Chapter 11
Resource Masters

GLENCOE MATHEMATICS

Mathematics

Applications and Concepts

Course 2

Mc Graw Hill Glencoe

New York, New York Columbus, Ohio Chicago, Illinois Peoria, Illinois Woodland Hills, California

NAME _____ DATE _____ PERIOD _____

11-1 Study Guide and Intervention

Squares and Square Roots

The product of a number and itself is the **square** of the number. Numbers like 4, 25, and 2.25 are called **perfect squares** because they are squares of rational numbers. The factors multiplied to form perfect squares are called **square roots**. Both $5 \cdot 5$ and $(-5)(-5)$ equal 25. So, 25 has two square roots, 5 and -5. A **radical sign**, $\sqrt{\ }$, is the symbol used to indicate the *positive* square root of a number. So, $\sqrt{25} = 5$.

EXAMPLES

1 **Find the square of 5.**

$5 \cdot 5 = 25$

2 **Find the square of 16.**

16 [x^2] [ENTER] **256**

3 **Find $\sqrt{49}$.**

$7 \cdot 7 = 49$, so $\sqrt{49} = 7$.

4 **Find $\sqrt{169}$.**

[2nd] [$\sqrt{\ }$] 169 [ENTER] **13**

So, $\sqrt{169} = 13$.

EXAMPLE 5 **A square tile has an area of 144 square inches. What are the dimensions of the tile?**

[2nd] [$\sqrt{\ }$] 144 [ENTER] **12** Find the square root of 144.

So, the tile measures 12 inches by 12 inches.

EXERCISES

Find the square of each number.

1. 2 **2.** 9 **3.** 14

4. 15 **5.** 21 **6.** 45

Find each square root.

7. $\sqrt{16}$ **8.** $\sqrt{36}$ **9.** $\sqrt{256}$

10. $\sqrt{1{,}024}$ **11.** $\sqrt{361}$ **12.** $\sqrt{484}$

Lesson 11–1

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11-1 Practice: Skills

Squares and Square Roots

Find the square of each number.

1. 3

2. 22

3. 25

4. 24

5. 35

6. 26

7. 37

8. 50

Find each square root.

9. $\sqrt{25}$

10. $\sqrt{100}$

11. $\sqrt{441}$

12. $\sqrt{900}$

13. $\sqrt{961}$

14. $\sqrt{784}$

15. $\sqrt{3{,}600}$

16. $\sqrt{1{,}936}$

17. What is the square of -37?

18. Find both square roots of 4,900.

19. Square 7.2.

20. Square 4.5.

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11-1 Practice: Word Problems

Squares and Square Roots

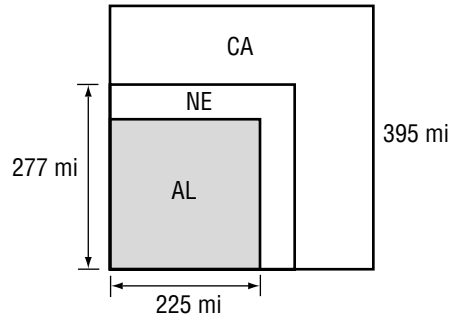
1. FERTILIZER John bought a bag of lawn fertilizer that will cover 400 square feet. What are the dimensions of the largest square plot of lawn that the bag of fertilizer will cover?

2. GEOMETRY The area A of a circle in square feet with a radius r in feet is given approximately by the formula $A \approx 3.14r^2$. What is the approximate area of a circle with a radius of 3 feet?

3. MOTION The time t in seconds for an object dropped from a height of h feet to hit the ground is given by the formula $t = \sqrt{\frac{2h}{32}}$. How long will it take an object dropped from a height of 500 feet to hit the ground? Round to the nearest tenth.

4. PACKAGING A cardboard envelope for a compact disc is a square with an area of 171.61 square centimeters. What are the dimensions of the envelope?

5. GEOGRAPHY Refer to the squares below. They represent the approximate areas of California, Alabama, and Nebraska. Find the area of Alabama.



6. Use the figure in Exercise 5. How much larger is California than Nebraska?

Lesson 11–1

NAME _____ DATE _____ PERIOD _____

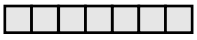
11-1 Reading to Learn Mathematics

Squares and Square Roots

Pre-Activity *Complete the Mini Lab at the top of page 470 in your textbook. Write your answers below.*

1. On grid paper, draw and label three other rectangles that have a perimeter of 16 units.

2. Summarize the dimensions and areas of the rectangles that you drew in a table like the one shown below.

Drawing	Dimensions (units)	Area (sq units)
	1×7	7

3. Draw three different rectangles that have a perimeter of 12 units and find their areas.

4. What do you notice about the rectangles with the greatest areas?

Reading the Lesson

5. In this lesson, the word *square* is used in several different ways. Tell the meaning of the word as it is used in each phrase or sentence.

a. Find the *square* of 3.

b. 9 units *squared*

c. A boxing ring is a *square* with an area of 400 ft^2.

Helping You Remember

6. Work with a partner. Use a calculator to find the squares of six numbers, some of them decimals. Then write only the squares in a list and exchange lists with your partner. Find the square roots of the squares in the list that you receive. Write your answers in the form $\sqrt{x} = y$.

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11-1 Enrichment

The Geometric Mean

The square root of the product of two numbers is called their **geometric mean**. The geometric mean of 12 and 48 is $\sqrt{12 \cdot 48} = \sqrt{576}$ or 24.

Find the geometric mean for each pair of numbers.

1. 2 and 8

2. 4 and 9

3. 9 and 16

4. 16 and 4

5. 16 and 36

6. 12 and 3

7. 18 and 8

8. 2 and 18

9. 27 and 12

Recall the definition of a **geometric sequence**. Each term is found by multiplying the previous term by the same number. A missing term in a geometric sequence equals the geometric mean of the two terms on either side.

Find the missing term in each geometric sequence.

10. 4, 12, ?, 108, 324

11. 10, ?, 62.5, 156.25, 390.625

12. 1, 0.4, ?, 0.064, 0.0256

13. 700, 70, 7, 0.7, ?, 0.007

14. 6, ?, 24

15. 18, ?, 32

Lesson 11–1

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11-2 Study Guide and Intervention

Estimating Square Roots

Recall that a perfect square is a square of a rational number. In Lesson 5-8, you learned that any number that can be written as a fraction is a rational number. A number that cannot be written as a fraction is an **irrational number**.

EXAMPLE 1 **Estimate $\sqrt{40}$ to the nearest whole number.**

List some perfect squares.

1, 4, 9, 16, 25, 36, 49, …

$36 < 40 < 49$ 40 is between the perfect squares 36 and 49.

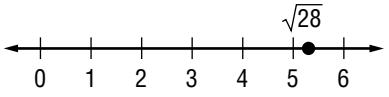
$\sqrt{36} < \sqrt{40} < \sqrt{49}$ Find the square root of each number.

$6 < \sqrt{40} < 7$ $\sqrt{36} = 6$ and $\sqrt{49} = 7$

So, $\sqrt{40}$ is between 6 and 7. Since 40 is closer to 36 than to 49, the best whole number estimate is 6.

EXAMPLE 2 **Use a calculator to find the value of $\sqrt{28}$ to the nearest tenth.**

2nd $\sqrt{\ }$ 28 ENTER **5.291502622**



$\sqrt{28} \approx 5.3$

Check Since $5^2 = 25$ and 25 is close to 28, the answer is reasonable.

EXERCISES

Estimate each square root to the nearest whole number.

1. $\sqrt{3}$

2. $\sqrt{8}$

3. $\sqrt{26}$

4. $\sqrt{41}$

5. $\sqrt{61}$

6. $\sqrt{94}$

7. $\sqrt{152}$

8. $\sqrt{850}$

Use a calculator to find each square root to the nearest tenth.

9. $\sqrt{2}$

10. $\sqrt{27}$

11. $\sqrt{73}$

12. $\sqrt{82}$

13. $\sqrt{105}$

14. $\sqrt{395}$

15. $\sqrt{846}$

16. $\sqrt{2{,}298}$

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11-2 Practice: Skills

Estimating Square Roots

Estimate each square root to the nearest whole number.

1. $\sqrt{5}$ **2.** $\sqrt{10}$ **3.** $\sqrt{21}$

4. $\sqrt{28}$ **5.** $\sqrt{78}$ **6.** $\sqrt{102}$

7. $\sqrt{179}$ **8.** $\sqrt{274}$ **9.** $\sqrt{303}$

10. $\sqrt{563}$ **11.** $\sqrt{592}$ **12.** $\sqrt{755}$

13. $\sqrt{981}$ **14.** $\sqrt{1{,}356}$ **15.** $\sqrt{1{,}688}$

16. $\sqrt{3{,}287}$ **17.** $\sqrt{3{,}985}$ **18.** $\sqrt{4{,}125}$

Use a calculator to find each square root to the nearest tenth.

19. $\sqrt{6}$ **20.** $\sqrt{19}$ **21.** $\sqrt{30}$

22. $\sqrt{77}$ **23.** $\sqrt{114}$ **24.** $\sqrt{125}$

25. $\sqrt{149}$ **26.** $\sqrt{182}$ **27.** $\sqrt{212}$

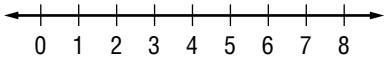
28. $\sqrt{436}$ **29.** $\sqrt{621}$ **30.** $\sqrt{853}$

31. $\sqrt{918}$ **32.** $\sqrt{1{,}004}$ **33.** $\sqrt{1{,}270}$

34. $\sqrt{5{,}438}$ **35.** $\sqrt{4{,}215}$ **36.** $\sqrt{5{,}786}$

37. Order $\frac{25}{7}$, 4.91, and $\sqrt{23}$ from least to greatest.

38. Graph $\sqrt{42}$ and $\sqrt{62}$ on the same number line.



Lesson 11–2

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11-2 Practice: Word Problems

Estimating Square Roots

1. GEOMETRY The diameter d of a circle with area A is given by the formula $d = \sqrt{\frac{4A}{\pi}}$. What is the diameter of a circle with an area of 56 square inches? Use 3.14 for π and round to the nearest tenth.

