

Chapter 3

Resource Masters



Mathematics

Applications and Concepts

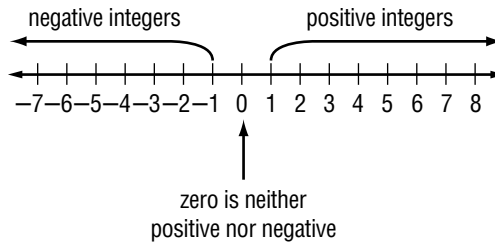
Course 2



New York, New York Columbus, Ohio Chicago, Illinois Peoria, Illinois Woodland Hills, California

3-1**Study Guide and Intervention****Integers and Absolute Value**

Integers less than zero are **negative integers**. Integers greater than zero are **positive integers**.



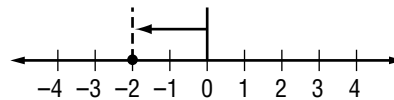
The **absolute value** of an integer is the distance the number is from zero on a number line. Two vertical bars are used to represent absolute value. The symbol for absolute value of 3 is $|3|$.

EXAMPLE 1 Write an integer that represents 160 feet below sea level.

Because it represents *below* sea level, the integer is -160 .

EXAMPLE 2 Evaluate $|-2|$.

On the number line, the graph of -2 is 2 units away from 0. So, $|-2| = 2$.

**EXERCISES**

Write an integer for each situation.

- | | |
|---------------------------------|-------------------|
| 1. 12°C above 0 | 2. a loss of \$24 |
| 3. a gain of 20 pounds | 4. falling 6 feet |

Evaluate each expression.

- | | |
|------------|-------------|
| 5. $ 12 $ | 6. $ -150 $ |
| 7. $ -8 $ | 8. $ 75 $ |
| 9. $ -19 $ | 10. $ 84 $ |

3-1**Practice: Skills*****Integers and Absolute Value*****Write an integer for each situation.**

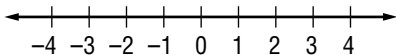
1. 15°C below 0
2. a profit of \$27
3. 2010 A.D.
4. average attendance is down 38 people
5. 376 feet above sea level
6. a withdrawal of \$200
7. 3 points lost
8. a bonus of \$150
9. a deposit of \$41
10. 240 B.C.
11. a wage increase of \$120
12. 60 feet below sea level

Evaluate each expression.

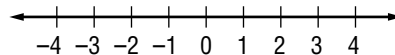
13. $|-1|$
14. $|9|$
15. $|23|$
16. $|-107|$
17. $|-45|$
18. $|19|$
19. $|0|$
20. $|6| - |-2|$
21. $|-8| + |4|$
22. $|-12| - |12|$

Graph each set of integers on a number line.

23. $\{0, 2, -3\}$



24. $\{-4, -1, 3\}$



3-1**Practice: Word Problems*****Integers and Absolute Value***

<p>1. DEATH VALLEY The lowest point in the United States is Death Valley in California. Its altitude is 282 feet below sea level. Write an integer to represent the altitude of Death Valley.</p>	<p>2. RAIN A meteorologist reported that in the month of April there were 3 inches more rainfall than normal. Write an integer to represent the amount of rainfall above normal in April.</p>
<p>3. ARCHIMEDES A famous mathematician and physicist named Archimedes was born in 287 B.C. Write an integer to express the year of his birth.</p>	<p>4. TEMPERATURE In our world's tropical rain forests, the average temperature of every month is 64 degrees above zero or higher. Write an integer to express this temperature.</p>
<p>5. STOCK MARKET A certain stock gained 5 points in one day and lost 4 points the next day. Write integers to represent the stock's gains and losses for the two days.</p>	<p>6. ALTITUDE An airplane pilot changed his altitude by 100 meters. Describe what this could mean.</p>

3-1**Reading to Learn Mathematics*****Integers and Absolute Value***

Pre-Activity *Read the introduction at the top of page 106 in your textbook. Write your answers below.*

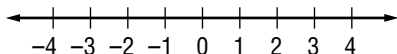
1. What does a value of -2 represent?
2. On which down did they lose the most yards?
3. How can you represent a gain of 9 yards?

Reading the Lesson

4. Express each of the following in words.

Symbols	Words
+7	
-7	
7	

5. On the following number line, draw a circle around the negative integers and label them *negative*. Draw a rectangle around the positive integers and label them *positive*.

**Helping You Remember**

6. Show a classmate how a number line can be used to show negative and positive integers. Explain the difference between some integers and the absolute values of those integers. Draw a number line to show what you mean.

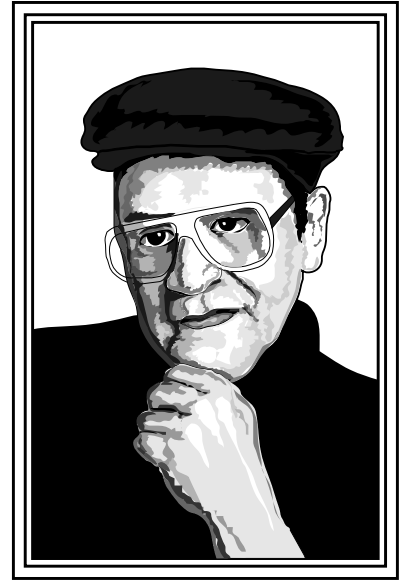
3-1**Enrichment****Jaime Escalante**

Jaime Escalante (1930–) was born in La Paz, Bolivia, and came to the United States in 1963. For ten years, he worked at odd jobs to support himself and his family while pursuing his dream—becoming certified to teach high school mathematics in California. As a mathematics teacher, he has become well known for his ability to inspire students to succeed in mathematics at levels they never thought possible. In 1988, the story of Mr. Escalante and a group of his students was the subject of the popular motion picture *Stand and Deliver*.

Mr. Escalante teaches concepts students must master if they are to succeed in high school and college mathematics. One of these is the concept of absolute value. For instance, a student should be able to solve an equation like $|y| = 6$ quickly using mental math. Here's how.

You know that $|6| = 6$ and $|-6| = 6$.

So, the equation $|y| = 6$ has *two* solutions: 6 and -6 .



Solve each equation. (Hint: One equation has no solution.)

1. $|a| = 8$

2. $|r| = 0$

3. $|j| = -3$

4. $|t| + 1 = 15$

5. $10 - |m| = 3$

6. $|c| - 4 = 16$

7. $5|z| = 60$

8. $12 \div |g| = 4$

9. $48 = 8|x|$

10. $2|d| + 3 = 5$

11. $4|p| - 9 = 59$

12. $7|z| + 12 = 12$

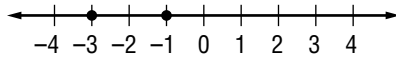
13. Suppose that the value of x can be selected from the set $\{-2, -1, 0, 1, 2\}$. Find all of the solutions of the equation $|x| = x$.

