ratios are proportional.

1) 10 boys for 15 girls and 14 boys for 21 girls

Method #1

$$\left. \begin{array}{l} \frac{10 \div 5}{15 \div 5} = \frac{2}{3} \\ \frac{14 \div 7}{21 \div 7} = \frac{2}{3} \end{array} \right\} \text{Yes}$$

Method #2

$$\begin{array}{l} \frac{10 \times 21}{15 \times 14} \\ 10 \cdot 21 \stackrel{?}{=} 15 \cdot 14 \\ 210 = 210 \checkmark \text{ Yes} \end{array}$$

Solve each proportion.

$$\begin{array}{l} 2) \quad \frac{7 \times 3}{x \times 6} \\ 3 \cdot x = 7 \cdot 6 \\ \frac{3x}{3} = \frac{42}{3} \\ \boxed{x = 14} \end{array}$$

$$\begin{array}{l} 3) \quad \frac{2.5}{4.5} = \frac{7.5}{x} \\ (2.5)x = (4.5)(7.5) \\ \frac{2.5x}{2.5} = \frac{33.75}{2.5} \\ \boxed{x = 13.5} \end{array}$$

Fractions and Percents

Write each percent as a fraction in simplest form.

$$1) 32.5\% = \frac{32.5 \times 10}{100 \times 10} = \frac{325 \div 5}{1,000 \div 5} = \frac{65 \div 5}{200 \div 5} = \boxed{\frac{13}{40}}$$

$$2) 6\frac{1}{4}\% = \frac{6\frac{1}{4}}{100} = 6\frac{1}{4} \div 100 = \frac{25}{4} \div \frac{100}{1}$$
$$= \frac{25}{4} \cdot \frac{1}{100} = \boxed{\frac{1}{16}}$$

Write each fraction as a percent. Round to the nearest hundredth if necessary.

$$3) \frac{120}{25} \rightarrow n$$

$$\frac{25n}{25} = \frac{12000}{25}$$

$$\boxed{n = 480\%}$$

$$4) \frac{16}{21} \rightarrow n$$

$$\frac{21n}{21} = \frac{1600}{21}$$

$$\boxed{n = 76.19\%}$$

Percent of a Number

1) Find 80% of 20.

Method #1

$$\frac{\overset{16}{80}}{\underset{\substack{5 \\ 1}}{100}} \cdot \frac{\overset{1}{20}}{1} = \frac{16}{1} = \boxed{16}$$

Method #2

$$80\% \times 20 = (0.8)(20) = \boxed{16}$$

2) Find 120% of 75.

Method #1

$$\frac{\overset{30}{120}}{\underset{\substack{4 \\ 1}}{100}} \cdot \frac{\overset{3}{75}}{1} = \frac{90}{1} = \boxed{90}$$

Method #2

$$120\% \times 75 = (1.2)(75) = \boxed{90}$$

3) Find 47.8% of 89. Round to the nearest tenth.

$$47.8\% \times 89 = (0.478)(89) = \boxed{42.5}$$

The Percent Proportion

Formula:

$$\frac{a}{b} = \frac{P}{100}$$

Part \uparrow base

$$\left(\frac{\text{is}}{\text{of}} = \frac{\%}{100} \right)$$

Ex. 1 What percent of 60 is 15?

$$\frac{a}{b} = \frac{P}{100}$$

$$\frac{15}{60} = \frac{P}{100}$$

$$\frac{100p}{60} = \frac{1500}{60}$$

$$p = 25\%$$

Ex. 2 75 is 25% of what number?

$$\frac{a}{b} = \frac{p}{100}$$

$$\frac{75}{b} = \frac{25}{100}$$

$$25b = 7500$$

$$\frac{25}{25} = \frac{25}{25}$$

$$b = 300$$

Ex. 3 What number is 32% of 28?

$$\frac{a}{b} = \frac{p}{100}$$

$$\frac{a}{28} = \frac{32}{100}$$

$$100a = 896$$

$$\frac{100}{100} = \frac{896}{100}$$

$$a = 8.96$$

Percent Estimation

$$10\% \text{ of } 80 = 8$$

$$10\% \text{ of } 120 = 12$$

$$10\% \text{ of } 980 = 98$$

Estimate.

1) $77\% \text{ of } 48$

2) $28\% \text{ of } 62$

$$80\% \text{ of } 50$$

$$30\% \text{ of } 60$$

$$10\% \text{ of } 50 = 5$$

$$10\% \text{ of } 60 = 6$$

$$5 \times 8 = \boxed{40}$$

$$6 \times 3 = \boxed{18}$$

3) $21.2\% \text{ of } 92$

$$20\% \text{ of } 90$$

$$10\% \text{ of } 90 = 9$$

$$9 \times 2 = \boxed{18}$$

The Percent Equation

Ex. 1, What number is 12% of 350?

$$x = .12 \cdot 350$$

$$x = (.12)(350)$$

$$x = 42$$

Ex. 2, 21 is what percent of 40?

$$21 = x \cdot 40$$

$$21 = x \cdot 40$$

$$21 = 40x$$

$$40 \quad 40$$

$$525 = x$$

$$x = 52.5\%$$

Ex. 3, 13 is 26% of what number?

$$13 = .26 \cdot x$$

$$13 = .26 \cdot x$$

$$.26 \quad .26$$

$$50 = x$$

Percent of Change

$$\frac{\text{percent of change}}{\text{change}} = \frac{\text{amount of change}}{\text{original amount}}$$

Find the percent of change. Round to the nearest tenth.

Ex. 1 Original: \$8.42/hr
New: \$9.57/hr

$$\begin{aligned} \% \text{ of Inc.} &= \frac{\text{amt. of change}}{\text{orig. amt.}} \\ &= \frac{9.57 - 8.42}{8.42} \\ &= \frac{1.15}{8.42} \\ &= 13.7\% \end{aligned}$$

Ex. 2 Original: In 2008, there were 535 students
New: In 2009, there were 512 students

$$\begin{aligned} \% \text{ of dec.} &= \frac{\text{amt. of change}}{\text{orig. amt.}} \\ &= \frac{535 - 512}{535} = \frac{23}{535} = 4.3\% \end{aligned}$$

Sales Tax and Discount

Goat: \$120.30

Tax Rate: 7%

Method #1

$$\text{Sales Tax} = (.07)(120.30) \\ = \$8.42$$

$$\text{Cost} = 120.30 + 8.42 \\ = \$128.72$$

Method #2

$$\text{Cost} = (1.07)(120.30) \\ = \$128.72$$

Goat: \$120.30

Discount: 40%

Method #1

$$\text{Discount} = (.40)(120.30) \\ = \$48.12$$

$$\text{Cost} = 120.30 - 48.12 \\ = \$72.18$$

Method #2

$$\text{Cost} = (.60)(120.30) \\ = \$72.18$$

100%

- 40%

60%

Simple Interest

Formula: $I = PRT$ ← Time (years)

↑ ↑ ↑
Interest Principal Rate as a decimal #
Earned

Ex. 1 $P = \$1,000$, $R = 3.2\%$, $T = 5$ years

$I = PRT$

$$I = (\$1,000)(.032)(5)$$

$$I = \$160$$

Ex. 2 $P = \$1,500$, $R = 2.75\%$, $T = 37$ months

$I = PRT$

$$I = (\$1,500)(.0275)\left(\frac{37}{12}\right)$$

$$I = \$127.19$$