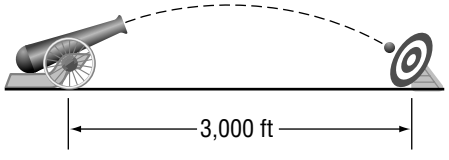
2. FENCING Carmen wants to buy fencing to enclose a square garden with an area of 500 square feet. How much fencing does Carmen need to buy? Round to the nearest tenth.

3. OCEANS The speed v in feet per second of an ocean wave in shallow water of depth d in feet is given by the formula $v = \sqrt{32d}$. What is the speed of an ocean wave at a depth of 10 feet? Round to the nearest tenth.

4. LIGHTING A new flashlight has a beam whose width w at a distance d from the flashlight is given by the formula $w = 1.2\sqrt{d}$. What is the width of the beam at a distance of 30 feet? Round to the nearest tenth.

5. SOUND The speed of sound in air c in meters per second at a temperature T in degrees Celsius is given approximately by the formula $c = \sqrt{402(T + 273)}$. What is the speed of sound in air at a temperature of 25 degrees Celsius? Round to the nearest tenth.

6. PROJECTILES The muzzle velocity v in feet per second necessary for a cannon to hit a target x feet away is estimated by the formula $v = \sqrt{32x}$. What muzzle velocity is required to hit a target 3,000 feet away? Round to the nearest tenth.



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11-2 Reading to Learn Mathematics

Estimating Square Roots

Pre-Activity

Complete the Mini Lab at the top of page 475 in your textbook. Write your answers below.
Use algebra tiles to estimate the square root of each number to the nearest whole number.

1. 40 **2.** 28 **3.** 85 **4.** 62

5. Describe another method that you could use to estimate the square root of a number.

Reading the Lesson

6. Why is $\sqrt{4}$ a rational number and $\sqrt{2}$ an irrational number?

7. How do you read the statement $\sqrt{64} < \sqrt{75} < \sqrt{81}$?

8. Why are $\sqrt{64}$ and $\sqrt{81}$ used in Example 1?

Helping You Remember

9. The key to estimating square roots without a calculator is to be familiar with common perfect squares. Complete the following table of common perfect squares then test yourself to see how many you can remember without using a calculator.

Number	5	6	7	8	9	10	11	12	13	14	15	16	20	25
Square	25													

Lesson 11–2

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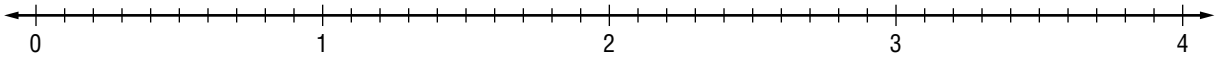
11-2 Enrichment

World Series Records

Each problem gives the name of a famous baseball player. To find who set each record, graph the points on the number line.

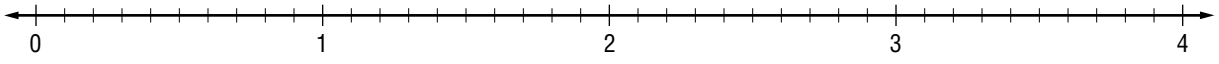
1. pitched 23 strikeouts in one World Series

U at $\sqrt{3}$, X at 3.3, K at 0.75, O at $\frac{3}{2}$, F at $\sqrt{6}$, A at $2\frac{7}{8}$



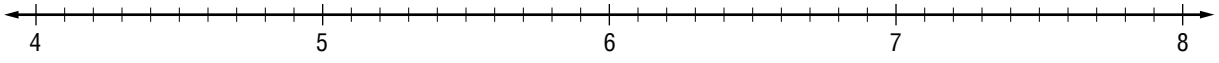
2. 71 base hits in his appearances in World Series

B at $\sqrt{5}$, R at $\sqrt{12}$, A at 3.75, G at $\frac{16}{13}$, E at $\frac{5}{2}$, Y at 0.375, R at $\frac{13}{4}$, I at 1.6, and O at $0.\overline{7}$



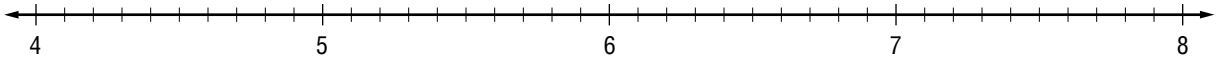
3. 10 runs in a single World Series

N at $\sqrt{60}$, K at $\sqrt{30}$, A at 4.3, S at 6.2, C at $\frac{46}{9}$, O at $\sqrt{45}$, and J at $\sqrt{17}$



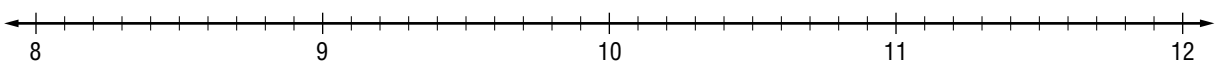
4. batting average of 0.625 in a single World Series

E at $\sqrt{32}$, U at $6\frac{5}{6}$, A at $\frac{14}{3}$, T at $\sqrt{55}$, B at 5.3, R at $\sqrt{40}$, H at 7.75, B at $\frac{21}{5}$



5. 42 World Series runs in his career

E at $\sqrt{140}$, Y at 9.6, I at 8.6, E at $\sqrt{90}$, A at $\frac{21}{2}$, M at $\sqrt{70}$, C at $8\frac{7}{8}$, M at $\sqrt{100}$, N at 10.7, K at $9\frac{1}{11}$, T at $\sqrt{120}$, L at 11.4



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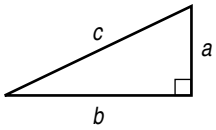
11-3 Study Guide and Intervention

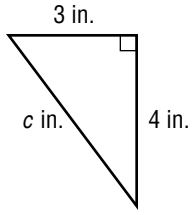
The Pythagorean Theorem

The sides of a right triangle have special names. The sides adjacent to the right angle are the **legs**. The side opposite the right angle is the **hypotenuse**. The **Pythagorean Theorem** describes the relationship between the length of the hypotenuse and the lengths of the legs. In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

$c^2 = a^2 + b^2$



EXAMPLE 1 **Find the missing measure of a right triangle if $a = 4$ inches and $b = 3$ inches.**



$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 4^2 + 3^2$	Replace a with 4 and b with 3.
$c^2 = 16 + 9$	Evaluate 4^2 and 3^2.
$c^2 = 25$	Add.
$\sqrt{c^2} = \sqrt{25}$	Take the square root of each side.
$c = 5$	Simplify.

The length of the hypotenuse is 5 inches.

EXAMPLE 2 **Determine whether a triangle with side lengths of 6 meters, 9 meters, and 12 meters is a right triangle.**

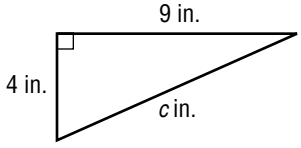
$c^2 = a^2 + b^2$	Pythagorean Theorem
$12^2 \stackrel{?}{=} 6^2 + 9^2$	Replace a with 6, b with 9, and c with 12.
$144 \stackrel{?}{=} 36 + 81$	Simplify.
$144 \neq 117$	Add.

The triangle is *not* a right triangle.

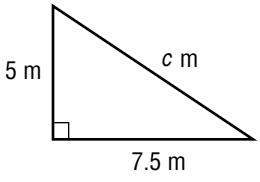
EXERCISES

Find the missing measure of each right triangle. Round to the nearest tenth if necessary.

1.



2.



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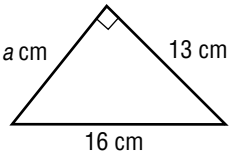

Determine whether each triangle with the given side lengths is a right triangle. Write *yes* or *no*.

4. 15 ft, 8 ft, 17 ft

5. 5 in., 13 in., 17 in.

6. 9 yd, 40 yd, 41 yd

Lesson 11–3

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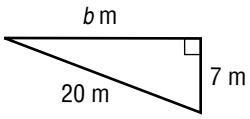
11-3 Practice: Skills

The Pythagorean Theorem

Find the missing measure of each right triangle. Round to the nearest tenth if necessary.

1.



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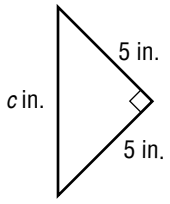

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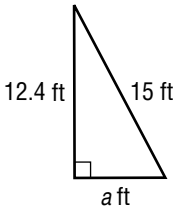

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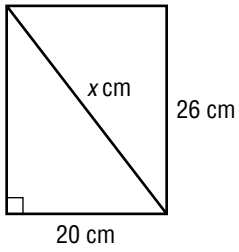
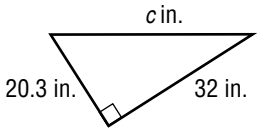


5.



