

Chapter 10

Resource Masters



Mathematics

Applications and Concepts

Course 2



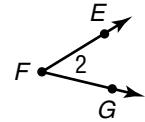
New York, New York Columbus, Ohio Chicago, Illinois Peoria, Illinois Woodland Hills, California

10-1

Study Guide and Intervention

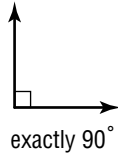
Angles

An **angle** is formed by two rays with a common endpoint. The endpoint is called the **vertex** of the angle. Angles can be named by using a number, using the vertex, or using the vertex and a point from each side. For example, the angle at the right could be named $\angle 2$, $\angle F$, or $\angle EFG$. Angles are measured in **degrees**, where 1 degree is one of 360 equal parts of a circle.

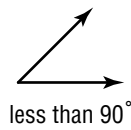


Angles are classified according to their measure.

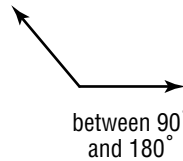
Right Angle



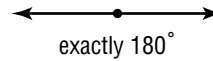
Acute Angle



Obtuse Angle

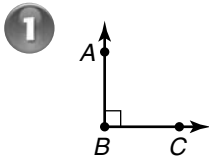


Straight Angle

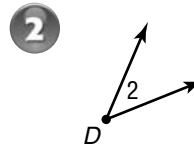


EXAMPLES

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.



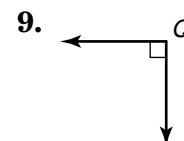
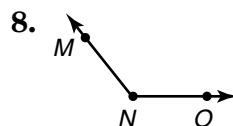
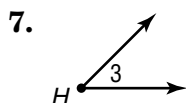
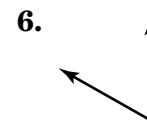
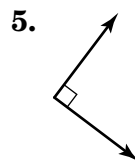
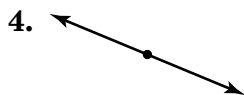
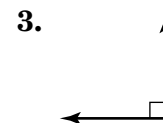
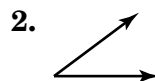
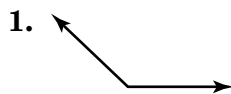
The angle is a right angle. Its measure is 90° .



The angle is less than 90° , so it is an acute angle.

EXERCISES

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.

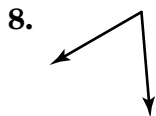
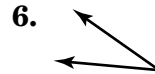
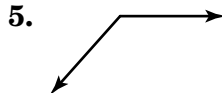
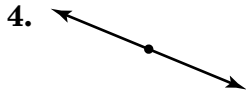
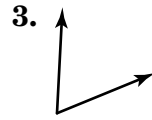
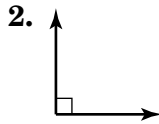
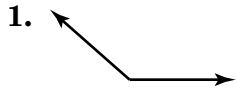


10-1

Practice: Skills

Angles

Classify each angle as *acute*, *obtuse*, *right*, or *straight*.



Draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, *right*, or *straight*.

10. 42°

11. 125°

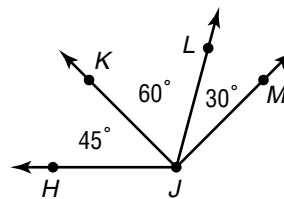
12. 90°

Use the figure at the right.

13. Name the acute angles.

14. Find $m\angle KJM$.

15. Name an angle adjacent to $\angle LJM$.



10-1

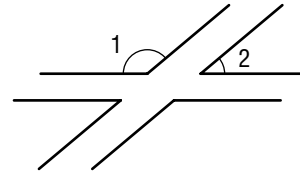
Practice: Word Problems

Angles

1. CLOCKS The time shown on the clock is 11:05. Starting at this time, approximately what time will it be when the hands form an obtuse angle?



2. AIRPORT The runways at a local airport are sketched in the figure. Classify $\angle 1$ and $\angle 2$ as *acute*, *obtuse*, *right*, or *straight*.



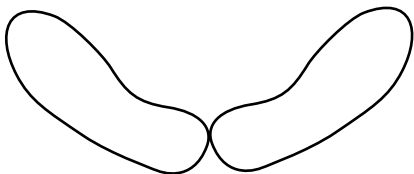
3. ALPHABET Which of the following letters contain at least one acute angle?

A E L W

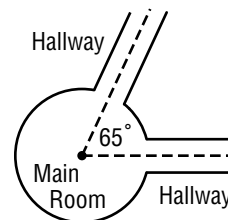
4. CLOCKS The time shown on the clock is 12:07. After 20 minutes have gone by, will the angle formed by the hour and minute hands be *acute*, *obtuse*, *right*, or *straight*?



5. BALLET When a ballet dancer's feet are in first position, the heels are touching, and the feet are turned out. A dancer with excellent technique can position his or her feet so that they are nearly in a straight line. Isabella is practicing her technique. Classify the angle her feet form as *acute*, *obtuse*, or *right*.



6. ARCHITECTURE The plans for a new aquarium call for several hallways of exhibits leading out of a circular main room. Because of the size of the tanks that will be used, the angle formed between two adjacent hallways can be no smaller than 65° . What is the maximum number of hallways that can be built leading out of the main room?



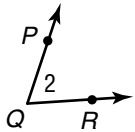
10-1**Reading to Learn Mathematics****Angles**

Pre-Activity *Read the introduction at the top of page 413 in your textbook.
Write your answers below.*

1. Name other times in which the hands of a clock form an angle less than 90° , equal to 90° , and greater than 90° .
2. How many degrees is the angle that is formed by clock hands at 6:00?

Reading the Lesson

3. Give three possible names for the angle shown.



4. Draw $\angle Y$ with a measure of 120° .

Helping You Remember

5. Name the four types of angles and draw an example of each one.

10-2**Study Guide and Intervention****Making Circle Graphs**

A graph used to compare parts of a whole is called a **circle graph**. In a circle graph, the percents add up to 100. When percents are not given, you must first determine what part of the whole each item represents.

EXAMPLE 1 **ENERGY** Make a circle graph of the data in the table.

Step 1 Find the total number of reactors:
 $104 + 59 + 54 + 222 = 439$.

Step 2 Find the ratio that compares each number with the total. Write the ratio as a decimal rounded to the nearest hundredth.

$$\text{United States: } \frac{104}{439} \approx 0.24$$

$$\text{Japan: } \frac{54}{439} \approx 0.12$$

$$\text{France: } \frac{59}{439} \approx 0.13$$

$$\text{Other: } \frac{222}{439} \approx 0.51$$

Step 3 Find the number of degrees for each section of the graph.

$$\text{United States: } 0.24 \cdot 360^\circ \approx 86^\circ$$

$$\text{Japan: } 0.12 \cdot 360^\circ \approx 43^\circ$$

$$\text{France: } 0.13 \cdot 360^\circ \approx 47^\circ$$

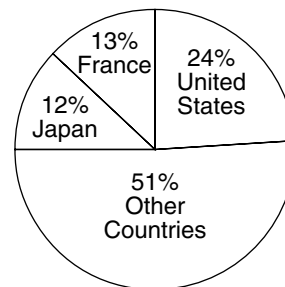
$$\text{Other: } 0.51 \cdot 360^\circ \approx 183^\circ$$

Step 4 Use a compass to draw a circle with a radius. Then use a protractor to draw an 86° angle. This represents the percent of nuclear reactors in the United States.

Step 5 From the new radius, draw a 47° angle for France. Repeat this step for the other two sections. Label each section and give the graph a title.

| Country | Number of Reactors |
|-----------------|--------------------|
| United States | 104 |
| France | 59 |
| Japan | 54 |
| Other Countries | 222 |

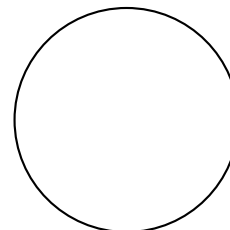
Nuclear Reactors in Operation, 2001

**EXERCISES**

1. **SWIMMING** The table shows the number of members of the swim team who competed at the swim meet. Each competed in only one event. Make a circle graph of the data.

| Event | Number |
|--------------|--------|
| Freestyle | 18 |
| Breaststroke | 7 |
| Backstroke | 5 |
| Butterfly | 2 |

Swim Team Member Participation



10-2

Practice: Skills

Making Circle Graphs

For each table, find the number of degrees in each section of a circle graph. Then make a circle graph of the data.

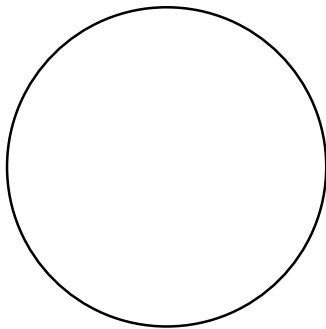
1.

| Category | Percent |
|---------------------------|---------|
| Commercial and Industrial | 52% |
| Residential | 20% |
| Transportation | 27% |
| Other | 1% |

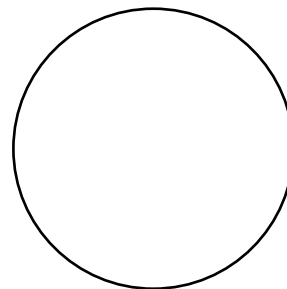
2.

| Family Member | Percent |
|-------------------|---------|
| Mom | 52% |
| Dad | 17% |
| Brother/Sister | 16% |
| Grandparent/Other | 15% |

United States Energy Usage, 2001



Family Members Students Confide In



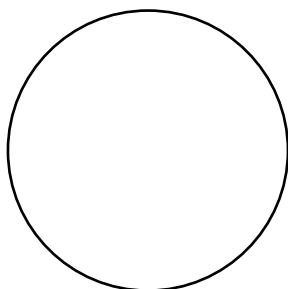
3.

| Country | Number |
|-----------------------|--------|
| India | 2 |
| United States | 23 |
| European Space Agency | 7 |
| China | 1 |

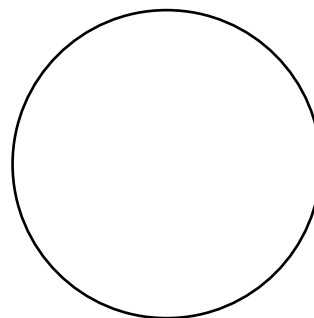
4.

| Coast | Length (mi) |
|----------|-------------|
| Atlantic | 2,100 |
| Pacific | 7,600 |
| Gulf | 1,600 |
| Arctic | 1,100 |

Successful Space Launches, 2001



United States Coastline



10-2**Practice: Word Problems*****Making Circle Graphs***

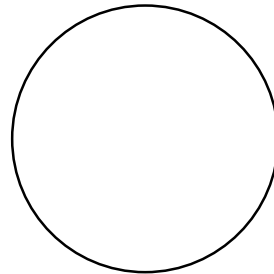
LANGUAGES For Exercises 1 and 2, use the table that shows the number of people that speak the five languages that are spoken by the most people in the world.