14. One of these statements is false. Which one is it? Explain.

- The absolute value of every integer is positive.
- There is at least one integer whose absolute value is zero.
- The absolute value of an integer is never negative.

3-2**Study Guide and Intervention****Comparing and Ordering Integers**

When two numbers are graphed on a number line, the number to the left is always less than ($<$) the number to the right. The number to the right is always greater than ($>$) the number to the left.

Model**Words**

-3 is less than -1 . -1 is greater than -3 .

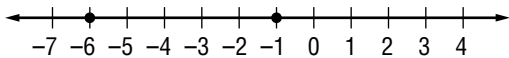
Symbols

$-3 < -1$ $-1 > -3$

The symbol points to the lesser number.

EXAMPLE 1 Replace the \bullet with $<$ or $>$ to make $-1 \bullet -6$ a true sentence.

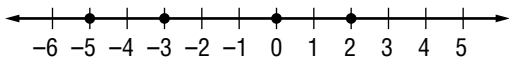
Graph each integer on a number line.



Since -1 is to the right of -6 , $-1 > -6$.

EXAMPLE 2 Order the integers $2, -3, 0, -5$ from least to greatest.

To order the integers, graph them on a number line.



Order the integers by reading from left to right: $-5, -3, 0, 2$.

EXERCISES

1. Replace the \bullet with $<$ or $>$ to make $-5 \bullet -10$ a true sentence.
2. Order $-1, 5, -3,$ and 2 from least to greatest.
3. Order $0, -4, -2,$ and 7 from greatest to least.
4. Order $-3, |-2|, 4, 0,$ and -5 from greatest to least.

3-2**Practice: Skills****Comparing and Ordering Integers**

Replace each ● with $<$ or $>$ to make a true sentence.

1. $-15 \bullet -16$

2. $-8 \bullet -7$

3. $0 \bullet -2$

4. $-2 \bullet -5$

5. $-25 \bullet 3$

6. $-14 \bullet |-20|$

7. $|-4| \bullet 3$

8. $|-6| \bullet |-7|$

9. $|-7| \bullet |2|$

10. $-8 \bullet |-9|$

Determine whether each sentence is *true* or *false*. If *false*, change one number to make the sentence true.

11. $-7 < 3$

12. $2 > 0$

13. $-20 < -22$

14. $12 < 15$

15. $3 > |-5|$

16. $|-2| < -3$

17. $|8| < |-10|$

18. $|-11| = 11$

19. $-4 < 4$

20. $|-9| < |-10|$

Order the integers from least to greatest.

21. 12, -6, 20, -47, -11

22. 9, -6, 0, -4, 17, -11

Order the integers from greatest to least.

23. -40, 65, -7, 24, -6, 15

24. $|-13|$, 0, 7, -8, -5, $|2|$

3-2**Practice: Word Problems****Comparing and Ordering Integers**

HISTORY OF WRITING For Exercises 1 and 2, use the table below. It shows important events in the history of writing.

Event	Aprox. Year
The <i>Iliad</i> and the <i>Odyssey</i> are composed by Homer.	700 BC
T'sai Lun invents paper.	105 AD
Date of oldest existing papyrus	2200 BC
Ovid wrote <i>Metamorphosis</i> .	5 AD
Torah is compiled.	450 BC
Metal type developed in Korea	1241 AD

EXTREME TEMPERATURES For Exercises 3–5, use the table below. It shows the extreme temperatures for four states. Temperatures are in degrees Fahrenheit.

Extreme Temperatures (°F)

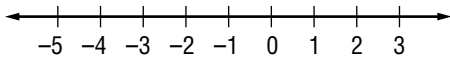
State	Highest	Lowest
Alabama	104	3
Nebraska	118	-47
Maine	101	-30
Florida	109	-2

1. Write each year as an integer.	2. Order the integers from Exercise 1 from least to greatest. Write a sentence describing the earliest and most recent events in the table.
3. Arrange the highest temperatures from greatest to least.	4. What is the median low temperature for these four states?
5. Nebraska's lowest temperature was -47°F , and Maine's lowest temperature was -30°F . Write a true statement using the two temperatures with the symbol $>$ or $<$.	6. MONEY Mr. Firewalks pays close attention to how much money is in his checking account. One week he deposited \$230, spent \$15 on a lunch, and loaned \$25 to a friend. Write each transaction as an integer, and list them from least to greatest.

3-2**Reading to Learn Mathematics****Comparing and Ordering Integers**

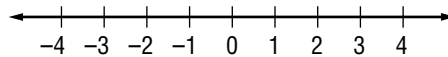
Pre-Activity *Read the introduction at the top of page 109 in your textbook. Write your answers below.*

1. What is the wind chill if there is a wind at 20 miles per hour and the temperature is 5° ?
2. Which is colder, a temperature of 15° with a 20 mile-per-hour wind or a temperature of 10° with a 10 mile-per-hour wind?
3. Graph both wind chills found in Exercise 2 on a number line.

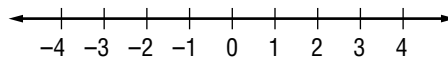
**Reading the Lesson**

For Exercises 4 and 5, express each of the following in words. Then graph the numbers on a number line.

4. $-1 < 0$



5. $3 > -2$



6. When two numbers are graphed on a number line, what can you tell about the number to the left? the number to the right?

Helping You Remember

7. Write a set of six numbers, some positive and some negative. Explain how you can use a number line to order the numbers from least to greatest.

3-2**Enrichment****Quantitative Comparisons**

An unusual type of problem is found on some standardized multiple-choice tests. This problem type is called the *quantitative comparison*.

In each quantitative comparison question, you are given two quantities, one in Column A and one in Column B. You are to compare the two quantities and shade one of four circles on an answer sheet.

- Shade circle A if the quantity in Column A is greater;
 Shade circle B if the quantity in Column B is greater;
 Shade circle C if the two quantities are equal;
 Shade circle D if the relationship cannot be determined from the information given.

Shade the correct oval to the left of each problem number.