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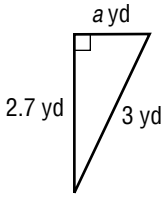

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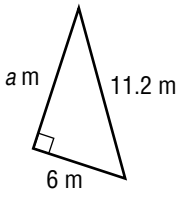
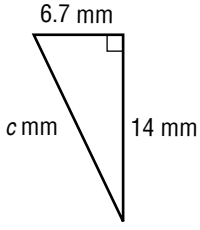


8.



9. $a = 15$ cm, $b = 20$ cm

10. $a = 2$ yd, $b = 12$ yd

11. $a = 13$ in., $c = 16.5$ in.

12. $b = 8$ mm, $c = 17$ mm

13. $a = 1.3$ ft, $b = 4.6$ ft

14. $a = 14.7$ m, $c = 23$ m

Determine whether each triangle with the given side lengths is a right triangle. Write *yes* or *no*.

15. 10 ft, 24 ft, 26 ft

16. 5 in., 8 in., 9 in.

17. 6 cm, 9 cm, 12 cm

18. 4.5 mm, 6.0 mm, 7.5 mm

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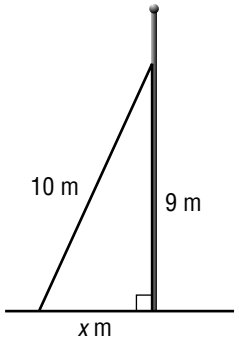
11-3 Practice: Word Problems

The Pythagorean Theorem

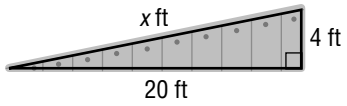
1. ORIGAMI Chee has a piece of paper measuring 8.5 inches by 8.5 inches. If she folds the paper diagonally in half, how long is the folded side? Round to the nearest tenth.

2. COMPUTERS In a computer catalog, a computer monitor is said to be 19 inches. This distance is the diagonal distance across the screen. If the screen is 10 inches high, what is the width of the screen? Round to the nearest tenth.

3. ANTENNAS A wire 10 meters long is supporting a utility pole. The wire is anchored to the ground and is attached to the pole 9 meters above the ground. What is the distance from the bottom of the pole to the point where the wire is attached to the ground? Round to the nearest tenth.



4. RAMPS Crystal wants to build a ramp that will rise 4 feet over a horizontal distance of 20 feet. How long will the ramp be? Round to the nearest tenth.



5. POOLS Salomon swims diagonally across his pool every day. If Salomon's pool is 4 meters wide and 16 meters diagonally across, how long is his pool, to the nearest tenth of a meter?

6. FRAMES Rosa has a picture frame that measures 12 inches by 18 inches. What is the diagonal distance across the frame? Round to the nearest tenth.

Lesson 11–3

NAME _____ DATE _____ PERIOD _____

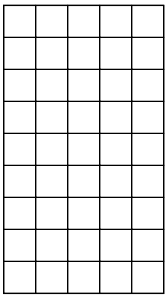
11-3 Reading to Learn Mathematics

The Pythagorean Theorem

Pre-Activity *Read the introduction at the top of page 479 in your textbook. Write your answers below.*

1. Can the mirror fit through the doorway? Explain.

2. Make a scale drawing on grid paper to solve the problem.



Reading the Lesson

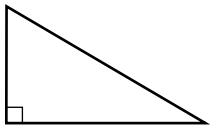
3. In the Pythagorean Theorem $c^2 = a^2 + b^2$, which letter represents the length of the hypotenuse?

4. How do you know that the diagonal of a rectangle is the hypotenuse of two right triangles?

5. In Examples 4 and 5 on page 481, how do you know which length is c?

Helping You Remember

6. Summarize what you learned in this lesson by labeling the sides of the right triangle with the letters a, b, and c and then completing the table.



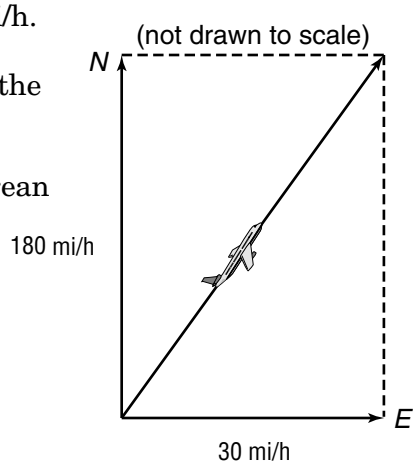
You can find	If you know the lengths
a	
b	
c	

NAME _____ DATE _____ PERIOD _____

11-3 Enrichment

Pythagoras in the Air

In the diagram at the right, an airplane heads north at 180 mi/h. But, the wind is blowing towards the east at 30 mi/h. So, the airplane is really traveling east of north. The middle arrow in the diagram shows the actual direction of the airplane.



The actual speed of the plane can be found using the Pythagorean Theorem.

$$\begin{aligned}\sqrt{30^2 + 180^2} &= \sqrt{900 + 32{,}400} \\ &= \sqrt{33{,}300} \\ &\approx 182.5\end{aligned}$$

The plane's actual speed is about 182.5 mi/h.

Find the actual speed of each airplane. Round answers to the nearest tenth. (You might wish to draw a diagram to help you solve the problem.)

1. An airplane travels at 240 mi/h east. A wind is blowing at 20 mi/h toward the south.

2. An airplane travels at 620 mi/h west. A wind is blowing at 35 mi/h toward the south.

3. An airplane travels at 450 mi/h south. A wind is blowing at 40 mi/h toward the east.

4. An airplane travels at 1,200 mi/h east. A wind is blowing at 30 mi/h toward the north.

Lesson 11–3

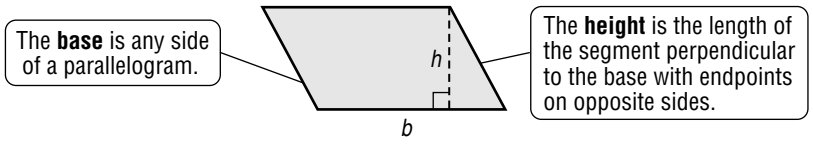
NAME _____ DATE _____ PERIOD _____

11-4 Study Guide and Intervention

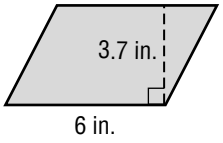
Area of Parallelograms

The area A of a parallelogram equals the product of its base b and its height h.

$$A = bh$$



EXAMPLE 1 **Find the area of a parallelogram if the base is 6 inches and the height is 3.7 inches.**

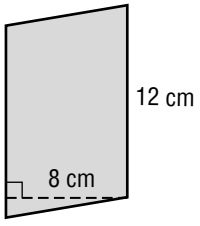


Estimate $A = 6 \cdot 4$ or 24 in^2

$A = bh$	Area of a parallelogram
$A = 6 \cdot 3.7$	Replace b with 6 and h with 3.7.
$A = 22.2$	Multiply.

The area of the parallelogram is 22.2 square inches. This is close to the estimate.

EXAMPLE 2 **Find the area of the parallelogram at the right.**



Estimate $A = 10 \cdot 10$ or 100 cm^2

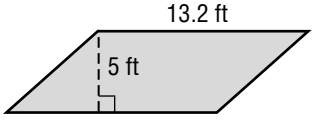
$A = bh$	Area of a parallelogram
$A = 12 \cdot 8$	Replace b with 12 and h with 8.
$A = 96$	Multiply.

The area of the parallelogram is 96 square centimeters. This is close to the estimate.

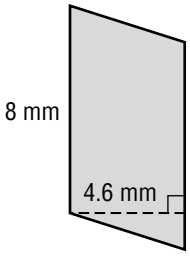
EXERCISES

Find the area of each parallelogram. Round to the nearest tenth if necessary.

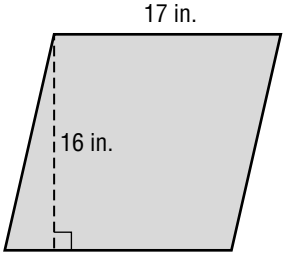
1.



2.



3.



NAME _____ DATE _____ PERIOD _____

11-4 Practice: Skills

Area of Parallelograms

Find the area of each parallelogram. Round to the nearest tenth if necessary.

1. base = 5 ft
height = 12 ft

2. base = 9 in.
height = 2 in.

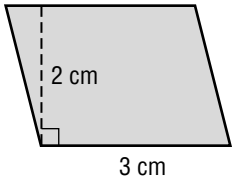
3. base = 6 cm
height = 5.5 cm

4. base = $4\frac{2}{5}$ yd
height = 2 yd

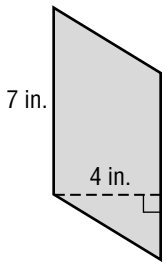
5. base = 15.3 mm
height = 8 mm

6. base = 19.6 m
height = 14.5 m

7.



8.



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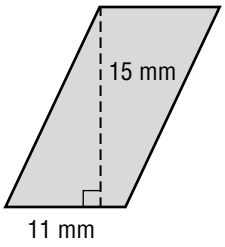

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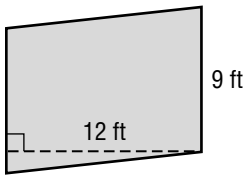

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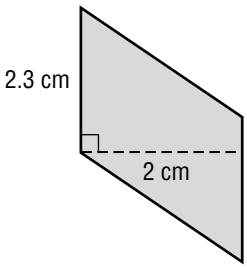

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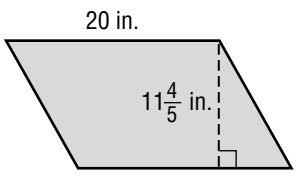
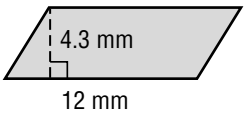
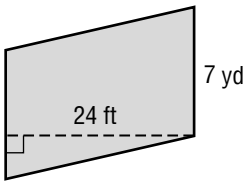


13.