| Languages Spoken by the Most People | |
|--|----------------------------|
| Language | Speakers (millions) |
| Chinese, Mandarin | 874 |
| Hindi | 366 |
| English | 341 |
| Spanish | 322 |
| Bengali | 207 |

1. Find the degrees for each part of a circle graph that shows the data.

2. Make a circle graph of the data. Which three languages account for 41% of the total?

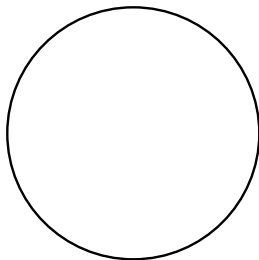
Languages Spoken by the Most People



MILITARY For Exercises 3 and 4, use the table that shows the number of people active in the United States military in 2002.

| United States Military, Active Duty, 2002 | |
|--|------------------------------|
| Branch | Personnel (thousands) |
| Army | 486 |
| Navy | 385 |
| Marine Corps | 174 |
| Air Force | 368 |
| Coast Guard | 38 |

3. Make a circle graph of the data.
United States Military Personnel Active Duty, 2002



4. Which two branches taken together account for almost half of the total?

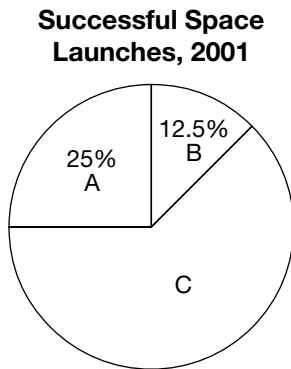
10-2**Reading to Learn Mathematics****Making Circle Graphs**

Pre-Activity *Read the introduction at the top of page 418 in your textbook. Write your answers below.*

1. Explain how you know that each person surveyed chose only one shade of blue.
2. If 500 people took part in the survey, how many preferred aquamarine?

Reading the Lesson

3. In the following circle graph, what is the percent represented by section C? How do you know?



4. As stated in Example 2 on page 419, when you construct a circle graph you can check your work by measuring the last section of a circle graph to verify that the angles have the correct measures. Why will this work as a check?

Helping You Remember

5. If you are given the results of a survey and the results are given in percents, how do you draw a circle graph to represent the results of the survey? Describe each step.

10-2

Enrichment

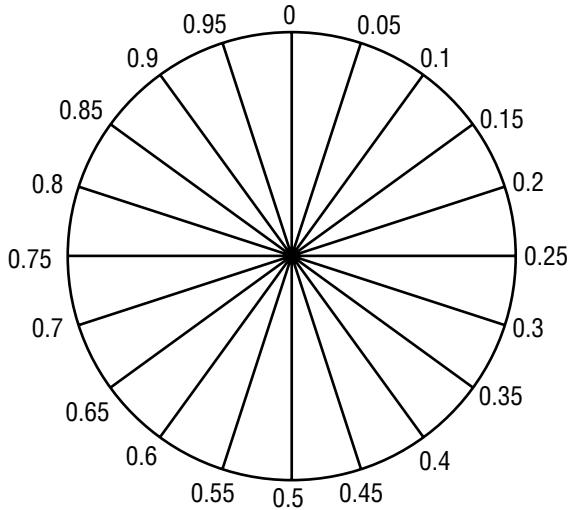
Relative Frequency and Circle Graphs

The **relative frequency** tells how the frequency of one item compares to the total of all the frequencies. Relative frequencies are written as fractions, decimals, or percents.

For example, in Exercise 1 below, the total of all the frequencies is 50. So, the relative frequency of the grade A is $8 \div 50$, or 0.16.

The circle at the right is divided into 20 equal parts. You can trace this circle and then use relative frequencies to make circle graphs.

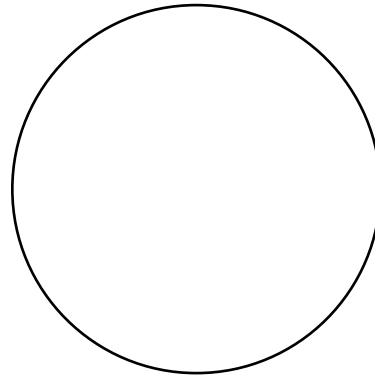
Complete each chart to show the relative frequencies. Then sketch a circle graph for the data. Use decimals rounded to the nearest hundredth.



1. History Grades for 50 Students

| Grade | Frequency | Relative Frequency |
|-------|-----------|--------------------|
| A | 8 | 0.16 |
| B | 16 | |
| C | 18 | |
| D | 6 | |
| F | 2 | |

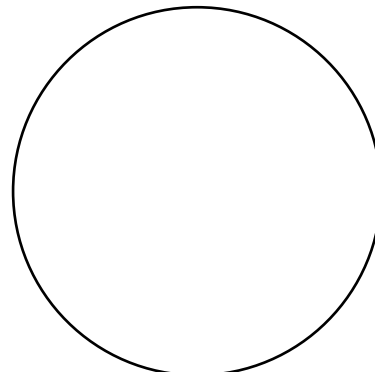
History Grades for 50 Students



2. Steve's Budget

| Item | Amount Spent | Relative Spending |
|-----------|--------------|-------------------|
| Telephone | \$26 | |
| Movies | \$46 | |
| Books | \$24 | |
| Car | \$38 | |
| Other | \$66 | |

Steve's Budget



10-3

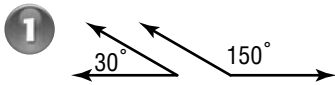
Study Guide and Intervention

Angle Relationships

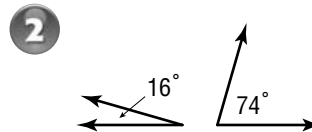
Angles that have the same measure are called **congruent angles**. Two angles are **supplementary** if the sum of their measures is 180° . Two angles are **complementary** if the sum of their measures is 90° . When two lines intersect, they form two pairs of opposite angles called **vertical angles**, which are always congruent.

EXAMPLES

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.



$30^\circ + 150^\circ = 180^\circ$
The angles are supplementary.



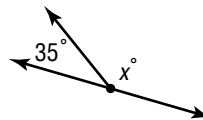
$16^\circ + 74^\circ = 90^\circ$
The angles are complementary.

EXAMPLE 3 Find the value of x in the figure below.

The two angles are supplementary, so the sum of their measures is 180° .

$$\begin{array}{r} x + 35 = 180 \\ - 35 \quad -35 \\ \hline x = 145 \end{array}$$

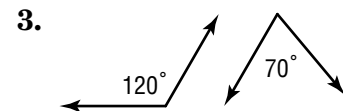
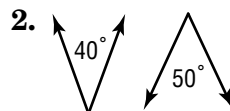
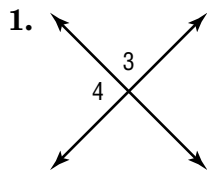
Write the equation.
Subtract 35 from each side.
Simplify.



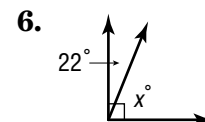
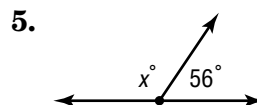
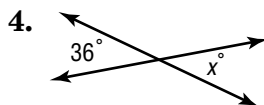
So, the angle is 145° .

EXERCISES

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.



Find the value of x in each figure.

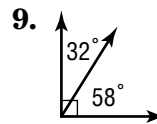
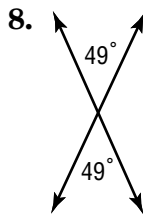
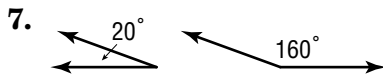
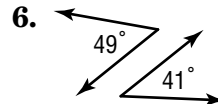
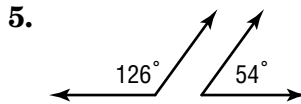
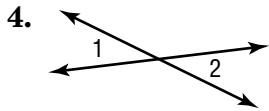
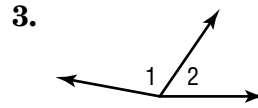
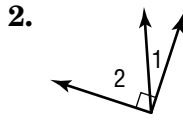
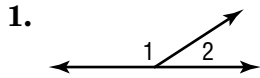


10-3

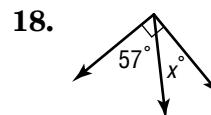
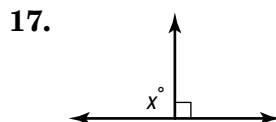
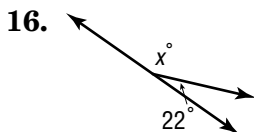
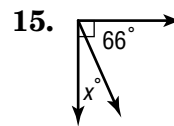
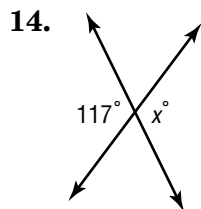
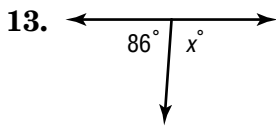
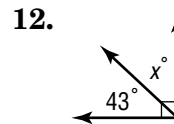
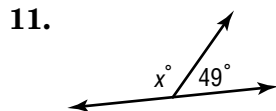
Practice: Skills

Angle Relationships

Classify each pair of angles as *complementary*, *supplementary*, or *neither*.



Find the value of x in each figure.

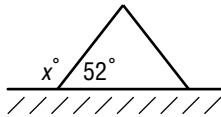


10-3

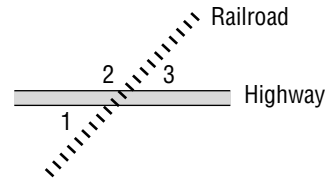
Practice: Word Problems

Angle Relationships

1. PYRAMIDS A side view of the Great Pyramid at Giza is shown below. The sides of the pyramid make an angle of 52° with respect to the ground. What is the value of x ?

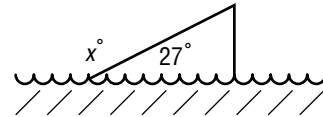


2. RAILROAD A map shows a railroad crossing a highway, as shown below. Which of the numbered angles are vertical angles?

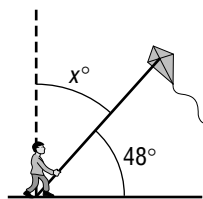


3. RAILROAD Refer to the map shown in Exercise 2. If $m\angle 1$ is 64° , what is the measure of $\angle 2$?

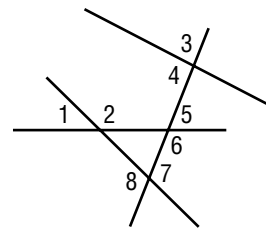
4. SKIING A ski jump makes an angle of 27° with respect to the water as shown below. How are the 27° angle and the unknown angle related? What is the value of x ?



5. KITES A kite string makes an angle of 48° with respect to the ground as shown below. The dashed line is vertical and the ground is horizontal. How are the 48° angle and the unknown angle related? What is the value of x ?



6. GAMES In a game of pick-up-sticks, the last 4 sticks are shown below. Which of the numbered angles are vertical angles?