	Column A	Column B
(A) (B) (C) (D) 1.	$0.006 + 2$	$0.002 + 6$
(A) (B) (C) (D) 2.	ten billion dollars	1,000 million dollars
(A) (B) (C) (D) 3.	20 inches	the perimeter of a square with an area of 25 square inches
(A) (B) (C) (D) 4.	half of one third	one fifth
(A) (B) (C) (D) 5.	the greatest possible product of two odd positive numbers less than 20	the greatest possible product of two even positive numbers less than 20
(A) (B) (C) (D) 6.	0.000000001	$-x$ is x if greater than 0
(A) (B) (C) (D) 7.	$ x $	$ x + 1 $
(A) (B) (C) (D) 8.	$ y $	$ -y $
(A) (B) (C) (D) 9.	$2 x $ if $x \neq 0$	$ x $ if $x \neq 0$
(A) (B) (C) (D) 10.	$-x$ if x is less than 0	$ x $ if x is less than 0

3-3

Study Guide and Intervention

The Coordinate Plane

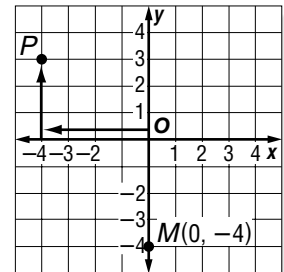
The **coordinate plane** is used to locate points. The horizontal number line is the **x-axis**. The vertical number line is the **y-axis**. Their intersection is the **origin**.

Points are located using **ordered pairs**. The first number in an ordered pair is the **x-coordinate**; the second number is the **y-coordinate**.

The coordinate plane is separated into four sections called **quadrants**.

EXAMPLE 1 Name the ordered pair for point P. Then identify the quadrant in which P lies.

- Start at the origin.
 - Move 4 units left along the x-axis.
 - Move 3 units up on the y-axis.
- The ordered pair for point P is $(-4, 3)$.
P is in the upper left quadrant or quadrant II.



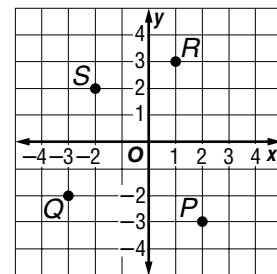
EXAMPLE 2 Graph and label the point $M(0, -4)$.

- Start at the origin.
- Move 0 units along the x-axis.
- Move 4 units down on the y-axis.
- Draw a dot and label it $M(0, -4)$.

EXERCISES

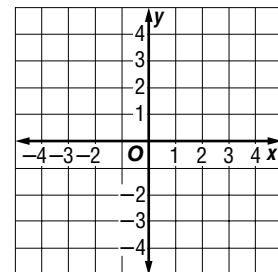
Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.

- | | |
|------|------|
| 1. P | 2. Q |
| 3. R | 4. S |



Graph and label each point on the coordinate plane.

- | | |
|---------------|----------------|
| 5. $A(-1, 1)$ | 6. $B(0, -3)$ |
| 7. $C(3, 2)$ | 8. $D(-3, -1)$ |
| 9. $E(1, -2)$ | 10. $F(1, 3)$ |

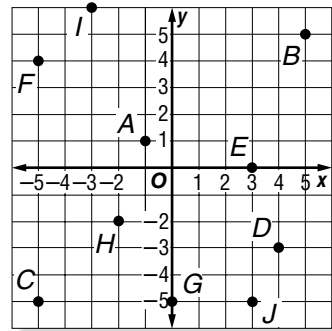


3-3

Practice: Skills

The Coordinate Plane

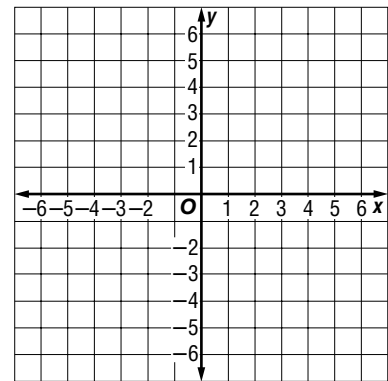
Name the ordered pair for each point graphed at the right. Then identify the quadrant in which each point lies.



- | | |
|-------------|--------------|
| 1. <i>A</i> | 2. <i>B</i> |
| 3. <i>C</i> | 4. <i>D</i> |
| 5. <i>E</i> | 6. <i>F</i> |
| 7. <i>G</i> | 8. <i>H</i> |
| 9. <i>I</i> | 10. <i>J</i> |

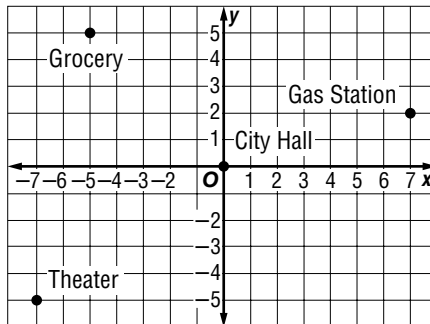
Graph and label each point on the coordinate plane.

- | | |
|-----------------|----------------|
| 11. $N(-1, 3)$ | 12. $V(2, -4)$ |
| 13. $C(4, 0)$ | 14. $P(-6, 2)$ |
| 15. $M(-5, 0)$ | 16. $K(-1, 5)$ |
| 17. $I(-3, -3)$ | 18. $A(5, -3)$ |
| 19. $D(0, -5)$ | |



Name the ordered pair for each point on the city map at the right.

20. City Hall
21. Theater
22. Gas Station
23. Grocery

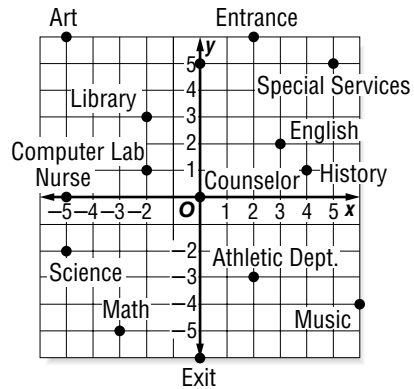


3-3

Practice: Word Problems

The Coordinate Plane

SCHOOL For Exercises 1–4, use the coordinate plane at the right. It shows a map of the rooms in a junior high school.



<p>1. Thalia is in the room located at $(-2, 1)$. What room is she in? Describe in words how to get from the origin to this point.</p>	<p>2. Thalia's next class is 8 units to the right and 5 units down on the map from where she is now. In what room is Thalia's next class? Find the ordered pair that represents the location of that room.</p>
<p>3. Tyrone is in the Art room, but his next class is in the History room. Give Tyrone directions on how to get to the History room.</p>	<p>4. On the map, which classrooms are located in the third quadrant? Describe the coordinates of all points in the third quadrant.</p>
<p>5. NEIGHBORHOOD Delsin made a map of his neighborhood in such a way that each intersection is a point on a coordinate plane. Right now, Delsin stands at point $(-4, -3)$. Give the ordered pair of where he will be if moves 5 units to the right and 7 units up on the map.</p>	<p>6. NEIGHBORHOOD Refer to Exercise 5. In which quadrant is Delsin when he is done walking? Describe this quadrant.</p>

3-3**Reading to Learn Mathematics****The Coordinate Plane**

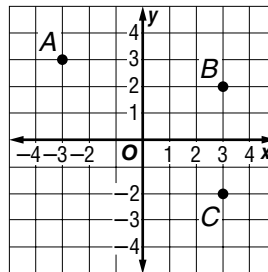
Pre-Activity Read the introduction at the top of page 112 in your textbook.
Write your answers below.