14.



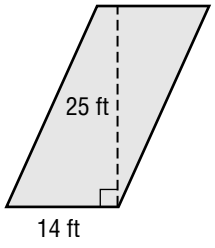
Lesson 11–4

NAME _____ DATE _____ PERIOD _____

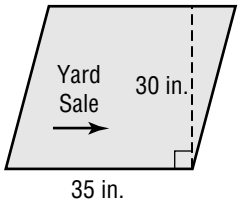
11-4 Practice: Word Problems

Area of Parallelograms

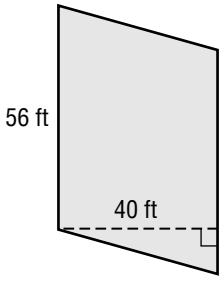
1. SAILS Joyce wants to construct a sail with the dimensions shown. How much material will be used?



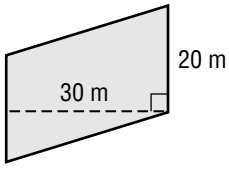
2. SIGNS Pedro wants to make the sign in the shape shown and needs to know how much material will be needed. What is the area of the sign?



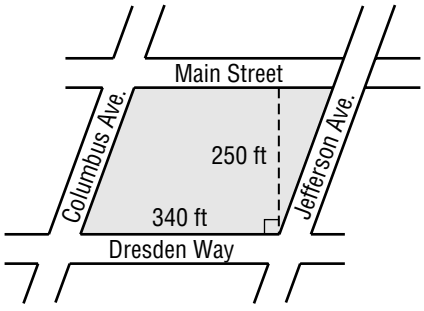
3. SHADING Alma's engineering firm must determine the area of the largest noontime shadow that a proposed building design will create. What is the area of the shadow?



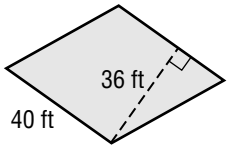
4. POOLS Tamika has designed a pool in the shape shown. What is the area of the bottom of the pool if the surface is perfectly flat?



5. CITY PLANNING Two parallel streets are cut across by two other parallel streets as shown in the figure, cutting off a parcel of land in the shape of a parallelogram. Find the area of the parcel of land.



6. TARPS Neka wants to cut a tarp in the shape shown. What is the minimum amount of canvas cloth that he will need?



NAME _____ DATE _____ PERIOD _____

11-4 Reading to Learn Mathematics

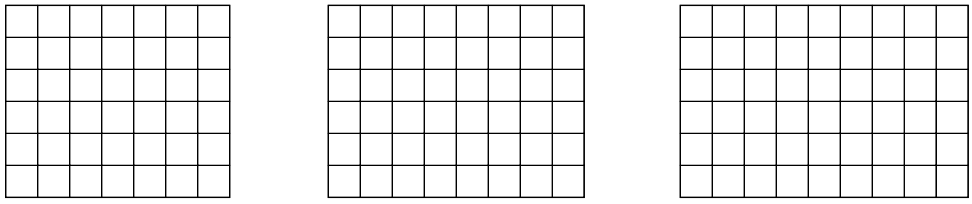
Area of Parallelograms

Pre-Activity *Complete the Mini Lab at the top of page 483 in your textbook. Write your answers below.*

1. What is the value of x and y for each parallelogram?

2. Count the grid squares to find the area of each parallelogram.

3. On grid paper, draw three different parallelograms in which $x = 5$ units and $y = 4$ units. Find the area of each.

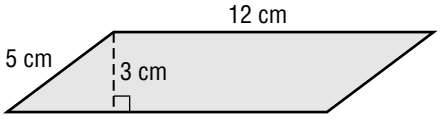


4. **Make a conjecture** about how to find the area of a parallelogram if you know the values of x and y.

Reading the Lesson

5. Explain how to find the height of a parallelogram.

6. Suppose you are asked to find the area of the parallelogram below. Is the given solution correct? Explain.



$A = bh$
$A = 12 \cdot 5$
$A = 60$
The area of the parallelogram is 60 square centimeters.

Helping You Remember

7. Because rectangles, rhombuses, and squares are all parallelograms, the formula for finding the area of a parallelogram is also used to find the areas of each of these figures. Think of a way to remember that the area of a parallelogram is the product of its base and height. For example, draw several parallelograms, rectangles, rhombuses, and squares and label the base and height for each. Write the formula for the area below each model.

Lesson 11–4

NAME _____ DATE _____ PERIOD _____

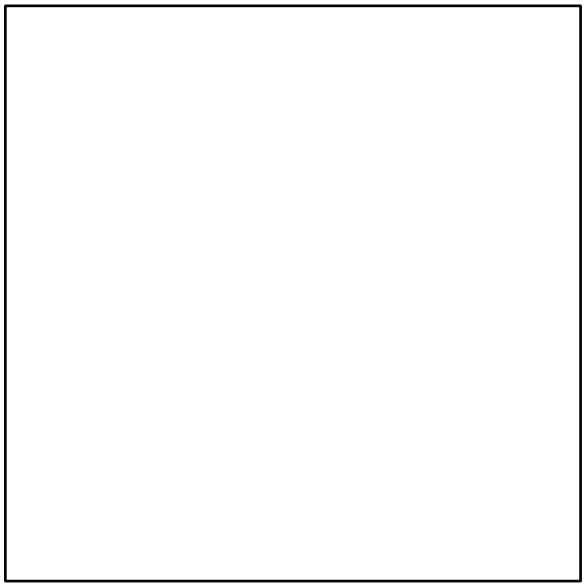
11-4 Enrichment

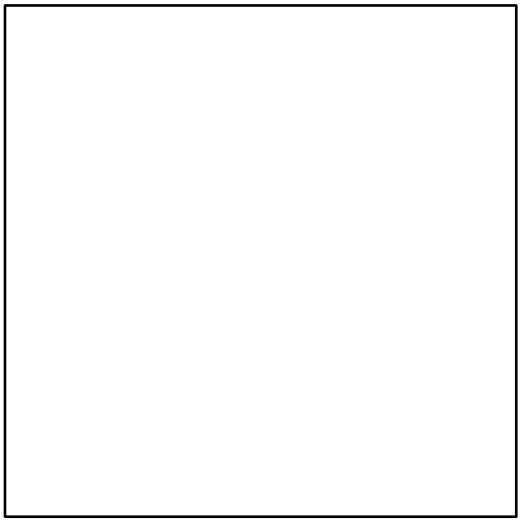
Two Area Puzzles

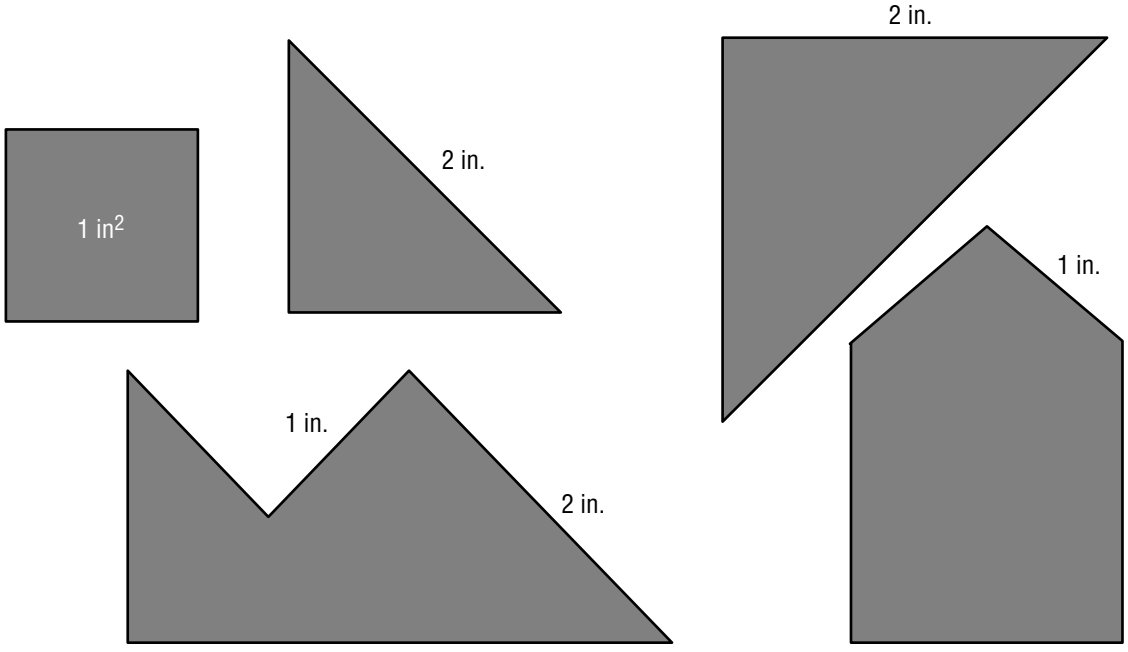
Cut out the five puzzle pieces at the bottom of this page. Then use them to solve these two puzzles.

1. Use all five puzzle pieces to make a square with an area of 9 square inches. Record your solution below.

2. Use the four largest pieces to make a square with an area of 8 square inches. Record your solution below.







NAME _____ DATE _____ PERIOD _____

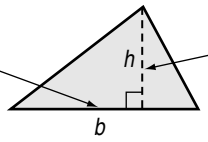
11-5 Study Guide and Intervention

Area of Triangles and Trapezoids

The area A of a triangle equals half the product of its base b and its height h.

$$A = \frac{1}{2}bh$$

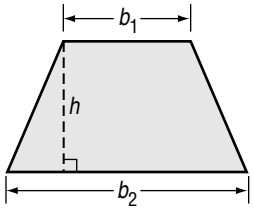
The **base** of a triangle can be any of its sides.