10-3**Reading to Learn Mathematics*****Angle Relationships***

Pre-Activity *Complete the Mini Lab at the top of page 422 in your textbook. Write your answers below.*

1. Which angles have the same measure?
2. Draw two other pairs of intersecting lines. Measure their angles. **Make a conjecture** involving four angles created by intersecting lines.
3. What is the relationship between the measures of $\angle 1$ and $\angle 2$? $\angle 3$ and $\angle 4$?
4. Do other pairs of angles share the same relationship? Explain.

Reading the Lesson

5. Write the sentence $\angle 1 \cong \angle 4$ in words.
6. Write the equation $m\angle 1 = 35^\circ$ in words.
7. What do you know about the sum of the measures of supplementary angles? Complementary angles?
8. In Example 4 on page 423, why does the sum of the three angles equal 180° ?

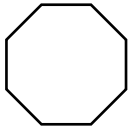
Helping You Remember

9. On a separate piece of paper, make a table or diagram that describes the angle relationships discussed in this lesson: vertical angles, congruent angles, complementary angles, and supplementary angles. Share your work with your class.

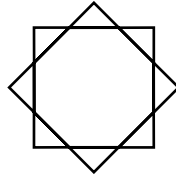
10-3 Enrichment

Star Polygons

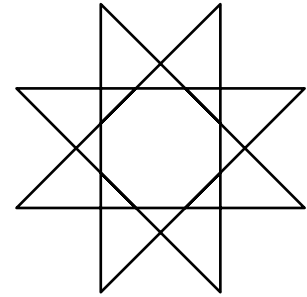
Any polygon can be turned into a *star polygon* by extending its sides. A star polygon is also called a *stellated* polygon.



Octagon



Extend the sides to make the first star.



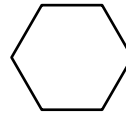
Extend the sides again to make a second star.

Make a star by extending the sides of each polygon.

1. pentagon

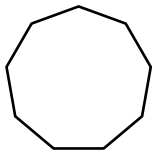


2. hexagon

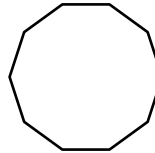


Trace each polygon on a separate sheet of paper. Then, make three different stars by extending the sides three times.

3. nonagon

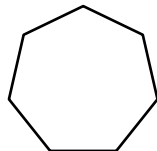


4. decagon

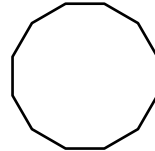


Show all the different stars that can be made from each polygon.

5. heptagon



6. dodecagon



10-4**Study Guide and Intervention****Triangles**

A **triangle** is a figure with three sides and three angles. The sum of the measures of the angles of a triangle is 180° . You can use this to find a missing angle measure in a triangle.

EXAMPLE 1 Find the value of x in $\triangle ABC$.

$$x + 66 + 52 = 180$$

$$x + 118 = 180$$

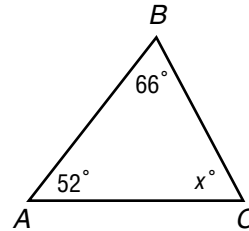
$$\underline{- 118 \quad - 118}$$

$$x = 62$$

The sum of the measures is 180.

Simplify.

Subtract 118 from each side.



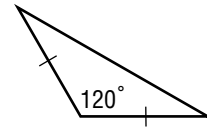
The missing angle is 62° .

Triangles can be classified by the measures of their angles. An **acute triangle** has three acute angles. An **obtuse triangle** has one obtuse angle. A **right triangle** has one right angle.

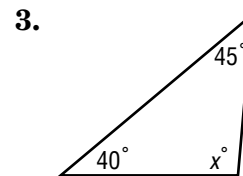
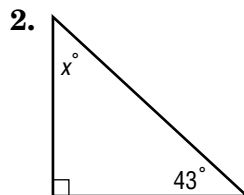
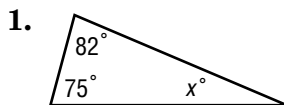
Triangles can also be classified by the lengths of their sides. Sides that are the same length are **congruent segments** and are often marked by tick marks. In a **scalene triangle**, all sides have different lengths. An **isosceles triangle** has at least two congruent sides. An **equilateral triangle** has all three sides congruent.

EXAMPLE 2 Classify the triangle by its angles and by its sides.

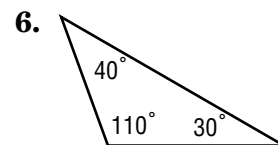
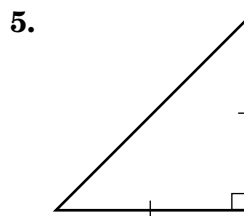
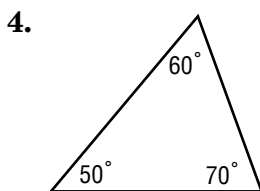
The triangle has one obtuse angle and two sides the same length. So, it is an obtuse, isosceles triangle.

**EXERCISES**

Find the missing measure in each triangle. Then classify the triangle as *acute*, *right*, or *obtuse*.



Classify each triangle by its angles and by its sides.



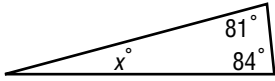
10-4

Practice: Skills

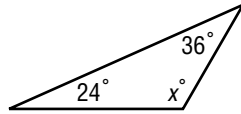
Triangles

Find the missing measure in each triangle. Then classify the triangle as *acute*, *right*, or *obtuse*.

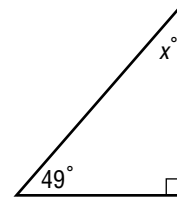
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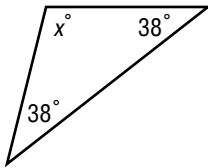
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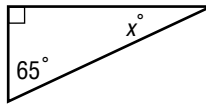
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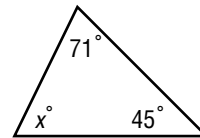
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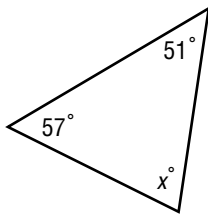
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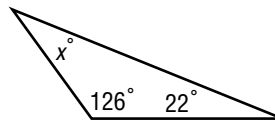
6.



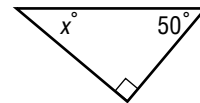
7.



8.

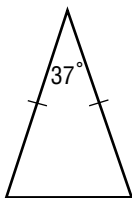


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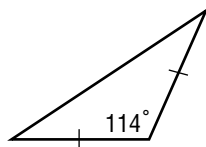


Classify each triangle by its angles and by its sides.

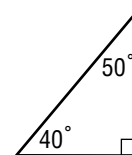
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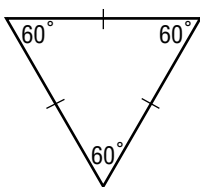
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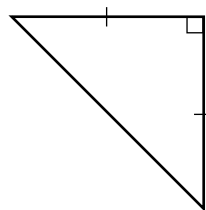
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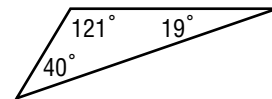
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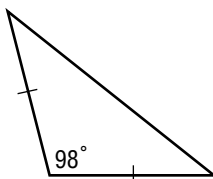
14.



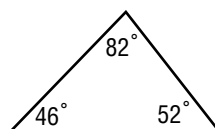
15.



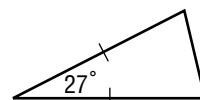
16.



17.



18.



10-4

Practice: Word Problems

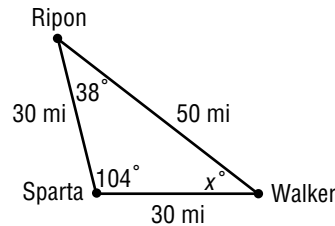
Triangles

1. TAILORING Each lapel on a suit jacket is in the shape of a triangle. The three angles of each triangle measure 47° , 68° , and 65° . Classify the triangle by its angles.

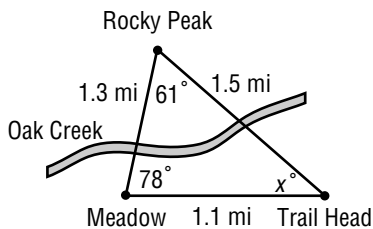
2. FLAGS A naval distress signal flag is in the shape of a triangle. The three sides of the triangle measure 5 feet, 9 feet, and 9 feet. Classify the triangle by its sides.

3. CARPENTRY The supports of a wood table are in the shape of a right triangle. Find the third angle of the triangle if the measure of one of the angles is 23° .

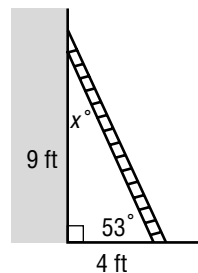
4. MAPS The three towns of Ripon, Sparta, and Walker form a triangle as shown below. Classify the triangle by its angles and by its sides. What is the value of x in the triangle?



5. HIKING The figure shows the Oak Creek trail, which is shaped like a triangle. Classify the triangle by its angles and by its sides. What is the value of x in the figure?



6. LADDER The figure shows a ladder leaning against a wall, forming a triangle. Classify the triangle by its angles and by its sides. What is the value of x in the figure?



10-4**Reading to Learn Mathematics****Triangles**

Pre-Activity Complete the Mini Lab at the top of page 428 in your textbook. Write your answers below.

1. What kind of angle is formed where the three vertices meet?
2. Repeat the activity with another triangle. **Make a conjecture** about the sum of the measures of three angles of any triangle.

Reading the Lesson

3. How can you indicate that two sides of a triangle are congruent?
4. Write the following equation in words: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.
5. If you know the measures of two angles of a triangle, how can you find the measure of the third angle?

Helping You Remember

6. Complete the table to help you remember the ways to classify triangles.

| Type of Triangle | Classified by Angles or Sides | Description |
|------------------|-------------------------------|--------------------|
| acute | angles | |
| obtuse | | |
| | sides | no congruent sides |
| | | 1 right angle |
| equilateral | | |
| isosceles | | |

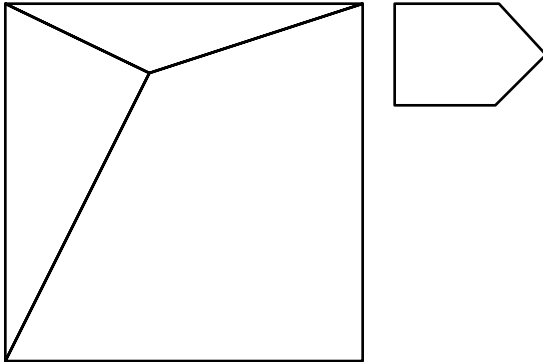
10-4

Enrichment

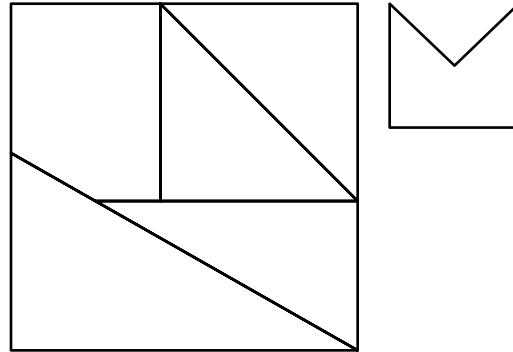
Dissecting Squares

In a *dissection puzzle*, the pieces of one shape are rearranged to make a different shape. Draw a square and then make a set of pieces to solve each dissection puzzle. Record your answers.