- Suppose Terrell starts at the corner of Russel and Main and walks 1 block north and 2 blocks east. Name the intersection of his location.
- Using the words *north*, *south*, *west*, and *east*, write directions to go from the corner of School and Highland to the corner of Main and Oak.

Reading the Lesson

- The word *coordinate* comes from two Latin words that mean “to arrange together.” How are coordinates used together to locate a point in a coordinate plane?

- Look at the coordinate plane at the right. Name the ordered pair for each point graphed.
- In the coordinate plane in Exercise 4, tell which quadrant each of the points is in.

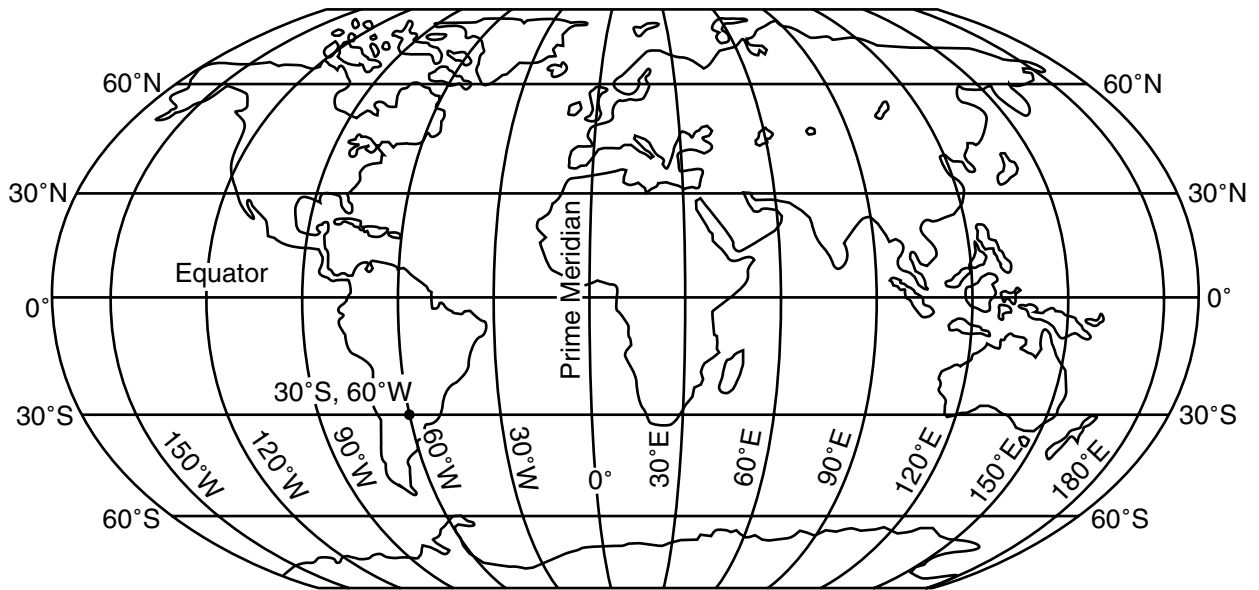
**Helping You Remember**

- Write a way to remember the names of the four quadrants of the coordinate plane.

3-3**Enrichment****Latitude and Longitude**

This world map shows some of the latitude and longitude lines. Latitude is measured in degrees north and south of the equator. Longitude is measured in degrees east and west of the prime meridian, a line passing through Greenwich, England. (Greenwich is a suburb of London.)

The latitude is usually given first. For example, the location of 30°S , 60°W is lower South America.



Name a place near each location. Use an atlas or other reference source to check your answers.

- | | | |
|---|--|---|
| 1. 30°N , 30°W | 2. 30°S , 30°E | 3. 60°N , 120°W |
| 4. 15°N , 150°W | 5. 30°S , 140°E | 6. 25°N , 100°W |
| 7. 40°N , 120°W | 8. 45°N , 90°W | 9. 40°N , 5°W |
| 10. 60°N , 45°W | 11. 35°N , 140°E | 12. 0° , 60°E |

3-4**Study Guide and Intervention****Adding Integers**

For integers with the same sign:

- the sum of two positive integers is positive.
- the sum of two negative integers is negative.

For integers with different signs, subtract their absolute values. The sum is:

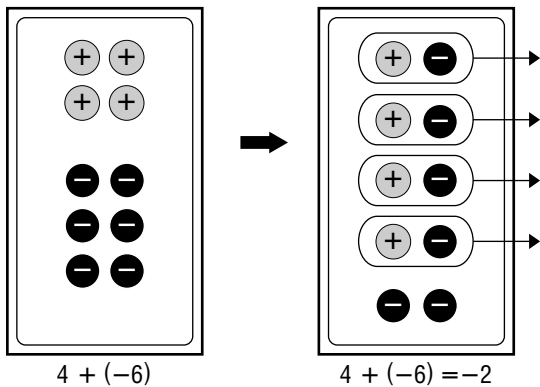
- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

To add integers, it is helpful to use counters or a number line.

EXAMPLE 1 Find $4 + (-6)$.

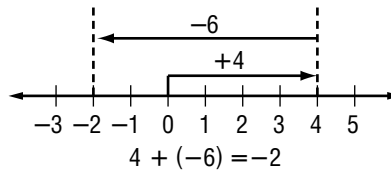
Method 1 Use counters.

Combine a set of 4 positive counters and a set of 6 negative counters on a mat.



Method 2 Use a number line.

- Start at 0.
- Move 4 units right.
- Then move 6 units left.

**EXERCISES**

Add.

1. $-5 + (-2)$

2. $8 + 1$

3. $-7 + 10$

4. $16 + (-11)$

5. $-22 + (-7)$

6. $-50 + 50$

7. $-10 + (-10)$

8. $100 + (-25)$

9. $-35 + -20$

Evaluate each expression if $a = 8$, $b = -8$, and $c = 4$.

10. $a + 15$

11. $b + (-9)$

12. $a + b$

13. $b + c$

14. $-10 + c$

15. $12 + b$

3-4**Practice: Skills*****Adding Integers***

Tell whether each sum is positive, negative, or zero without adding.

Add.