The **height** is the distance from a base to the opposite vertex.

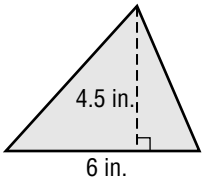
A trapezoid has two bases, b_1 and b_2. The height of a trapezoid is the distance between the two bases. The area A of a trapezoid equals half the product of the height h and the sum of the bases b_1 and b_2.

$$A = \frac{1}{2}h(b_1 + b_2)$$



EXAMPLE 1 **Find the area of the triangle.**

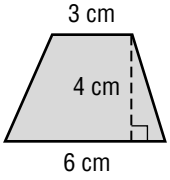
Estimate $\frac{1}{2}(6)(5) = 15$



$A = \frac{1}{2}bh$	Area of a triangle
$A = \frac{1}{2}6 \cdot 4.5$	Replace b with 6 and h with 4.5.
$A = 13.5$	Multiply.

The area of the triangle is 13.5 square inches. This is close to the estimate.

EXAMPLE 2 **Find the area of the trapezoid.**



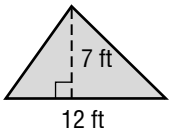
$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$A = \frac{1}{2}(4)(3 + 6)$	Replace h with 4, b_1 with 3, and b_2 with 6.
$A = 18$	Simplify.

The area of the trapezoid is 18 square centimeters.

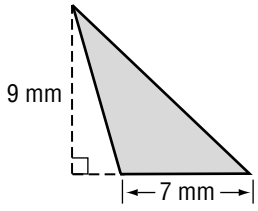
EXERCISES

Find the area of each figure. Round to the nearest tenth if necessary.

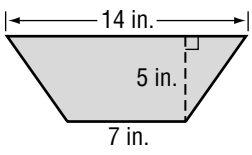
1.



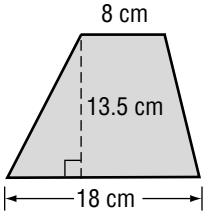
2.



3.



4.



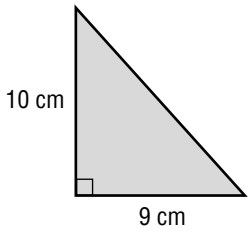
NAME _____ DATE _____ PERIOD _____

11-5 Practice: Skills

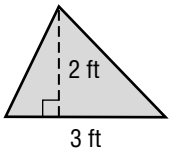
Area of Triangles and Trapezoids

Find the area of each figure. Round to the nearest tenth if necessary.

1.



2.



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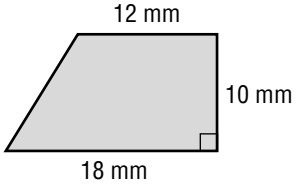
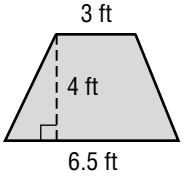
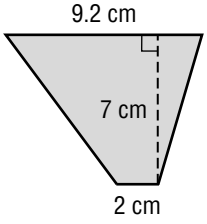


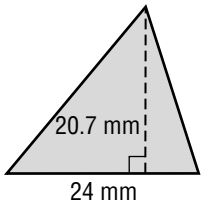
4.



5.



6.



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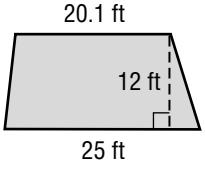
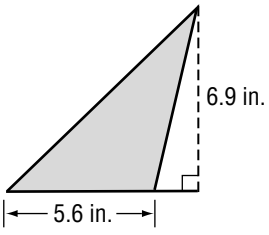


8.



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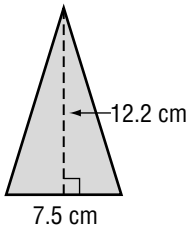
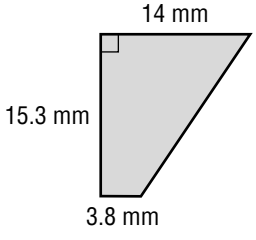


10.



11. triangle: base = 16 cm, height = 9.4 cm

12. triangle: base = 13.5 in., height = 6.4 in.

13. trapezoid: bases 22.8 mm and 19.7 mm, height 36 mm

14. trapezoid: bases 5 ft and $3\frac{1}{2}$ yd, height 7 ft

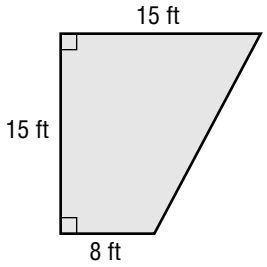
NAME _____ DATE _____ PERIOD _____

11-5 Practice: Word Problems

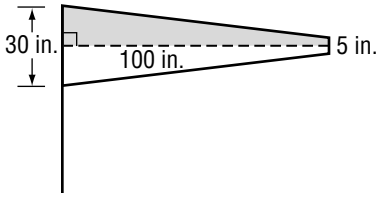
Area of Triangles and Trapezoids

1. **GEOGRAPHY** Arkansas has a shape that is similar to a trapezoid with bases of about 182 miles and 267 miles and a height of about 254 miles. Estimate the area of the state.

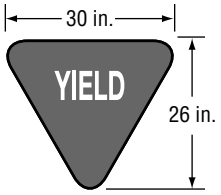
2. **PATIOS** Greta is making a patio with the dimensions given in the figure. What is the area of the patio?



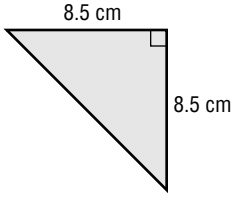
3. **FLAGS** Malila wants to make the International Marine Signal flag shown which represents the number six. What is the area of the flag?



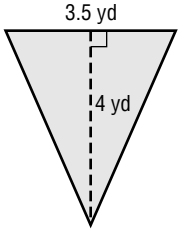
4. **SIGNS** Estimate the area of the yield sign.



5. **TILING** A ceramics company wants to produce tiles in the shape shown. What is the area of the surface of each tile?



6. **GARDENING** Kinu wants to buy topsoil for a section of her garden that has the dimensions shown in the figure. What is the area of this section of Kinu's garden?



NAME _____ DATE _____ PERIOD _____

11-5

Reading to Learn Mathematics

Area of Triangles and Trapezoids

Pre-Activity

Complete the Mini Lab at the top of page 489 in your textbook. Write your answers below.

1. What is the area of the parallelogram?

2. Cut along the diagonal. What is true about the triangles formed?

3. What is the area of each triangle?

4. If the area of a parallelogram is bh, then write an expression for the area A of each of the two congruent triangles that form the parallelogram.

Reading the Lesson

5. In a triangle, which side is the base?

6. How do you find the height of a triangle?

7. For what kind of triangle might the height be found outside of the triangle?

8. How is the height of a trapezoid similar to the height of a triangle or parallelogram?

Helping You Remember

9. The Mini Lab in this lesson gave you a good way to remember the formula for the area of a triangle by showing you that it is half the area of a parallelogram, so $A = \frac{1}{2}bh$. Think of a way to help you remember the formula for the area of a trapezoid. Do you recognize anything in the formula $A = \frac{1}{2}h(b_1 + b_2)$?

NAME _____ DATE _____ PERIOD _____

11-5 Enrichment

Heron's Formula

A formula named after Heron of Alexandria, Egypt, can be used to find the area of a triangle given the lengths of its sides.

Heron's formula states that the area A of a triangle whose sides measure a, b, and c is given by

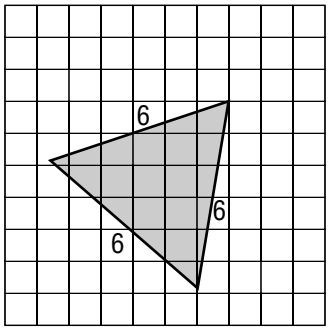
$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter:

$$s = \frac{a + b + c}{2}.$$

Estimate the area of each triangle by finding the mean of the inner and outer measures. Then use Heron's Formula to compute a more exact area. Give each answer to the nearest tenth of a square unit.

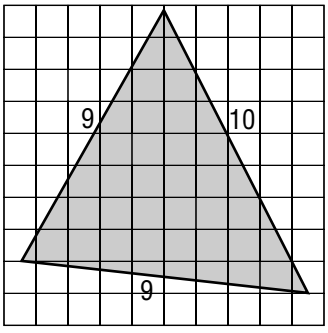
1.



Estimated area:

Computed area:

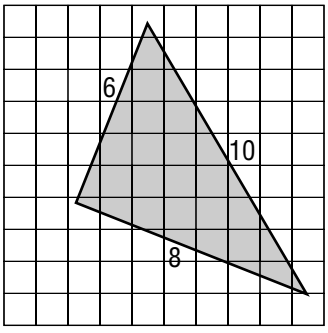
2.



Estimated area:

Computed area:

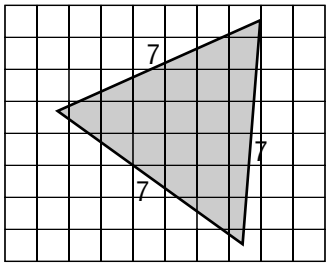
3.



Estimated area:

Computed area:

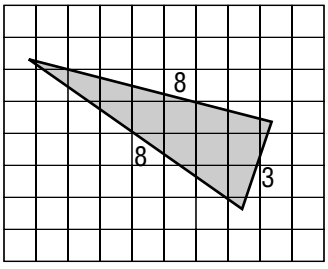
4.