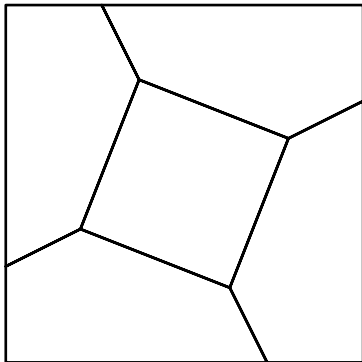
1. Rearrange the pieces to make a figure shaped like the one at the right.



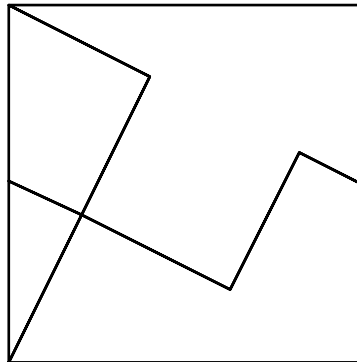
2. Rearrange the pieces to make a figure shaped like the one at the right.



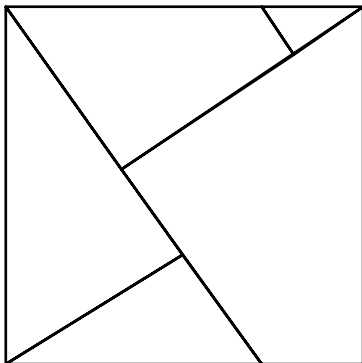
3. Rearrange the pieces to make an octagon with sides of equal length.



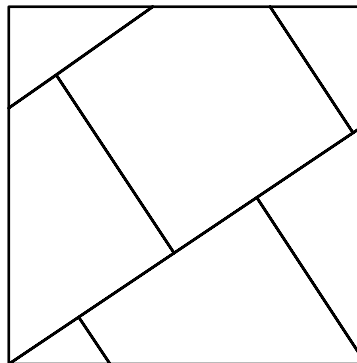
4. Rearrange the pieces to make a figure shaped like a plus sign.



5. Rearrange the pieces to make two new squares.



6. Rearrange the pieces to make three squares of equal size.

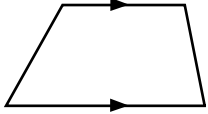
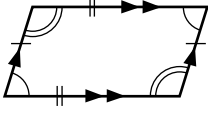

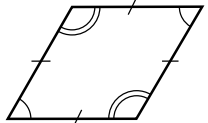
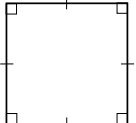


10-5

Study Guide and Intervention

Quadrilaterals

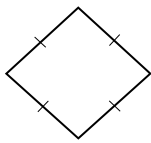
Quadrilaterals can be classified using their angles and sides. The best description of a quadrilateral is the one that is the most specific.

| | | | | |
|---|---|---|--|--|
|  <p>Trapezoid quadrilateral with one pair of parallel sides</p> |  <p>Parallelogram quadrilateral with opposite sides parallel and opposite sides congruent</p> |  <p>Rectangle parallelogram with 4 right angles</p> |  <p>Rhombus parallelogram with 4 congruent sides</p> |  <p>Square parallelogram with 4 right angles and 4 congruent sides</p> |
|---|---|---|--|--|

EXAMPLES

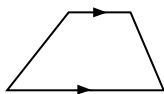
Classify the quadrilateral using the name that *best* describes it.

1



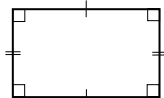
The quadrilateral is a parallelogram with 4 congruent sides. It is a rhombus.

2



The quadrilateral has one pair of parallel sides. It is a trapezoid.

3

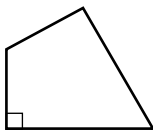


The quadrilateral is a parallelogram with 4 right angles. It is a rectangle.

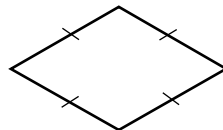
EXERCISES

Classify the quadrilateral using the name that *best* describes it.

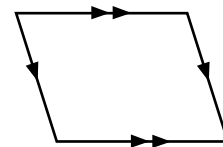
1.



2.

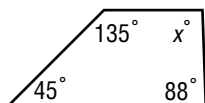


3.



Find the missing measure in each quadrilateral.

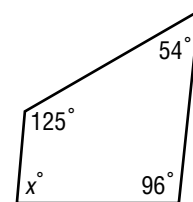
4.



5.



6.

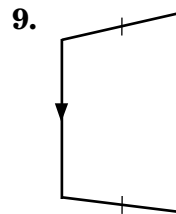
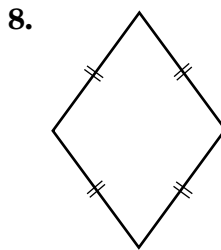
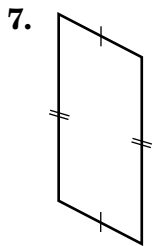
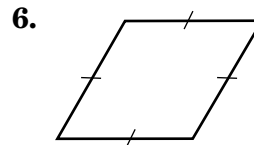
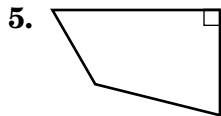
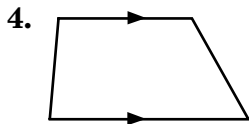
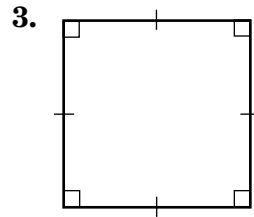
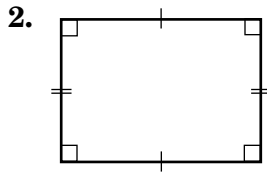
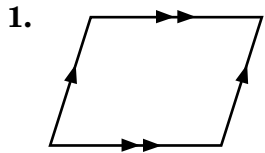


10-5

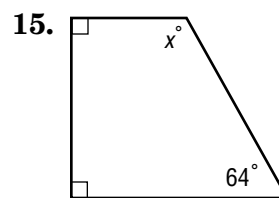
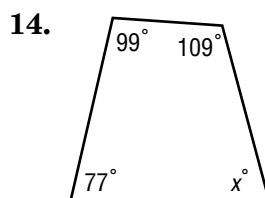
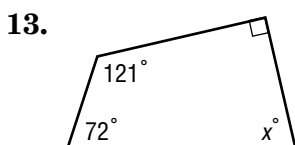
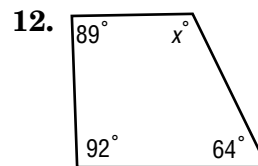
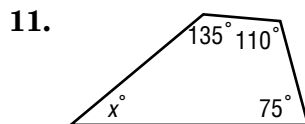
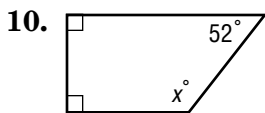
Practice: Skills

Quadrilaterals

Classify the quadrilateral using the name that *best* describes it.



Find the missing angle measure of each quadrilateral.

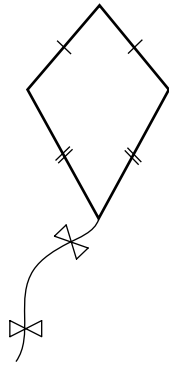


10-5

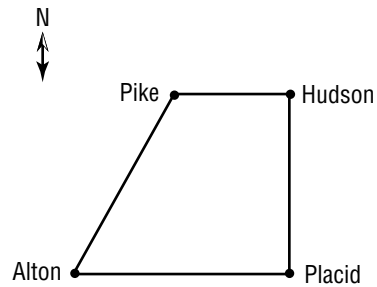
Practice: Word Problems

Quadrilaterals

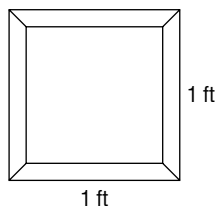
1. KITES A kite is shown below. What is the best name to classify the shape of the kite? Explain.



2. MAPS A map showing the road connecting the towns of Pike, Hudson, Placid, and Alton is shown. The road connecting Pike and Hudson is parallel to the road connecting Alton and Placid. What is the best name to classify the shape of the roads connecting the four towns? Explain.



3. ART A picture frame is shown below. What is the *best* name to classify the shape of the frame?



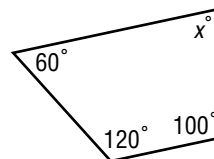
4. SCHOOL SUPPLIES The side view of an eraser is shown below. What is the best name to classify the shape of the eraser?



5. PARTY The front of a birthday party invitation is shown below. Find the measure of the missing angle.



6. TABLE The top of Mr. Bautista's new coffee table is shown below. Find the measure of the missing angle.



10-5**Reading to Learn Mathematics*****Quadrilaterals***

Pre-Activity *Read the introduction at the top of page 434 in your textbook. Write your answers below.*

1. Describe the angles inside the 4-sided figure.
2. Which sides of the figure appear to be parallel?
3. Which sides of the figure appear to be congruent?

Reading the Lesson

4. In the diagram of quadrilaterals on page 434, what do the arrowheads on the parallelogram and trapezoid indicate?
5. Why are all the figures on the page called quadrilaterals?
6. How is a trapezoid different from a rhombus, a square, and a rectangle?
7. In the diagrams of the parallelogram, rhombus, square, and rectangle, how do you know that certain sides are congruent?

Helping You Remember

8. Work with a partner. Take turns drawing quadrilaterals. Have the other person give all possible names for the quadrilateral. Then decide on the best name for the quadrilateral. For example, if one person draws a square, the other person should say that it is a quadrilateral, a parallelogram, a rectangle, and a rhombus, but the best name for the figure is a square.

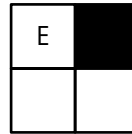
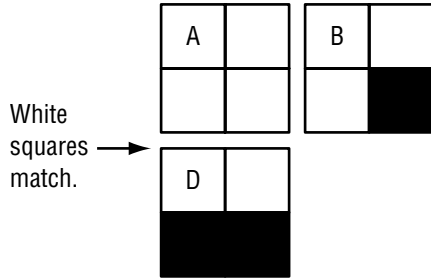
10-5

Enrichment

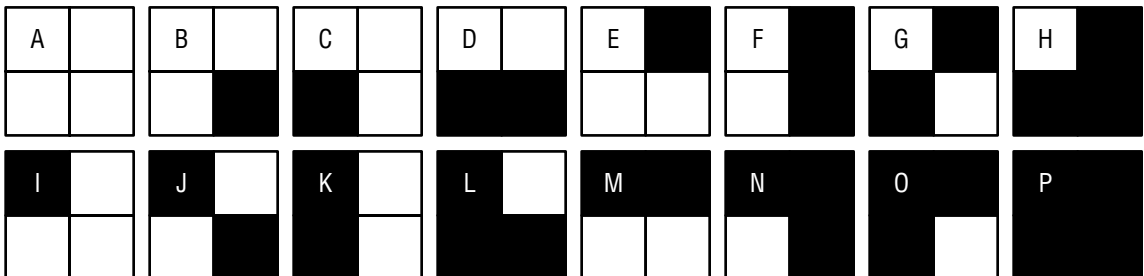
The Colormatch Square

To work this puzzle, cut out the 16 tiles at the bottom of this page. The goal of the puzzle is to create a square so that the sides of any pair of adjacent tiles match. You are not allowed to rotate any of the tiles.