1. $5 + (-8)$

2. $-3 + 3$

3. $-3 + (-8)$

4. $-7 + (-7)$

5. $-8 + 10$

6. $-7 + 13$

7. $15 + (-10)$

8. $-11 + (-12)$

9. $25 + (-12)$

10. $-14 + (-13)$

11. $14 + (-27)$

12. $-28 + 16$

Evaluate each expression if $a = -8$, $b = 12$, and $c = -4$.

13. $5 + a$

14. $b + (-9)$

15. $c + (-5)$

16. $a + b$

17. $a + 0$

18. $b + c$

19. $-12 + b$

20. $a + (-7)$

21. $21 + c$

22. $a + c$

3-4**Practice: Word Problems*****Adding Integers***

Write an addition expression to describe each situation. Then find each sum.

<p>1. FOOTBALL A team gains 20 yards. Then they lose 7 yards.</p>	<p>2. MONEY Roger owes his mom \$5. He borrows another \$6 from her.</p>
<p>3. GOLF Juanita's score was 5 over par on the first 9 holes. Her score was 4 under par on the second 9 holes.</p>	<p>4. HOT AIR BALLOON A balloon rises 340 feet into the air. Then it descends 130 feet.</p>
<p>5. CYCLING A cyclist travels downhill for 125 feet. Then she travels up a hill 50 feet.</p>	<p>6. AIRPLANE A plane descends 1,200 feet. Then it descends another 500 feet.</p>

3-4**Reading to Learn Mathematics*****Adding Integers***

Pre-Activity *Read the introduction at the top of page 120 in your textbook. Write your answers below.*

1. What is the charge at the top of a cloud where there are more protons than electrons?
2. What is the charge at the bottom of a cloud where there are more electrons than protons?

Reading the Lesson

For Exercises 3 and 4, tell how you would solve each of the following on a number line. Then solve.

3. $-7 + (-9)$
4. $-7 + 9$
5. When you use counters to add integers, what property are you applying when you remove zero pairs?
6. How many units away from 0 is the number 17? How many units away from 0 is the number -17 ? What are 17 and -17 called?

Helping You Remember

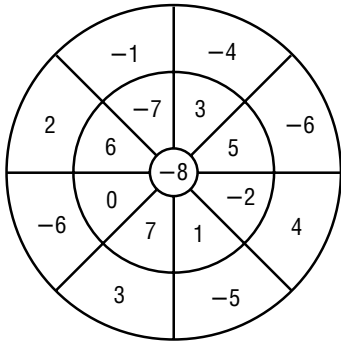
7. Work with a partner. Tell your partner how to use absolute values to add integers with different signs when the positive integer has the greater absolute value. Then have your partner explain to you how to use absolute values to add integers with different signs when the negative integer has the greater absolute value.

3-4 Enrichment

Dartboard Puzzles

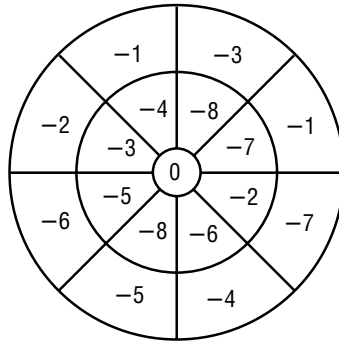
Three darts are thrown. Each dart must land on a different space in order to count. Find the highest and the lowest possible scores.

1.



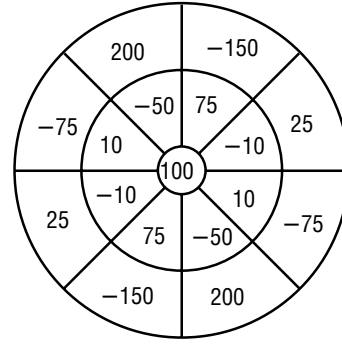
highest score:
lowest score:

2.



highest score:
lowest score:

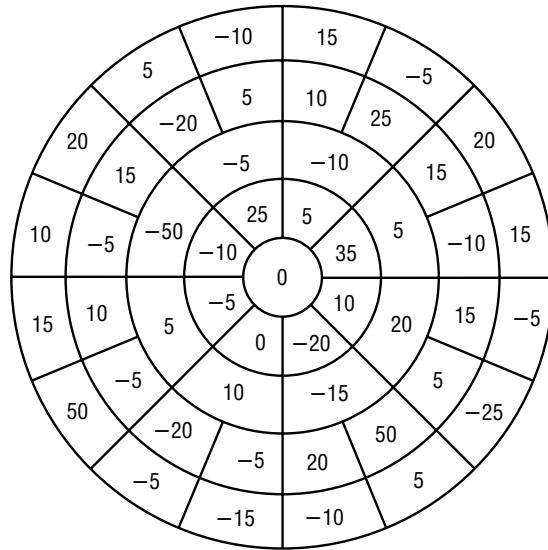
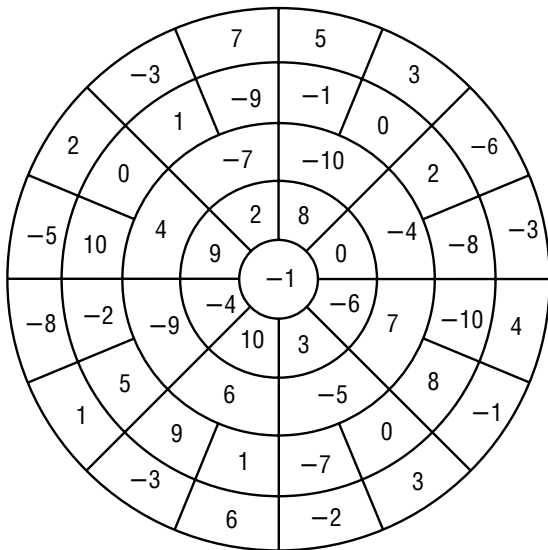
3.



highest score:
lowest score:

In these problems, five darts are thrown. Each dart must land on a different space in order to count. Solve each puzzle.

4. Find three ways to make the score -5 . 5. Find three ways to make the score 0.



3-5**Study Guide and Intervention*****Subtracting Integers***

To subtract an integer, add its opposite.

EXAMPLE 1 Find $6 - 9$.

$$\begin{aligned} 6 - 9 &= 6 + (-9) \\ &= -3 \end{aligned}$$

To subtract 9, add -9 .
Simplify.

EXAMPLE 2 Find $-10 - (-12)$.

$$\begin{aligned} -10 - (-12) &= -10 + 12 \\ &= 2 \end{aligned}$$

To subtract -12 , add 12.
Simplify.

EXAMPLE 3 Evaluate $a - b$ if $a = -3$ and $b = 7$.

$$\begin{aligned} a - b &= -3 - 7 \\ &= -3 + (-7) \\ &= -10 \end{aligned}$$

Replace a with -3 and b with 7.
To subtract 7, add -7 .
Simplify.