Estimated area:

Computed area:

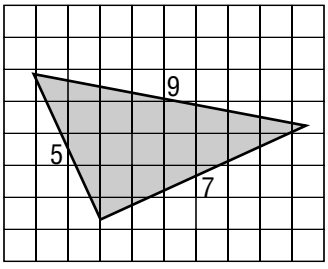
5.



Estimated area:

Computed area:

6.



Estimated area:

Computed area:

NAME _____ DATE _____ PERIOD _____

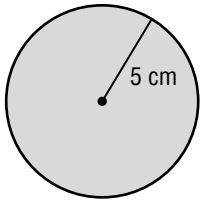
11-6 Study Guide and Intervention

Area of Circles

The area A of a circle equals the product of pi (π) and the square of its radius r.

$$A = \pi r^2$$

EXAMPLE 1 **Find the area of the circle.**



$A = \pi r^2$ Area of circle

$A = \pi \cdot 5^2$ Replace r with 5.

π $\times$ 5 x^2 ENTER **78.53981634**

The area of the circle is approximately 78.5 square centimeters.

EXAMPLE 2 **Find the area of a circle that has a diameter of 9.4 millimeters.**

$A = \pi r^2$ Area of a circle

$A = \pi \cdot 4.7^2$ Replace r with 9.4 ÷ 2 or 4.7.

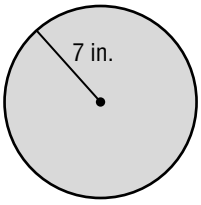
$A \approx 69.4$ Use a calculator.

The area of the circle is approximately 69.4 square millimeters.

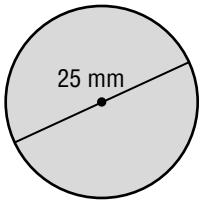
EXERCISES

Find the area of each circle. Round to the nearest tenth.

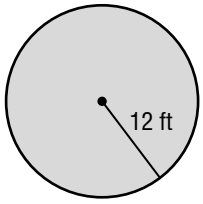
1.



2.



3.



4. radius = 2.6 cm

5. radius = 14.3 in.

6. diameter = $5\frac{1}{2}$ yd

7. diameter = $4\frac{3}{4}$ mi

8. diameter = 7.9 mm

9. radius = $2\frac{1}{5}$ ft

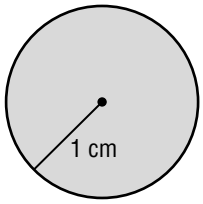
NAME _____ DATE _____ PERIOD _____

11-6 Practice: Skills

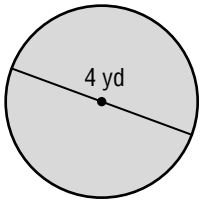
Area of Circles

Find the area of each circle. Round to the nearest tenth.

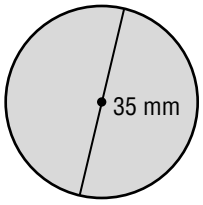
1.



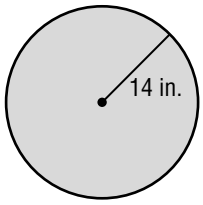
2.



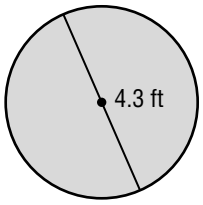
3.



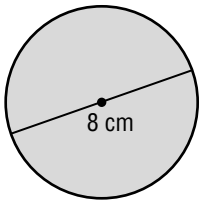
4.



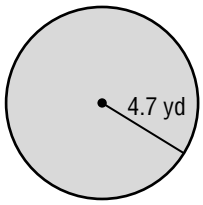
5.



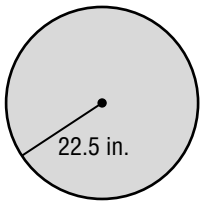
6.



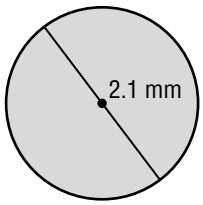
7.



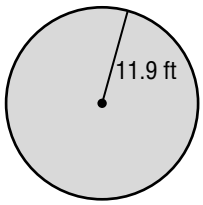
8.



9.



10.



11. radius = 5.7 mm

12. radius = 8.2 ft

13. diameter = $3\frac{1}{4}$ in.

14. diameter = 15.6 cm

15. radius = 1.1 in.

16. diameter = $12\frac{3}{4}$ yd

Lesson 11–6

NAME _____ DATE _____ PERIOD _____

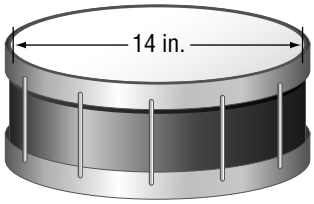
11-6 Practice: Word Problems

Area of Circles

1. POOLS Susan designed a circular pool with a diameter of 25 meters. What is the area of the bottom of the pool? Round to the nearest tenth.

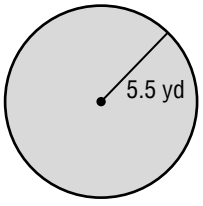
2. MONEY Find the area of the coin to the nearest tenth.

3. DRUMS What is the area of the drumhead on the drum shown below? Round to the nearest tenth.



4. PIZZA Estimate the area of the top of a round pizza that has a diameter of 16 inches. Round to the nearest tenth.

5. GARDENING Jane needs to buy mulch for the garden with the dimensions shown in the figure. For how much area does Jane need to buy mulch? Round to the nearest tenth.



6. UTILITIES What is the area of the top surface of a circular manhole cover that has a radius of 30 centimeters? Round to the nearest tenth.

NAME _____ DATE _____ PERIOD _____

11-6 Reading to Learn Mathematics

Area of Circles

Pre-Activity *Complete the Mini Lab at the top of page 493 in your textbook. Write your answers below.*

1. What is the measurement of the base and the height?

2. Substitute these values into the formula for the area of a parallelogram.

3. Replace C with the expression for the circumference of a circle, $2\pi r$. Simplify the equation and describe what it represents.

Reading the Lesson

4. The formula for the area of a circle uses the number π. How does this affect the value of the area of a circle found using the formula?

5. If you are given the length of the diameter of a circle, how can you find its area?

Helping You Remember

6. Think about the formulas you have learned that involve circles: $C = 2\pi r$ or $C = \pi d$ and $A = \pi r^2$. To help you remember the difference between the formulas for circumference and the formula for area, think about the differences in the units used for each measurement. What kinds of units are used for each? How can this help you remember the formula for the area of a circle?

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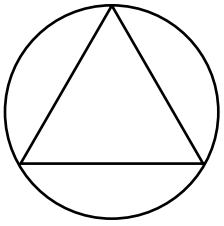
11-6 Enrichment

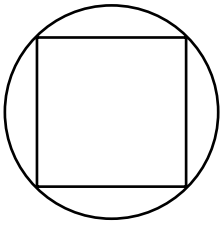
Seki Kowa

Japanese mathematician Seki Kowa (c. 1642–1708) is called The Arithmetical Sage because of his many contributions to the development of mathematics in Japan. Before Seki, mathematics in Japan was considered a form of art to be enjoyed by intellectuals in their leisure time. Seki demonstrated the practical uses of mathematics and introduced social reforms that made it possible for anyone, not just intellectuals, to study mathematics.



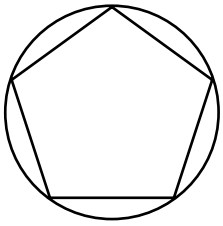
One of Seki's contributions to mathematics was his calculation of a value of π that was correct to eighteen decimal places.

$$\pi \approx 3.141592653589793238\ldots$$



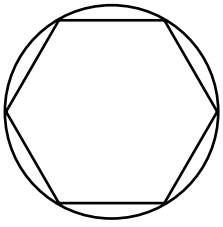
Seki had noticed the phenomenon that you see at the right: as the number of sides of a regular polygon increases, the polygon looks more and more like a circle. So, Seki calculated the following ratio for polygons of increasingly many sides.

$$\frac{\text{perimeter of regular polygon}}{\text{diameter of circle drawn around the polygon}}$$



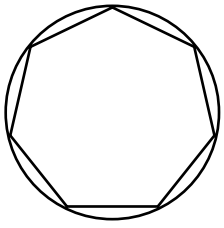
As the number of sides of the polygon gets larger, this ratio must get closer to the ratio of the circumference of the circle to the diameter of the circle. This ratio, of course, is π.

You are given information below about a regular polygon and the circle drawn around the polygon. Use a calculator to find Seki's ratio. (Give as many decimal places as there are in your calculator display.) What do you notice about your answers?



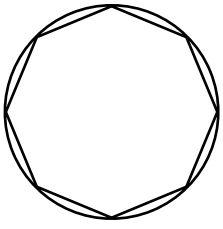
1. length of one side = 5
number of sides = 6
diameter of circle = 10

2. length of one side ≈ 4.5922
number of sides = 8
diameter of circle = 12



3. length of one side ≈ 3.7544
number of sides = 20
diameter of circle = 24

4. length of one side ≈ 37.5443
number of sides = 20
diameter of circle = 240



5. length of one side ≈ 1.6754
number of sides = 150
diameter of circle = 80

6. length of one side ≈ 2.6389
number of sides = 500
diameter of circle = 420

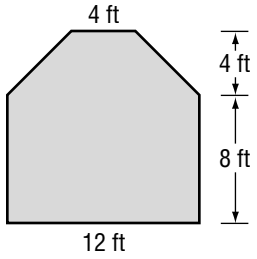
NAME _____ DATE _____ PERIOD _____

11-7 Study Guide and Intervention

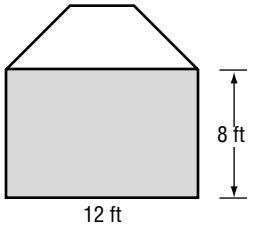
Area of Complex Figures

Complex figures are made of circles, rectangles, squares, and other two-dimensional figures. To find the area of a complex figure, separate it into figures whose areas you know how to find, and then add the areas.