- Complete the solution to the colormatch square puzzle below.



- Find at least one other solution in which the A tile is in the upper left corner.



10-6

Study Guide and Intervention

Similar Figures

Two figures are **similar** if corresponding angles are congruent and corresponding sides are proportional. The symbol \sim means *is similar to*. You can use proportions to find the missing length of a side in a pair of similar figures.

For example $\triangle ABC \sim \triangle DEF$.

Corresponding angles

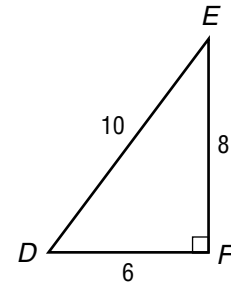
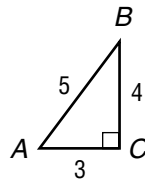
$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

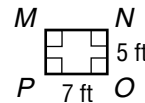
Corresponding sides

$$\frac{5}{10} = \frac{4}{8} = \frac{3}{6}$$



EXAMPLE 1 If $MNOP \sim RSTU$, find the length of \overline{ST} .

Since the two figures are similar, the ratios of their corresponding sides are equal. You can write and solve a proportion to find \overline{ST} .



$$\frac{PO}{UT} = \frac{NO}{ST}$$

Write a proportion.

$$\frac{7}{28} = \frac{5}{n}$$

Let n represent the length of \overline{ST} . Then substitute.

$$7n = 28(5)$$

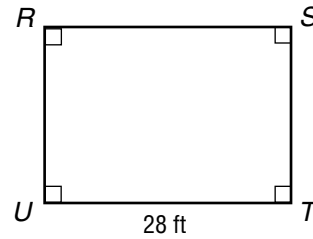
Find the cross products.

$$7n = 140$$

Simplify.

$$n = 20$$

Divide each side by 7.

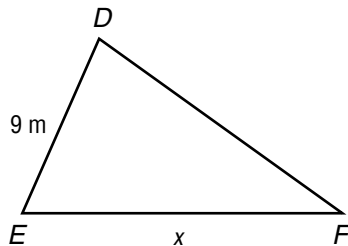
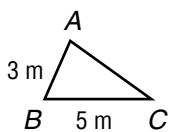


The length of \overline{ST} is 20 feet.

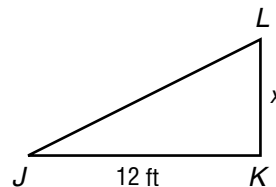
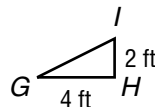
EXERCISES

Find the value of x in each pair of similar figures.

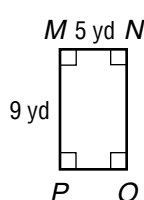
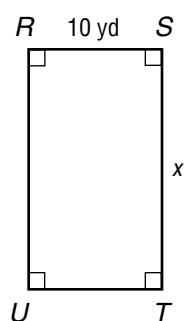
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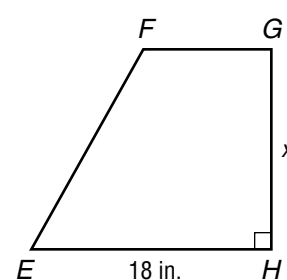
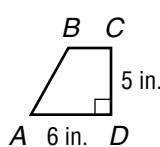
2.



3.



4.



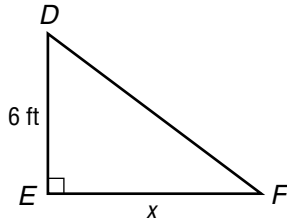
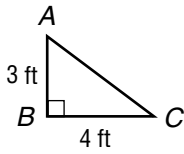
10-6

Practice: Skills

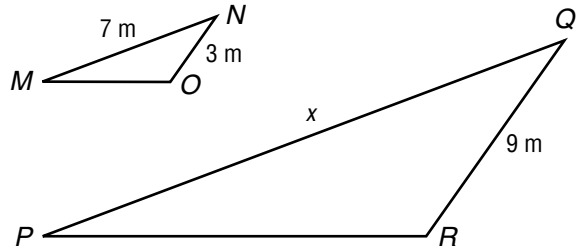
Similar Figures

Find the value of x in each pair of similar figures.

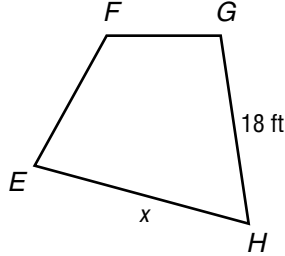
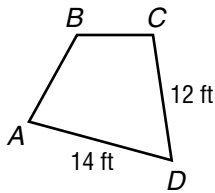
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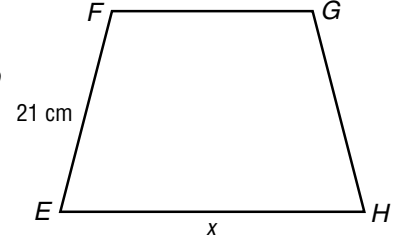
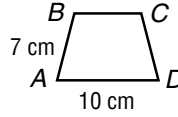
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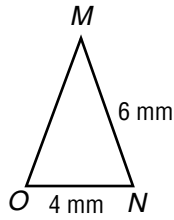
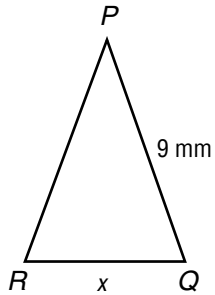
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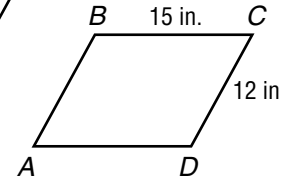
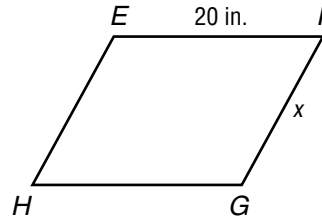
4.



5.

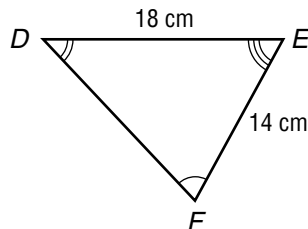
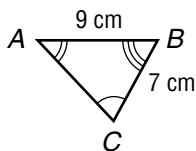


6.

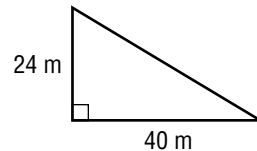
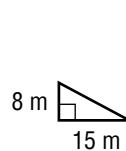


Determine whether each pair of figures is similar. Justify your answer.

7.

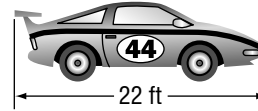


8.



10-6**Practice: Word Problems****Similar Figures**

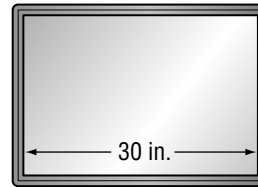
MODEL CARS For Exercises 1 and 2, use the following information. A scale model racing car is 11 inches long, 3 inches wide, and 2 inches tall. The actual racing car is shown at the right.



1. How wide is the actual racing car?

2. How tall is the actual racing car?

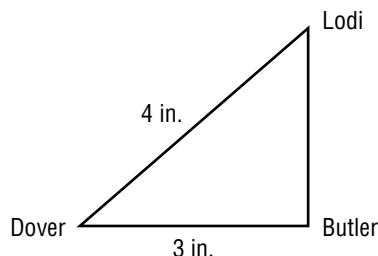
PHOTOGRAPHY For Exercises 3–4, use the given information. James wants to enlarge a photograph that is 6 inches wide and 4 inches tall so that it fits into the frame shown.



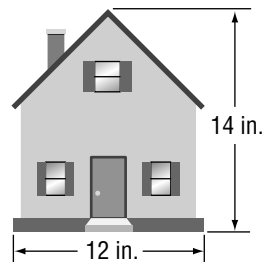
3. How tall must the frame be for the picture to fit?

4. Suppose James cuts 1 inch from the width of the photo, so that it is 5 inches wide, before he makes the enlargement. How tall will the frame have to be for the picture to fit?

5. **MAPS** A map below shows the towns of Dover, Butler, and Lodi. If the actual distance between Dover and Butler is 24 miles, how far is it from Dover to Lodi?



6. **BLUEPRINTS** A blueprint for a house is shown below. If the front of the house is actually 30 feet wide, how tall is the house?



10-6**Reading to Learn Mathematics****Similar Figures**

Pre-Activity Complete the Mini Lab at the top of page 440 in your textbook. Write your answers below.

- Write each fraction in simplest form.
 - $\frac{AB}{EF}, \frac{BC}{FG}, \frac{DC}{HG}, \frac{AD}{EH}$
 - $\frac{RS}{XY}, \frac{ST}{YZ}, \frac{RT}{XZ}$
- What do you notice about the ratios of corresponding sides?
- Measure the corresponding angles in the figures above. What do you notice about the measure of these angles?
- The rectangles are similar, and the triangles are similar. **Make a conjecture** about similar figures.

Reading the Lesson

- Write the statement $\triangle ABC \sim \triangle DEF$ in words.
- Write the statement $\angle A \cong \angle D$ in words.
- Suppose you know that two figures are similar, and that the following corresponding sides are proportional: side JK corresponds to side DE , and side KL corresponds to side EF . How would you write a proportion to find the length of side KL if the lengths of all other sides are known?

Helping You Remember

- Think of a real-life example that you could measure using indirect measurement if you were given a miniature replica of your example. Explain how you could find an unknown measurement using known measurements of your example and the measurements of the replica.

10-6

Enrichment

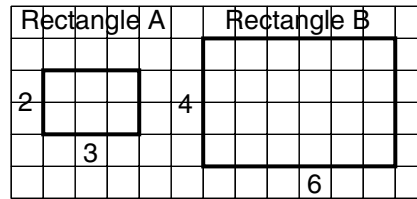
Similar Figures and Areas

The areas of two similar figures are related in a special way. Suppose that rectangle A is 2 units by 3 units and rectangle B is 4 units by 6 units.

The area of rectangle A is $2 \times 3 = 6$ units².

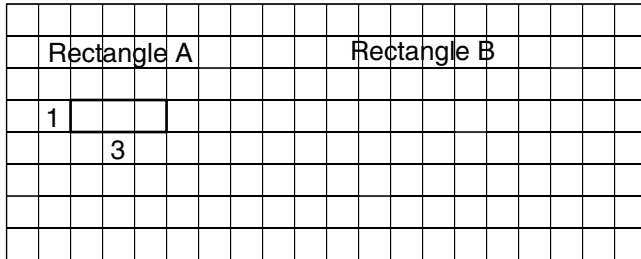
The area of rectangle B is $4 \times 6 = 24$ units².