EXERCISES**Subtract.**

1. $7 - 9$

2. $20 - (-6)$

3. $-10 - 4$

4. $0 - 12$

5. $-7 - 8$

6. $13 - 18$

7. $-20 - (-5)$

8. $-8 - (-6)$

9. $25 - (-14)$

10. $-75 - 50$

11. $15 - 65$

12. $19 - (-10)$

Evaluate each expression if $m = -2$, $n = 10$, and $p = 5$.

13. $m - 6$

14. $9 - n$

15. $p - (-8)$

16. $p - m$

17. $m - n$

18. $-25 - p$

3-5**Practice: Skills*****Subtracting Integers*****Subtract.**

1. $5 - 2$

2. $6 - (-7)$

3. $-3 - 2$

4. $8 - 13$

5. $-7 - (-7)$

6. $6 - 12$

7. $15 - (-7)$

8. $-15 - 6$

9. $-3 - 8$

10. $-10 - 12$

11. $13 - (-12)$

12. $14 - (-22)$

13. $10 - (-20)$

14. $-16 - 14$

15. $-25 - 25$

16. $6 - (-31)$

17. $-18 - (-40)$

18. $15 - (-61)$

Evaluate each expression if $r = -4$, $s = 10$, and $t = -7$.

19. $r - 7$

20. $t - s$

21. $s - (-8)$

22. $t - r$

23. $s - t$

24. $r - s$

3-5**Practice: Word Problems*****Subtracting Integers*****Subtract.**

<p>1. FOOTBALL A team gained 5 yards on their first play of the game. Then they lost 6 yards. Find the total change in yardage.</p>	<p>2. CHECKING Your checking account is overdrawn by \$50. You write a check for \$20. What is the balance in your account?</p>
<p>3. TEMPERATURE The average temperature in Calgary, Canada, is 22°C in July and -11°C in January. Find the range of the highest and lowest temperatures in Calgary.</p>	<p>4. ROLLER COASTER A roller coaster begins at 90 feet above ground level. Then it descends 105 feet. Find the height of the coaster after the first descent.</p>
<p>5. SAVINGS Sonia has \$235 in her savings account. She withdraws \$45. What is left in her savings account?</p>	<p>6. BEACH Wai and Kuri were digging in the sand at the beach. Wai dug a hole that was 15 inches below the surface, and Kuri dug a hole that was 9 inches below the surface. Find the difference in the depths of their holes.</p>

3-5**Reading to Learn Mathematics*****Subtracting Integers***

Pre-Activity *Complete the Mini Lab at the top of page 128 in your textbook.
Write your answers below.*

1. Write a related addition sentence for each subtraction sentence.

Use a number line to find each difference. Write an equivalent addition sentence for each.

2. $1 - 5$

3. $-2 - 1$

4. $-3 - 4$

5. $0 - 5$

6. **Compare and contrast** subtraction sentences with their related addition sentences.

Reading the Lesson

**Tell how you would solve each of the following on a number line.
Then solve.**

7. $-8 - (-6)$

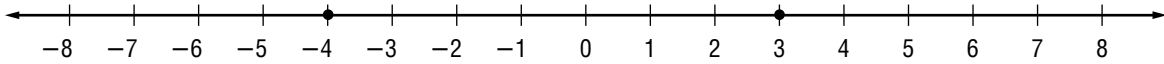
8. $6 - 8$

Helping You Remember

9. Write the rule that tells how to subtract integers. Then give an example.

3-5**Enrichment****Distance on the Number Line**

To find the distance between two points on a number line, subtract their coordinates. Then, take the absolute value of the difference.



$$-4 - 3 = -7$$

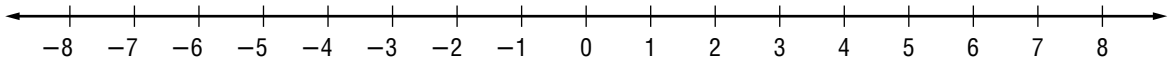
$$|-7| = 7$$

You can also find the distance by finding the absolute value of the difference of the coordinates.

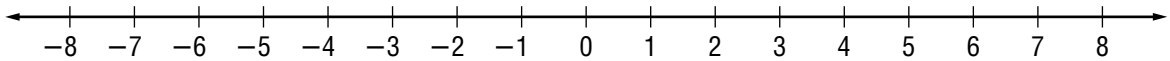
$$|-4 - 3| = 7$$

Graph each pair of points. Then write an expression using absolute value to find the distance between the points.

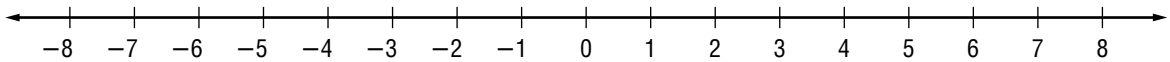
1. *A* at -5 and *B* at 2



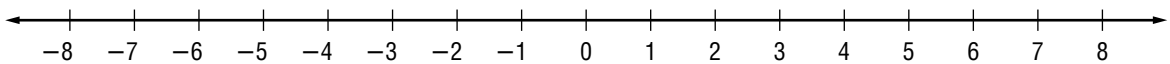
2. *C* at -7 and *D* at -1



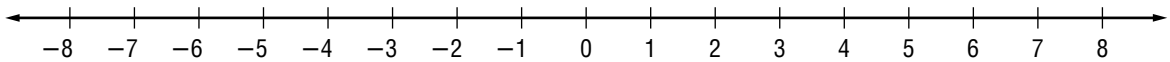
3. *E* at -5 and *F* at 5



4. *W* at 0 and *X* at 6



5. *Y* at -4 and *Z* at 0



3-6**Study Guide and Intervention*****Multiplying Integers***

The product of two integers with **different** signs is **negative**.

The product of two integers with the **same** sign is **positive**.

EXAMPLE 1 Multiply $5(-2)$.

$$5(-2) = -10 \quad \text{The integers have different signs. The product is negative.}$$

EXAMPLE 2 Multiply $-3(7)$.

$$-3(7) = -21 \quad \text{The integers have different signs. The product is negative.}$$

EXAMPLE 3 Multiply $-6(-9)$.

$$-6(-9) = 54 \quad \text{The integers have the same sign. The product is positive.}$$

EXAMPLE 4 Multiply $(-7)^2$.

$$\begin{aligned} (-7)^2 &= (-7)(-7) && \text{There are 2 factors of } -7. \\ &= 49 && \text{The product is positive.} \end{aligned}$$

EXAMPLE 5 Simplify $-2(6c)$.

$$\begin{aligned} -2(6c) &= (-2 \cdot 6)c && \text{Associative Property of Multiplication.} \\ &= -12c && \text{Simplify.} \end{aligned}$$

EXAMPLE 6 Simplify $2(5x)$.

$$\begin{aligned} 2(5x) &= (2 \cdot 5)x && \text{Associative Property of Multiplication.} \\ &= 10x && \text{Simplify.} \end{aligned}$$

EXERCISES**Multiply.**

- | | | |
|------------|---------------|-------------|
| 1. $-5(8)$ | 2. $-3(-7)$ | 3. $10(-8)$ |
| 4. $-8(3)$ | 5. $-12(-12)$ | 6. $(-8)^2$ |

ALGEBRA Simplify each expression.