EXAMPLE 1 **Find the area of the figure at the right in square feet.**



The figure can be separated into a rectangle and a trapezoid. Find the area of each.

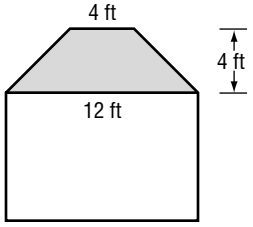


Area of Rectangle

$A = \ell w$ Area of a rectangle

$A = 12 \cdot 8$ Replace ℓ with 12 and w with 8.

$A = 96$ Multiply.



Area of Trapezoid

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(4)(4 + 12)$ Replace h with 4, b_1 with 4, and b_2 with 12.

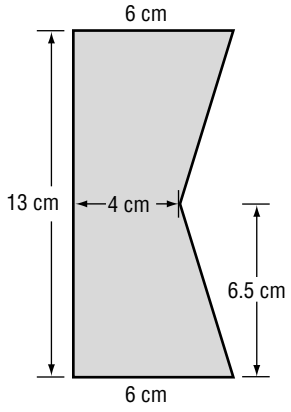
$A = 32$ Multiply.

The area of the figure is 96 + 32 or 128 square feet.

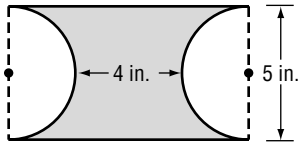
EXERCISES

Find the area of each figure. Round to the nearest tenth if necessary.

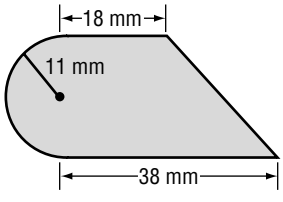
1.



2.



3.



Lesson 11–7

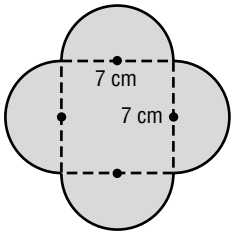
NAME _____ DATE _____ PERIOD _____

11-7 Practice: Skills

Area of Complex Figures

Find the area of each figure. Round to the nearest tenth if necessary.

1.



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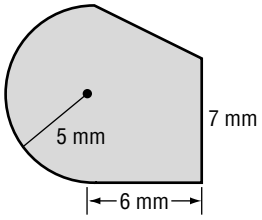

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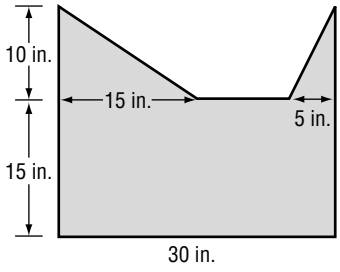

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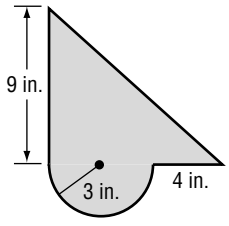

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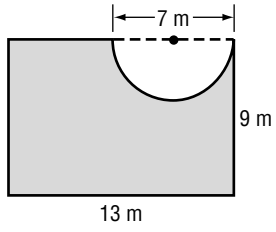

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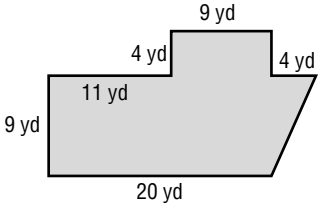

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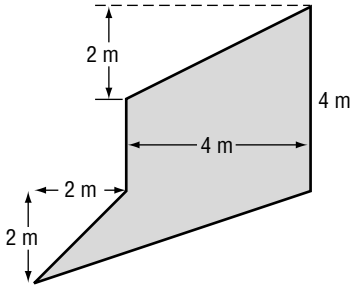

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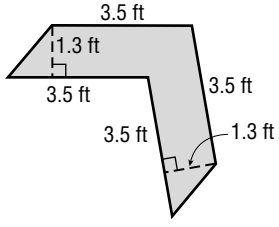
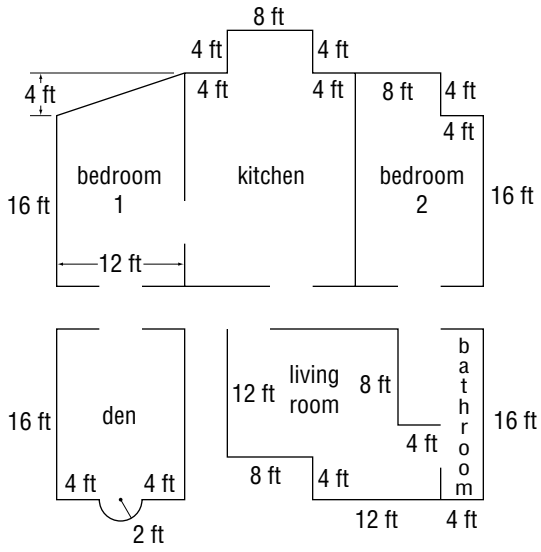


NAME _____ DATE _____ PERIOD _____

11-7 Practice: Word Problems

Area of Complex Figures

ARCHITECTURE For Exercises 1–6 use Jaco's preliminary design of his vacation house at the right. Round to the nearest tenth if necessary.



1. What type of figure is bedroom 1? Find the area of bedroom 1.	**2.** What is the area of the bedroom 2? What figures did you use to find the area?
3. What is the area of the bathroom? What are the dimensions of the figures you used to find this area?	**4.** What is the area of the living room? How many figures did you use to find this area?
5. What is the area of the den? What would the area of the den be if the semicircular window were removed and replaced with a flat window?	**6.** What is the area of the kitchen? If Jaco adds a rectangular cooking island in the middle of the kitchen with dimensions 6 feet by 4 feet, how many square feet of walking space will be left?

Lesson 11–7

NAME _____ DATE _____ PERIOD _____

11-7 Reading to Learn Mathematics

Area of Complex Figures

Pre-Activity

Read the introduction at the top of page 498 in your textbook. Write your answers below.

1. Describe the shape of the kitchen.

2. How could you determine the area of the kitchen?

3. How could you determine the total square footage of a house with rooms shaped like these?

Reading the Lesson

4. Look up the term *footage* in a dictionary. Write the meaning that matches the way the term is used in this lesson.

5. What do you think the term *square footage* means?

6. Which word of the compound *square footage* indicates area? Explain.

7. Look up the term *two-dimensional* in a dictionary.

8. Name two dimensions of each of the following figures.

a. rectangle **b.** parallelogram **c.** triangle

9. Refer to the figure in Example 2 on page 499. How do you know that the base and height of the triangle are each 4 inches long?

Helping You Remember

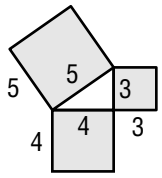
10. Look in a dictionary for the meanings of the word *complex* when used as an adjective. Write the meaning of the word as it is used in this lesson. Why can the figures in Examples 1 and 2 be considered complex figures?

NAME _____ DATE _____ PERIOD _____

11-7 Enrichment

Extending the Pythagorean Theorem

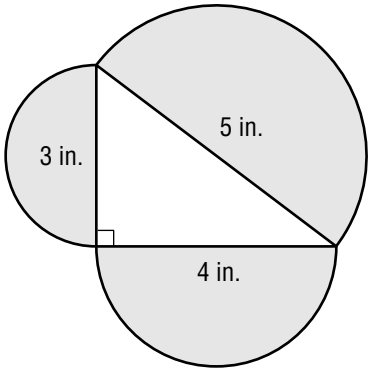
The Pythagorean Theorem says that the sum of the areas of the two smaller squares is equal to the area of the largest square. Show that the Pythagorean Theorem can be extended to include other shapes on the sides of a triangle. To do so, find the areas of the two smaller shapes. Then, check that their sum equals the area of the largest shape.