The lengths of the sides of rectangle B are twice those of rectangle A and the area of rectangle B is four times that of rectangle A.

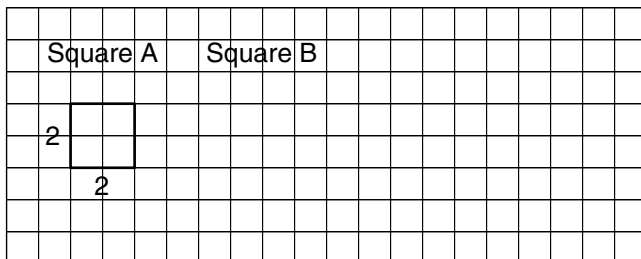


Sketch figure B similar to figure A and satisfying the given condition.

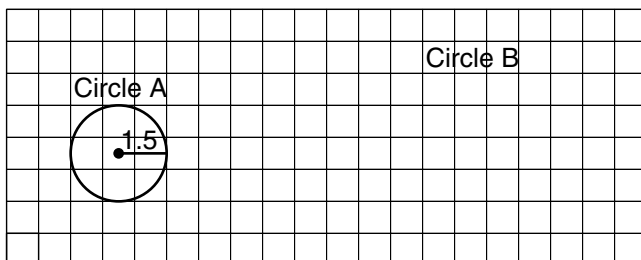
- Rectangle B has sixteen times the area of rectangle A.



- Square B has an area that is 4 times that of square A.



- Circle B has an area four times that of circle A.

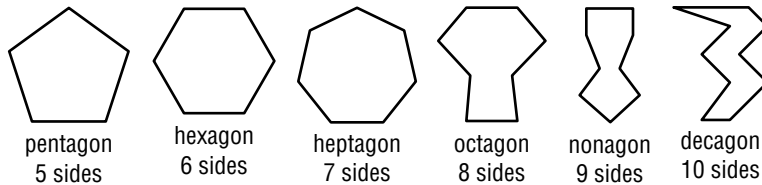


10-7

Study Guide and Intervention

Polygons and Tessellations

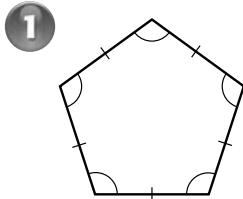
A **polygon** is a simple, closed figure formed by three or more straight lines. A simple figure does not have lines that cross each other. You have drawn a closed figure when your pencil ends up where it started. Polygons can be classified by the number of sides they have.



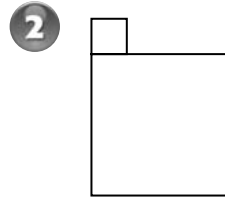
A polygon that has all sides congruent and all angles congruent is called a **regular polygon**.

EXAMPLES

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



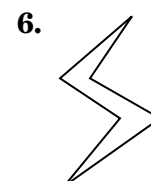
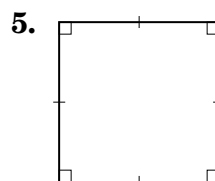
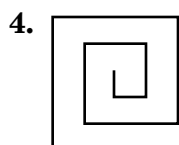
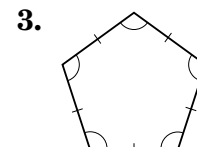
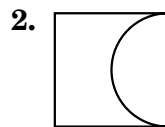
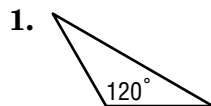
The figure has 5 congruent sides and 5 congruent angles. It is a regular pentagon.



The figure is not a polygon because it has sides that overlap.

EXERCISES

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



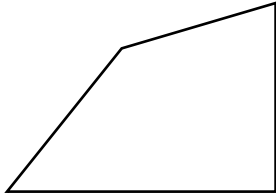
10-7

Practice: Skills

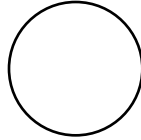
Polygons and Tessellations

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.

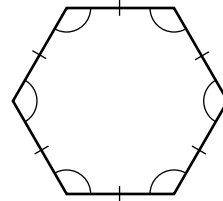
1.



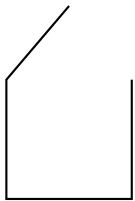
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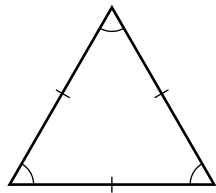
3.



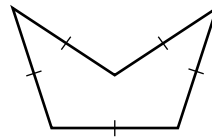
4.



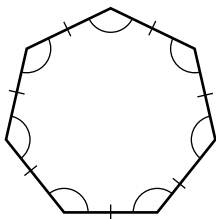
5.



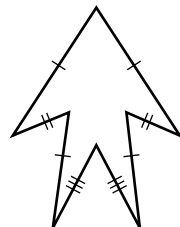
6.



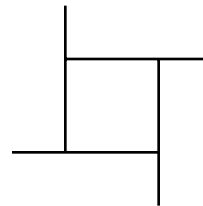
7.



8.



9.



Find the measure of an angle in each polygon.

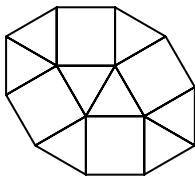
10. regular 15-gon

11. regular 18-gon

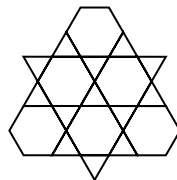
12. regular 24-gon

Identify the polygons that are used to create each tessellation.

13.



14.



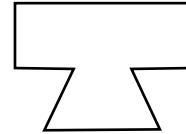
15. What is the perimeter of a regular pentagon with sides 8.4 inches long?

10-7**Practice: Word Problems*****Polygons and Tessellations***

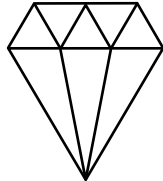
- 1. ROYALTY** The outline of a crown worn by a king is shown below. Is the figure a polygon? If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



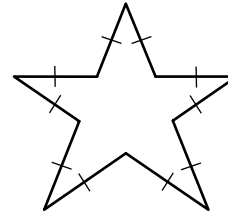
- 2. ALCHEMY** The symbol shown is one of the signs for *salt alkali* used in 17th-century chemistry. Is the symbol a polygon? If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



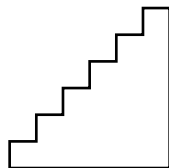
- 3. JEWELRY** The symbol shown is often used to represent gems. Is the symbol a polygon? If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



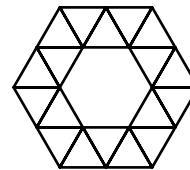
- 4. SYMBOLS** The 5-pointed star shown has sides of equal length. Is the symbol a polygon? If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



- 5. STAIRS** The figure shows a side view of a set of stairs. Is the figure a polygon? If it is, classify the polygon and state whether it is regular. If it is *not* a polygon, explain why.



- 6. TESSELLATIONS** Identify the polygons that are used to create the tessellation shown in the figure.



10-7**Reading to Learn Mathematics*****Polygons and Tessellations***

Pre-Activity *Read the introduction at the top of page 446 in your textbook. Write your answers below.*

1. Find the difference between the shapes of the states in Group 1 and the shapes of the states in Group 2.
2. Why do most states have boundaries that are not straight line segments?

Reading the Lesson

3. How many straight lines can a polygon have?
4. What is a simple figure?
5. When you draw a figure, how can you tell whether or not it is closed?
6. How do you find the sum of the angle measures in a regular polygon?
7. In this lesson, the terms *vertex* and *vertices* are used. How are the terms related?

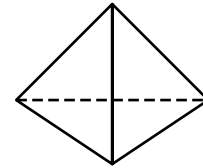
Helping You Remember

8. Using dot paper, draw a tessellation different from the ones shown in this lesson. You can use all the same shape or you can use combinations of shapes that form patterns. Share your work with your class.

10-7 Enrichment

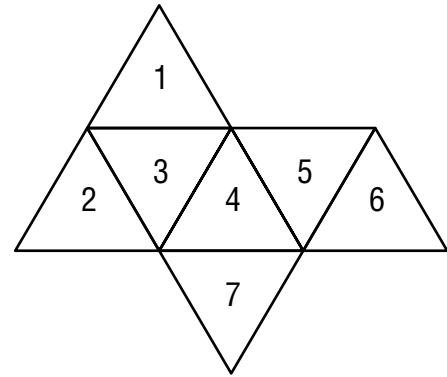
Tessellated Patterns for Solid Shapes

Tessellations made from equilateral triangles can be used to build three-dimensional shapes. In Exercise 1, you should get a shape like the one shown at the right. It is called a pyramid.

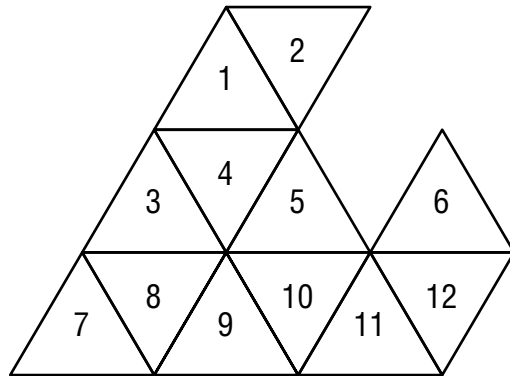


Copy each pattern. Crease the pattern along the lines. Then follow directions for folding the pattern. Use tape to secure the folded parts. When you have finished each model, describe it in words.

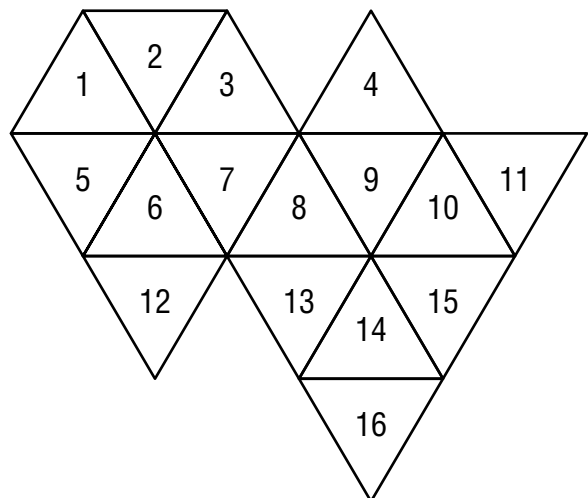
1. Fold 5 over 1.
Repeat, in this order:
fold 6 over 7,
fold 2 over 6.



2. Cut between 4 and 5. Then fold 5 over 3.
Repeat in this order:
fold 6 over 5,
fold 7 over 12, and
fold 2 over 9.



3. Cut between 1 and 2 and between 14 and 15. Then fold 15 over 14.
Repeat, in this order:
fold 1 over 2,
fold 4 over 3,
fold 11 over 1,
fold 16 over 5, and
fold 12 over 13.