- | | | |
|-------------|---------------|--------------|
| 7. $-5(7a)$ | 8. $3(-2x)$ | 9. $4(6f)$ |
| 10. $7(6b)$ | 11. $-6(-3y)$ | 12. $7(-8g)$ |

ALGEBRA Evaluate each expression if $a = -3$, $b = -4$, and $c = 5$.

- | | | |
|------------|-------------|-----------|
| 13. $-2a$ | 14. $9b$ | 15. ab |
| 16. $-3ac$ | 17. $-2c^2$ | 18. abc |

3-6**Practice: Skills*****Multiplying Integers*****Multiply.**

1. $-4(6)$

2. $-2(-8)$

3. $12(-4)$

4. $-6(5)$

5. $-10(-9)$

6. $-(5)^2$

7. $(-5)^2$

8. $-30(5)$

9. $20(-6)$

10. $-14(-6)$

11. $(-13)^2$

12. $-7(15)$

ALGEBRA Simplify each expression.

13. $-3(4y)$

14. $7(-3x)$

15. $7(5g)$

16. $7(7w)$

17. $3(-3y)$

18. $-2(-10h)$

ALGEBRA Evaluate each expression if $g = -5$, $h = -3$, and $k = 4$.

19. $-3g$

20. $5h$

21. $7gk$

22. $-2gh$

23. $-10h$

24. $-2h^2$

3-6**Practice: Word Problems*****Multiplying Integers*****Multiply.**

<p>1. TEMPERATURE Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature drop in 8 hours?</p>	<p>2. DIVING A deep-sea diver descends below the surface of the water at a rate of 60 feet each minute. What is the depth of the diver after 10 minutes?</p>
<p>3. STOCK A computer stock lost 2 points each hour for 6 hours. Find the total points the stock fell.</p>	<p>4. DROUGHT A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How far will it have dropped in 4 weeks?</p>
<p>5. MONEY Mrs. Rockwell lost money on an investment at a rate of \$4 per day. How much did she lose after two weeks?</p>	<p>6. TENNIS BALLS Josh purchased 8 cans of tennis balls. The cans came with 3 balls in each can. How many balls did Josh purchase?</p>

3-6**Reading to Learn Mathematics*****Multiplying Integers***

Pre-Activity Complete the Mini Lab at the top of page 134 in your textbook.
Write your answers below.

1. Write a multiplication sentence that describes the model.

Find each product using counters.

2. $3(-2)$

3. $4(-3)$

4. $1(-7)$

5. $5(-2)$

6. Write a rule for finding the sign of the product of a positive and negative integer.

Reading the Lesson

7. Give an example that shows how multiplication is the same as repeated addition. In your example, tell what the addend is.
8. How does the sentence $4(-2) = -2(4)$ illustrate the Commutative Property of Multiplication?
9. Complete each of the following sentences with the word *positive* or *negative*.
 - a. The product of two integers with different signs is _____.
 - b. The product of two integers with the same sign is _____.

Helping You Remember

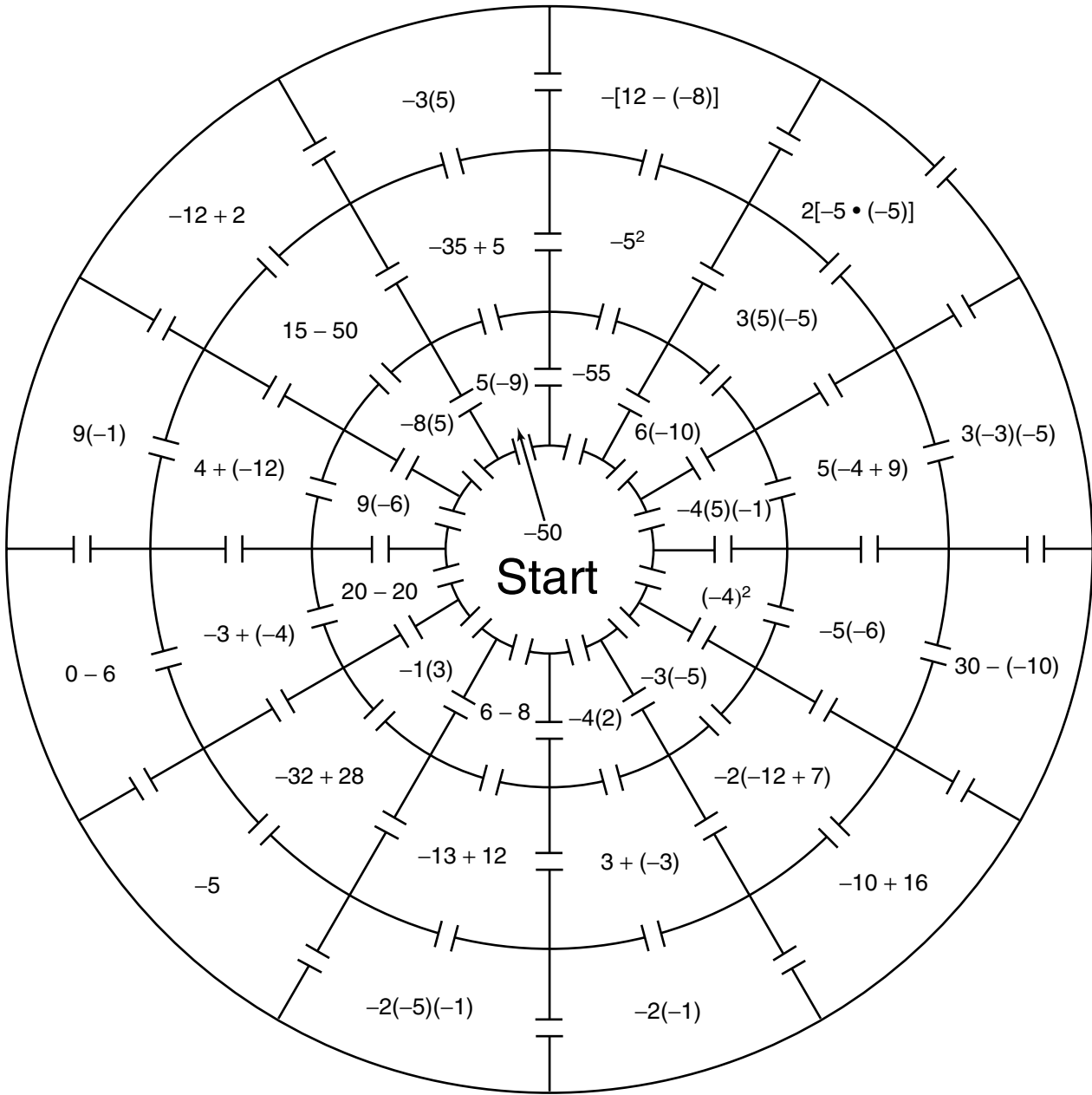
10. You know the rule for determining the sign of the product of two integers when the signs are alike or different. Consider the product of three integers. With a partner summarize the signs of the products of 3 integers when three, two, one or none of the integers are positive.