1. area of smallest shape:

area of middle shape:

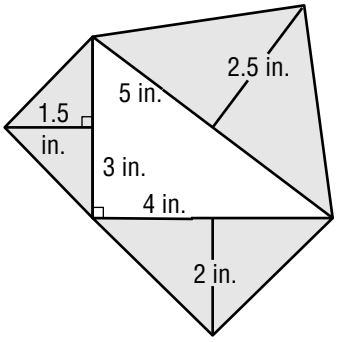
area of largest shape:



2. area of smallest shape:

area of middle shape:

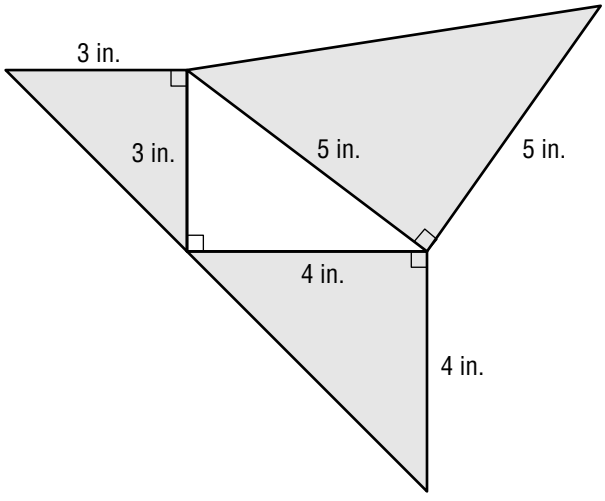
area of largest shape:



3. area of smallest shape:

area of middle shape:

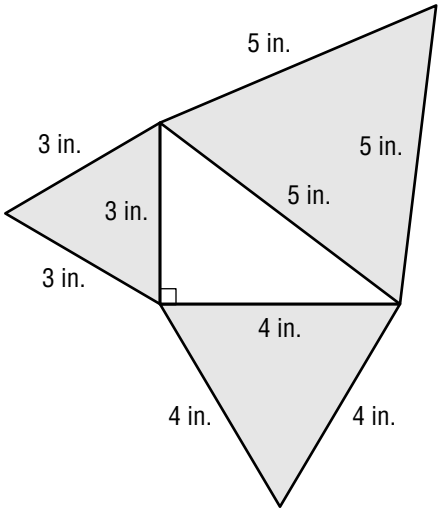
area of largest shape:



4. area of smallest shape:

area of middle shape:

area of largest shape:



Lesson 11–7

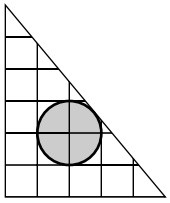
NAME _____ DATE _____ PERIOD _____

11-8 Study Guide and Intervention

Area Models and Probability

You can relate probability to the area of geometric shapes.

EXAMPLE 1 **A randomly-dropped counter falls somewhere in the squares. Find the probability that it falls on the shaded squares.**



$$\text{probability} = \frac{\text{number of ways to land in shaded squares}}{\text{number of ways to land on squares}}$$

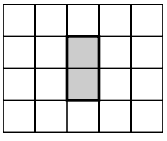
$$= \frac{\text{area of shaded squares}}{\text{area of all squares}}$$

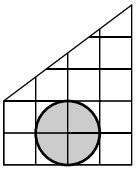
Area of Shaded Squares		Area of All Squares	
$A = \pi r^2$	Area of a circle	$A = \frac{1}{2}bh$	Area of a triangle
$A = \pi \cdot 1^2$	$r = 1$	$A = \frac{1}{2}(5)(6)$	$b = 5$ and $h = 6$
$A \approx 3.1$	Simplify.	$A = 15$	Simplify.

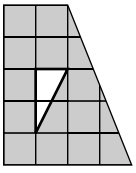
So, the probability of a counter falling in the shaded squares is about $\frac{3.1}{15}$ or about 20.7%.

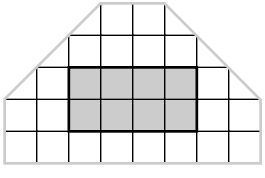
EXERCISES

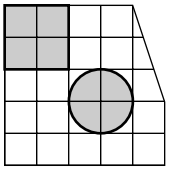
A randomly-dropped counter falls in the squares. Find the probability that it falls in the shaded squares. Write as a percent. Round to the nearest tenth if necessary.

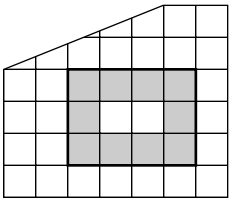
1. 

2. 

3. 

4. 

5. 

6. 

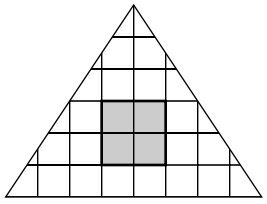
NAME _____ DATE _____ PERIOD _____

11-8 Practice: Skills

Area Models and Probability

A randomly-dropped counter falls in the squares. Find the probability that it falls in the shaded squares. Write as a percent. Round to the nearest tenth if necessary.

1.



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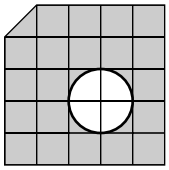

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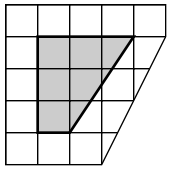

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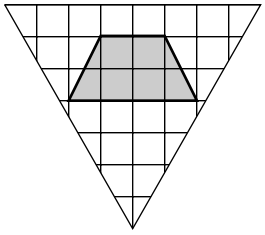

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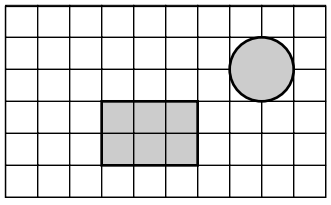

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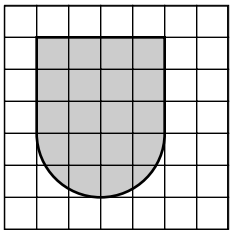

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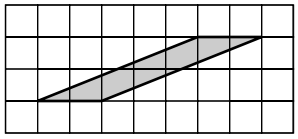

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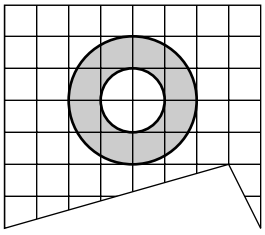


Lesson 11–8

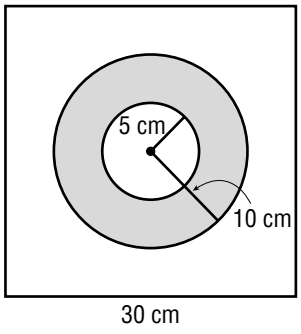
NAME _____ DATE _____ PERIOD _____

11-8 Practice: Word Problems

Area Models and Probability

GAMES Each figure represents a square dartboard. If it is equally likely that a thrown dart will land anywhere on the dartboard, find the probability that it lands in the shaded region. Round to the nearest tenth.

1.



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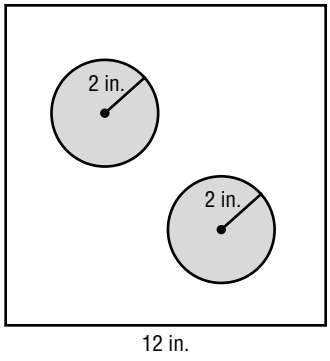

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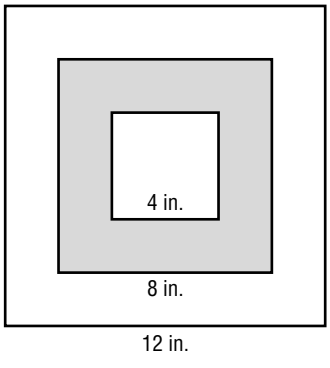

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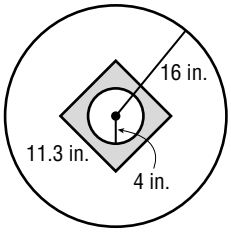


NAME _____ DATE _____ PERIOD _____

11-8 Reading to Learn Mathematics

Area Models and Probability

Pre-Activity *Complete the Mini Lab at the top of page 501 in your textbook. Write your answers below.*

1. Do certain products occur more often?

2. Make and complete the table below to find all the possible outcomes.

×	1	2	3	4	5	6
1	1	2	3			
2	2	4	6			
3						
4						
5						
6						

Reading the Lesson

3. How can you use the grid following the introduction in your textbook to determine that the probability of rolling two numbers whose product is 6 or 12 is $\frac{2}{9}$?

4. The formula for probability is $\frac{\text{desired area}}{\text{total area}}$. How does this lesson simplify the expression for probability?

Helping You Remember

5. Find the dimensions of a target for darts or for a bow and arrow. Draw a model that shows the measurements. Then show the probability of hitting the area that scores the most points per hit.

Lesson 11–8

NAME _____ DATE _____ PERIOD _____

11-8 Enrichment

Area Formulas for Regular Polygons

Recall that the sides of a regular polygon are all the same length. Here are some area formulas for four of the regular polygons. The variable s stands for the length of one side.

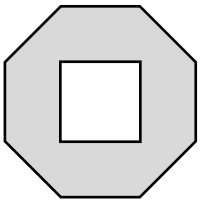
triangle	pentagon	hexagon	octagon
$A = \frac{s^2}{4}\sqrt{3}$	$A = \frac{s^2}{4}\sqrt{25 + 10\sqrt{5}}$	$A = \frac{3s^2}{2}\sqrt{3}$	$A = 2s^2(\sqrt{2} + 1)$

Find the area of each polygon with the side of given length. Use a calculator and round each answer to the nearest tenth.

	Length of a Side	Triangle	Pentagon	Hexagon	Octagon
1.	1 cm				
2.	2 cm				
3.	3 cm				
4.	4 cm				

Now use the table above to find the area of each shaded region below. Each small segment is 1 centimeter long.

5.



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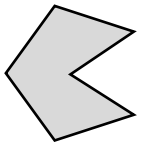

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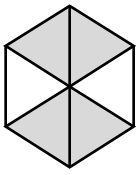

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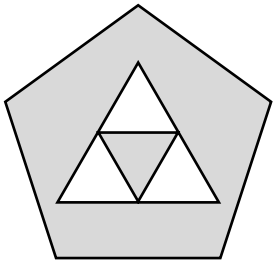

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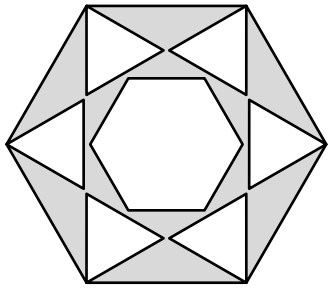

10.