10-8

Study Guide and Intervention

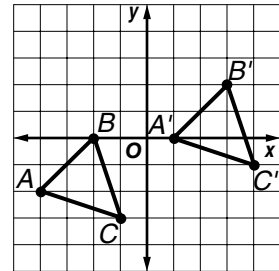
Translations

A **translation** is the movement of a geometric figure in some direction without turning the figure. When translating a figure, every point of the original figure is moved the same distance and in the same direction. To graph a translation of a figure, move each vertex of the figure in the given direction. Then connect the new vertices.

EXAMPLE 1 Triangle ABC has vertices $A(-4, -2)$, $B(-2, 0)$, and $C(-1, -3)$. Find the vertices of triangle $A'B'C'$ after a translation of 5 units right and 2 units up.

Add 5 to each x -coordinate. Add 2 to each y -coordinate.

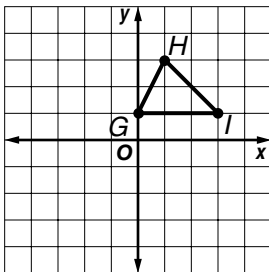
| Vertices of $\triangle ABC$ | $(x + 5, y + 2)$ | Vertices of $\triangle A'B'C'$ |
|-----------------------------|--------------------|--------------------------------|
| $A(-4, -2)$ | $(-4 + 5, -2 + 2)$ | $A'(1, 0)$ |
| $B(-2, 0)$ | $(-2 + 5, 0 + 2)$ | $B'(3, 2)$ |
| $C(-1, -3)$ | $(-1 + 5, -3 + 2)$ | $C'(4, -1)$ |



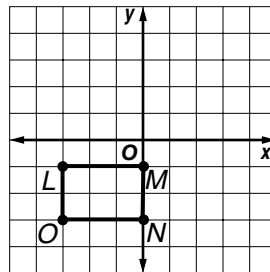
The coordinates of the vertices of $\triangle A'B'C'$ are $A'(1, 0)$, $B'(3, 2)$, and $C'(4, -1)$.

EXERCISES

1. Translate $\triangle GHI$ 1 unit left and 5 units down.

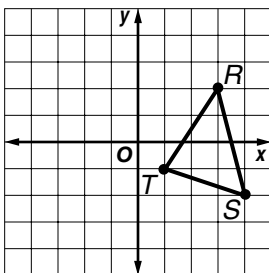


2. Translate rectangle $LMNO$ 4 units right and 3 units up.

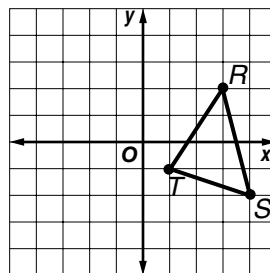


Triangle RST has vertices $R(3, 2)$, $S(4, -2)$, and $T(1, -1)$. Find the vertices of $R'S'T'$ after each translation. Then graph the figure and its translated image.

3. 5 units left, 1 unit up



4. 3 units left, 2 units down

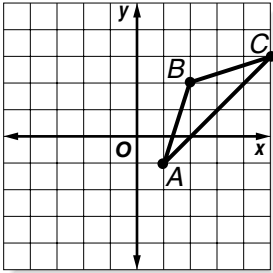


10-8

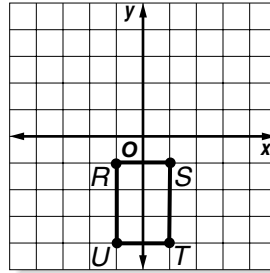
Practice: Skills

Translations

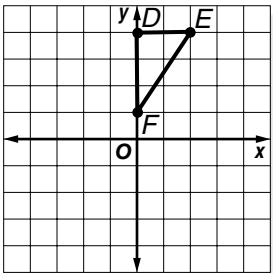
1. Translate $\triangle ABC$ 5 units left.



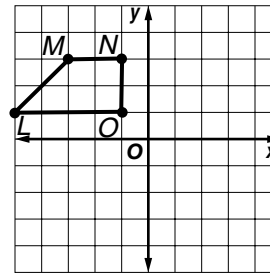
2. Translate rectangle $RSTU$ 2 units right and 5 units up.



3. Translate $\triangle DEF$ 4 units left and 4 units down.

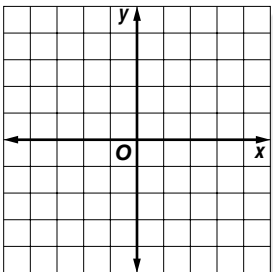


4. Translate trapezoid $LMNO$ 5 units right and 3 units down.

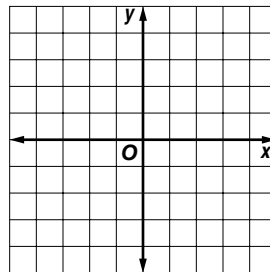


Triangle XYZ has vertices $X(-4, 5)$, $Y(-1, 3)$, and $Z(-2, 0)$. Find the vertices of $X'Y'Z'$ after each translation. Then graph the figure and its translated image.

5. 5 units down

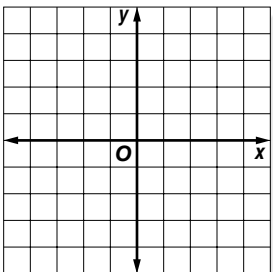


6. 4 units right, 3 units down

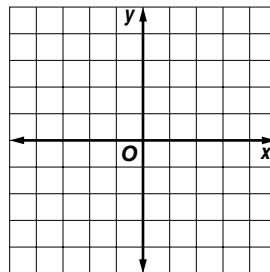


Parallelogram $RSTU$ has vertices $R(-1, -3)$, $S(0, -1)$, $T(4, -1)$, and $U(3, -3)$. Find the vertices of $R'S'T'U'$ after each translation. Then graph the figure and its translated image.

7. 3 units left, 3 units up



8. 1 unit right, 5 units up

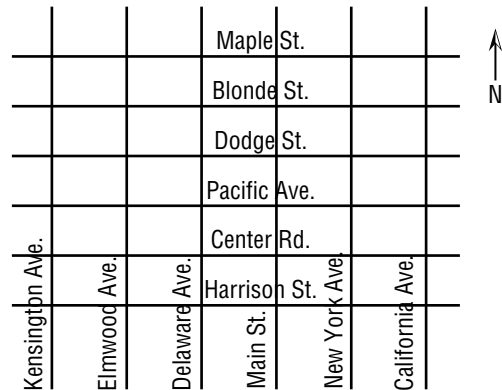


10-8

Practice: Word Problems

Translations

MAPS For Exercises 1–4, use the map at the right.



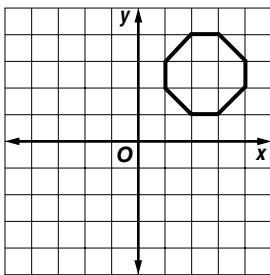
1. Stanley's school is located at the corner of Center and Elmwood. The library is located at the corner of Dodge and Delaware. Describe Stanley's walk from school to the library as an ordered pair of the number of blocks.

2. After he goes to the library, Stanley goes to his Aunt Jeanne's house at the corner of California and Harrison. Describe Stanley's walk from the library to his aunt's house as an ordered pair of the number of blocks.

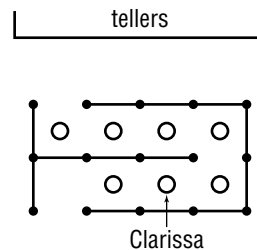
3. If a bus picks up passengers at the corner of New York and Maple and drives 2 blocks south and 3 blocks west, where does the bus end up?

4. Organizers of a walkathon want to map out a route that will lead people from the corner of Center and Kensington to the corner of California and Maple. Write a coordinate pair that describes the most direct route.

5. **GEOMETRY** The figure shows an octagon plotted on a coordinate system. The figure is to be translated by 5 units left and 5 units down. Graph the translated image of the figure.



6. **BANKS** Clarissa is waiting in line at the bank. There are several people in line in front of her. Describe the path Clarissa must take to get to the front of the line if each time she moves up in line by one position is considered one unit.



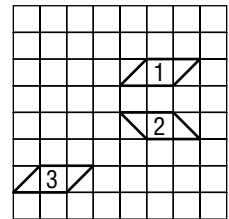
10-8**Reading to Learn Mathematics****Translations**

Pre-Activity Complete the Mini Lab at the top of page 451 in your textbook. Write your answers below.

1. Make your own tessellation. Use different colors to make an interesting design.

Reading the Lesson

2. When translating a figure, what do you know about every point of the original figure?
3. Can a figure be turned in a translation? Explain.
4. What notation is used to indicate the vertices of a translated figure?
5. Which figure is a translation of Figure 1—Figure 2 or Figure 3? Explain why one figure is a translation and why the other figure is not a translation.

**Helping You Remember**

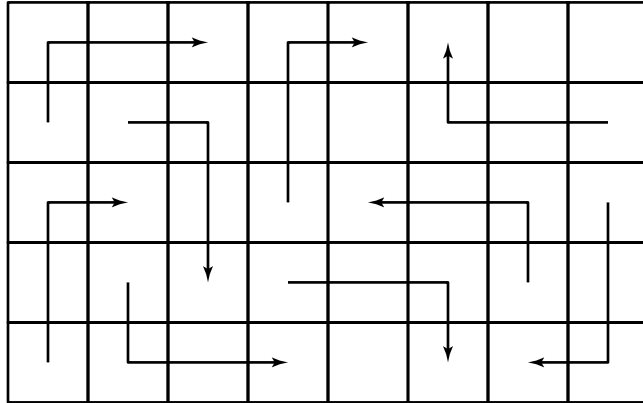
6. Describe the translation given by the ordered pair $(-7, 3)$. Think of a way to remember which direction to translate when the x -coordinate of the ordered pair describing the translation is negative.

10-8

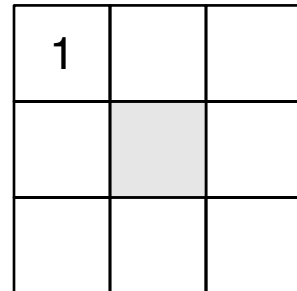
Enrichment

Chess Moves

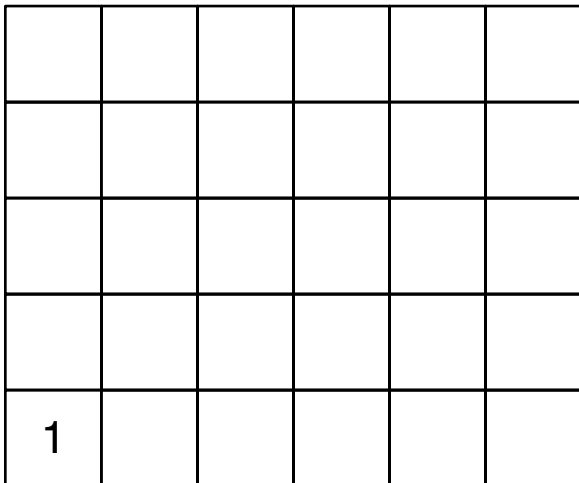
In the game of chess, a knight can move several different ways. It can move two spaces vertically or horizontally, then one space at a 90° angle. It can also move one space vertically or horizontally, then two spaces at a 90° angle. Several examples of a knight's moves are indicated on the grid at the right.