3-6

Enrichment

Integer Maze

Find your way through the maze by moving to the expression in an adjacent section with the next highest value.



3-7**Study Guide and Intervention*****Dividing Integers***

The quotient of two integers with different signs is negative.

The quotient of two integers with the same sign is positive.

EXAMPLE 1 Divide $30 \div (-5)$.

$30 \div (-5)$ The integers have different signs.

$30 \div (-5) = -6$ The quotient is negative.

EXAMPLE 2 Divide $-100 \div (-5)$.

$-100 \div (-5)$ The integers have the same sign.

$-100 \div (-5) = 20$ The quotient is positive.

EXERCISES**Divide.**

1. $-12 \div 4$

2. $-14 \div (-7)$

3. $\frac{18}{-2}$

4. $-6 \div (-3)$

5. $-10 \div 10$

6. $\frac{-80}{-20}$

7. $350 \div (-25)$

8. $-420 \div (-3)$

9. $\frac{540}{45}$

10. $\frac{-256}{16}$

ALGEBRA Evaluate each expression if $d = -24$, $e = -4$, and $f = 8$.

11. $12 \div e$

12. $40 \div f$

13. $d \div 6$

14. $d \div e$

15. $f \div e$

16. $e^2 \div f$

17. $\frac{-d}{e}$

18. $ef \div 2$

19. $\frac{f^2}{e^2}$

20. $\frac{de}{f}$

3-7**Practice: Skills*****Dividing Integers*****Divide.**

1. $-15 \div 3$

2. $-24 \div (-8)$

3. $22 \div (-2)$

4. $-49 \div (-7)$

5. $-8 \div (-8)$

6. $\frac{36}{-4}$

7. $225 \div (-15)$

8. $\frac{0}{-9}$

9. $-38 \div 2$

10. $\frac{64}{4}$

11. $-500 \div (-50)$

12. $-189 \div (-21)$

ALGEBRA Evaluate each expression if $m = -32$, $n = 2$, and $p = -8$.

13. $m \div n$

14. $p \div 4$

15. $p^2 \div m$

16. $m \div p$

17. $\frac{-p}{n}$

18. $p \div n^2$

19. $\frac{p^2}{n^2}$

20. $\frac{18 - n}{p}$

21. $m \div (np)$

22. $\frac{m}{p} + n$

3-7**Practice: Word Problems*****Dividing Integers*****Divide.**

<p>1. STOCK MARKET During a 5-day workweek, the stock market decreased by 65 points. Find the average daily change in the market for the week.</p>	<p>2. MOTION Mr. Diaz decreased the speed of his car by 30 miles per hour over a period of 10 seconds. Find the average change in speed each second.</p>
<p>3. WEATHER Over the past seven days, Mrs. Cho found that the temperature outside had dropped a total of 35 degrees. Find the average drop in temperature each day.</p>	<p>4. BASKETBALL The basketball team lost their last 6 games. They lost by a total of 48 points. Find the average number of points by which each game was lost.</p>
<p>5. POPULATION The enrollment at Davis Middle School dropped by 60 students over a 5-year period. What is the average yearly drop in enrollment?</p>	<p>6. SUBMARINE A submarine descends at a rate of 60 feet each minute. How long will it take it to descend to a depth of 660 feet below the surface?</p>

3-7**Reading to Learn Mathematics*****Dividing Integers***

Pre-Activity Complete the Mini Lab at the top of page 138 in your textbook.
Write your answers below.

Find each quotient using counters.

1. $-6 \div 2$

2. $-12 \div 3$

Reading the Lesson

Write two division sentences related to each of the following multiplication sentences.

3. $-6(-3) = 18$

4. $-21(-2) = 42$

5. $-6(3) = -18$

6. $2(-21) = -42$

7. Complete each of the following sentences with the word *positive* or *negative*.

a. The quotient of two integers with different signs is _____.

b. The quotient of two integers with the same sign is _____.

8. In the division sentence $-72 \div 8 = -9$, identify the dividend, the divisor, and the quotient.

Helping You Remember

9. Describe how the operations of multiplication and division are opposite of each other. Are these operations opposite in all cases? What is the one integer that cannot be a divisor?

3-7**Enrichment****Division by Zero?**

Some interesting things happen when you try to divide by zero. For example, look at these two equations.

$$\frac{5}{0} = x \qquad \frac{0}{0} = y$$

If you can write the equations above, you can also write the two equations below.

$$0 \cdot x = 5 \qquad 0 \cdot y = 0$$

However, there is no number that will make the left equation true. This equation has no solution. For the right equation, *every* number will make it true. The solutions for this equation are “all numbers.”

Because division by zero leads to impossible situations, it is not a “legal” step in solving a problem. People say that division by zero is undefined, or not possible, or simply not allowed.

Describe the solution set for each equation.

1. $4x = 0$

2. $x \cdot 0 = 0$

3. $x \cdot 0 = x$

4. $\frac{0}{x} = 0$

5. $\frac{0}{x} = x$

6. $\frac{0}{x} = 5$

What values for x must be excluded to prevent division by 0?

7. $\frac{1}{x^2}$

8. $\frac{1}{x-1}$

9. $\frac{1}{x+1}$

10. $\frac{0}{2x}$

11. $\frac{1}{2x-2}$

12. $\frac{1}{3x+6}$

Explain what is wrong with this “proof.”

13. Step 1 $0 \cdot 1 = 0$ and $0 \cdot (-1) = 0$

Step 2 Therefore, $\frac{0}{0} = 1$ and $\frac{0}{0} = -1$.

Step 3 Therefore, $1 = -1$.