1. Use the diagram at the right. Place a knight or other piece in the square marked 1. Move the knight so that it lands on each of the remaining white squares only once. Mark each square in which the knight lands with 2, then 3, and so on.



2. Use the diagram below. Place a knight or other piece in the square marked 1. Move the knight so that it lands on each of the remaining squares only once. Mark each square in which the knight lands with 2, then 3, and so on.



10-9

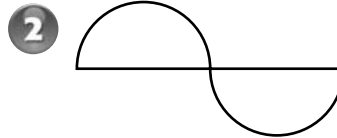
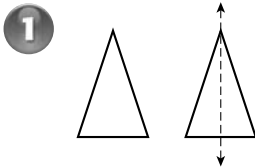
Study Guide and Intervention

Reflections

Figures that match exactly when folded in half have **line symmetry**. Each fold line is called a **line of symmetry**. Some figures have more than one line of symmetry.

EXAMPLES

Determine whether each figure has line symmetry. If so, draw all lines of symmetry.



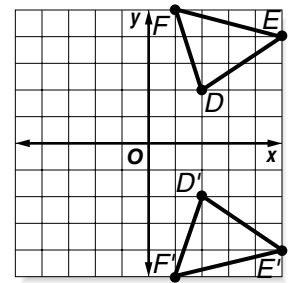
no symmetry

A type of transformation where a figure is flipped over a line of symmetry is a **reflection**. To draw the reflection of a polygon, find the distance from each vertex of the polygon to the line of symmetry. Plot the new vertices the same distance from the line of symmetry but on the other side of the line. Then connect the new vertices to complete the reflected image.

EXAMPLE 3

Triangle DEF has vertices $D(2, 2)$, $E(5, 4)$, and $F(1, 5)$. Find the coordinates of the vertices of DEF after a reflection over the x -axis. Then graph the figure and its reflected image.

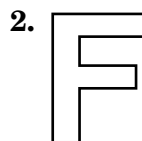
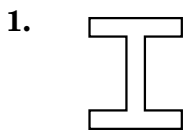
| Vertices of $\triangle DEF$ | Distance from x -axis | Vertices of $\triangle D'E'F'$ |
|-----------------------------|-------------------------|--------------------------------|
| $D(2, 2)$ | 2 | $D'(2, -2)$ |
| $E(5, 4)$ | 4 | $E'(5, -4)$ |
| $F(1, 5)$ | 5 | $F'(1, -5)$ |



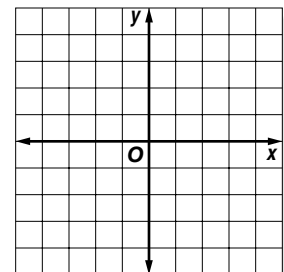
Plot the vertices and connect to form $\triangle DEF$. The x -axis is the line of symmetry. The distance from a point on $\triangle DEF$ to the line of symmetry is the same as the distance from the line of symmetry to the reflected image.

EXERCISES

For Exercises 1 and 2, determine which figures have line symmetry. Write *yes* or *no*. If *yes*, draw all lines of symmetry.



3. Triangle ABC has vertices $A(0, 4)$, $B(2, 1)$, and $C(4, 3)$. Find the coordinates of the vertices of ABC after a reflection over the x -axis. Then graph the figure and its reflected image.



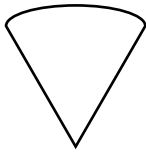
10-9

Practice: Skills

Reflections

Determine which figures have line symmetry. Then draw all lines of symmetry.

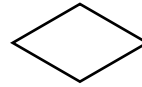
1.



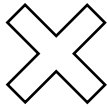
2.



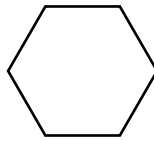
3.



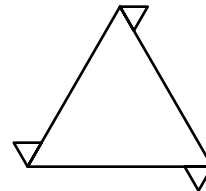
4.



5.

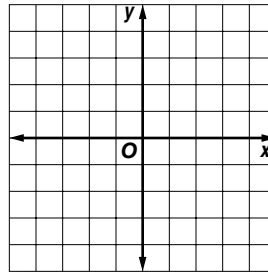


6.

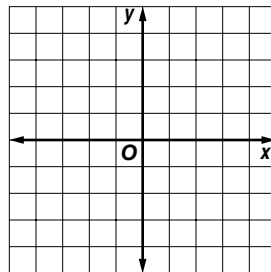


Find the coordinates of the vertices of each figure after a reflection over the x -axis. Then graph the figure and its reflected image.

7. triangle ABC with vertices $A(-3, 4)$, $B(1, 4)$, and $C(3, 1)$

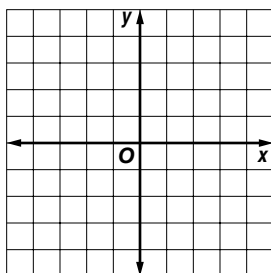


8. rectangle $MNOP$ with vertices $M(-2, -4)$, $N(-2, -1)$, $O(3, -1)$, and $P(3, -4)$

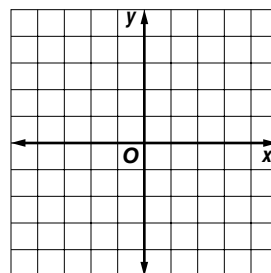


Find the coordinates of the vertices of each figure after a reflection over the y -axis. Then graph the figure and its reflected image.

9. triangle DEF with vertices $D(1, 4)$, $E(4, 3)$, and $F(2, 0)$



10. trapezoid $WXYZ$ with vertices $W(-1, 3)$, $X(-1, -4)$, $Y(-5, -4)$, and $Z(-3, 3)$

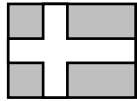


10-9

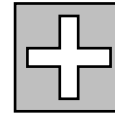
Practice: Word Problems

Reflections

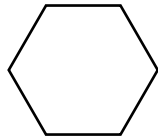
1. FLAGS The figure shows a flag similar to the national flag of Denmark. How many lines of symmetry does the flag have? Draw all lines of symmetry.



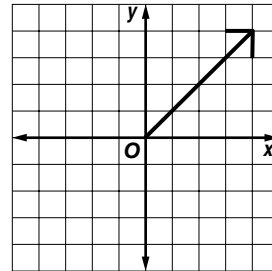
2. FLAGS The figure shows a flag similar to the national flag of Switzerland. How many lines of symmetry does the flag have? Draw all lines of symmetry.



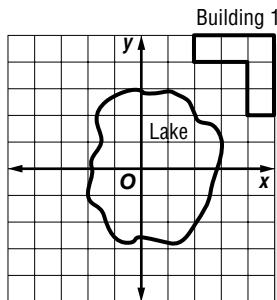
3. INTERIOR DESIGN An interior designer has been hired to decorate a room that has the shape of a regular hexagon. Before beginning work, the designer studies the symmetry of the room. How many lines of symmetry does the room have? Draw all lines of symmetry on the figure.



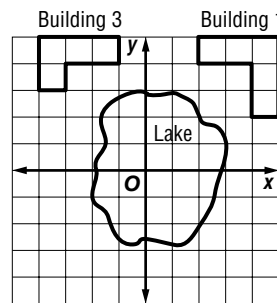
4. ASTROLOGY The figure shows the astrological symbol for Sagittarius plotted on a coordinate system. Reflect the symbol across the x -axis. Graph the reflected image.



5. ARCHITECTURE A corporate plaza is to be built around a small lake. Building 1 has already been built. Suppose there are axes through the lake as shown. Show where Building 2 should be built if it will be a reflection of Building 1 across the y -axis followed by a reflection across the x -axis.



6. ARCHITECTURE Use the information from Exercise 5. Suppose that a third building is to be built as shown. To complete the business park, show where a fourth building should be built if it is a reflection of Building 3 across the x - and y -axis.



Reading to Learn Mathematics

Reflections

Pre-Activity Complete the Mini Lab at the top of page 456 in your textbook. Write your answers below.

1. Describe how you drew the reflection of your last name.
2. List the capital letters that look the same as their reflections.
3. Explain why the line where the geomirror and paper meet is called the *line of symmetry*.

Reading the Lesson

4. Is the image of a reflection smaller, larger, or the same size as the original figure?
5. In Example 4 on page 457, how can you tell that one image is a reflection of the other across the x -axis?
6. Study the tables given in Examples 4 and 5 on page 457. Without graphing, how can you tell how many units a vertex is away from the x -axis if you know the coordinates of the vertex? How can you tell how many units a vertex is away from the y -axis if you know the coordinates of the vertex?

Helping You Remember

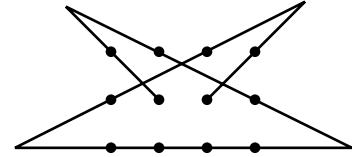
7. Work with a partner. Draw and cut out figures of regular polygons. Demonstrate which regular polygons have lines of symmetry and which do not. Mark the lines of symmetry with dashed lines on the models.

10-9

Enrichment

The Twelve Dot Puzzle

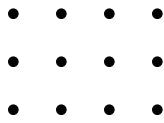
In this puzzle, a broken line made up of 5 segments must pass through each of 12 dots. The line cannot go through a dot more than once, although it may intersect itself. The line must start at one dot and end at a different dot.



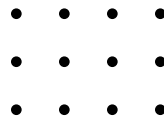
One solution to this puzzle is shown at the right. Two solutions to the puzzle are not “different” if one is just a reflection or rotation of the other.

Find 18 other solutions.

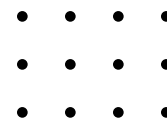
1.



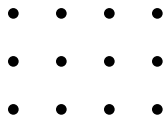
2.



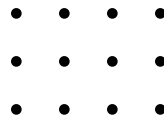
3.



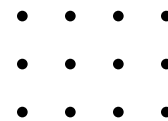
4.



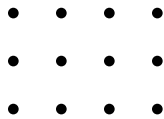
5.



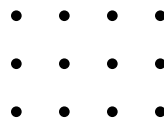
6.



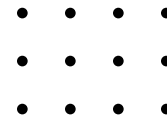
7.



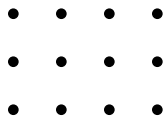
8.



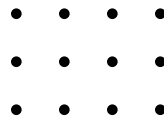
9.



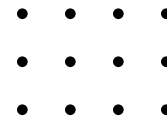
10.



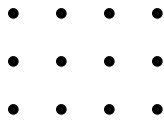
11.



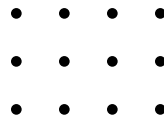
12.



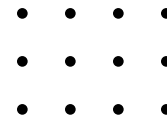
13.



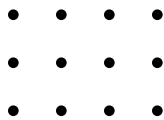
14.



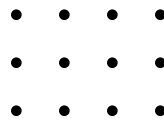
15.



16.



17.



18.

