Chapter 9
Resource Masters

GLENCOE MATHEMATICS

Mathematics

Applications and Concepts

Course 2

Glencoe

New York, New York   Columbus, Ohio   Chicago, Illinois   Peoria, Illinois   Woodland Hills, California

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9-1

Study Guide and Intervention

Simple Events

The **probability** of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes. Outcomes occur at **random** if each outcome occurs by chance.

Two events that are the only ones that can possibly happen are **complementary events**. The sum of the probabilities of complementary events is 1.

EXAMPLE 1 **What is the probability of rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?**

$P(\text{multiple of } 3) = \dfrac{\text{multiples of 3 possible}}{\text{total numbers possible}}$

$= \dfrac{2}{6}$ Two numbers are multiples of 3: 3 and 6.

$= \dfrac{1}{3}$ Simplify.

The probability of rolling a multiple of 3 is $\frac{1}{3}$ or about 33.3%.

EXAMPLE 2 **What is the probability of *not* rolling a multiple of 3 on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?**

$P(A) + P(\text{not } A) = 1$

$\frac{1}{3} + P(\text{not } A) = 1$ Substitute $\frac{1}{3}$ for $P(A)$.

$-\frac{1}{3} \qquad\qquad -\frac{1}{3}$ Subtract $\frac{1}{3}$ from each side

$P(\text{not } A) = \frac{2}{3}$ Simplify.

The probability of *not* rolling a multiple of 3 is $\frac{2}{3}$ or about 66.7%.

EXERCISES

A set of 30 cards is numbered 1, 2, 3, ..., 30. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

1. $P(12)$

2. $P(2 \text{ or } 3)$

3. $P(\text{odd number})$

4. $P(\text{a multiple of } 5)$

5. $P(\text{not a multiple of } 5)$

6. $P(\text{less than or equal to } 10)$

Lesson 9–1

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9-1 Practice: Skills

Simple Events

A set of 12 cards is numbered 1, 2, 3, ...12. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

1. $P(5)$

2. P(6 or 8)

3. P(a multiple of 3)

4. P(an even number)

5. P(a multiple of 4)

6. P(less than or equal to 8)

7. P(a factor of 12)

8. P(not a multiple of 4)

9. P(1, 3, or 11)

10. P(a multiple a 5)

The students at Job's high school were surveyed to determine their favorite foods. The results are shown in the table at the right. Suppose students were randomly selected and asked what their favorite food is. Find the probability of each event. Write as a fraction in simplest form.

Favorite Food	Responses
pizza	19
steak	8
chow mein	5
seafood	4
spaghetti	3
cereal	1

11. P(steak)

12. P(spaghetti)

13. P(cereal or seafood)

14. P(not chow mein)

15. P(pizza)

16. P(cereal or steak)

17. P(not steak)

18. P(not cereal or seafood)

19. P(chicken)

20. P(chow mein or spaghetti)

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9-1 Practice: Word Problems

Simple Events

COINS Susan opened her piggy bank and counted the number of each coin. The table at the right shows the results. For Exercises 1–3, assume that the coins are put in a bag and one is chosen at random.

Coin	Number
quarters	15
dimes	21
nickels	22
pennies	32

1. What is the probability that a quarter is chosen?

2. What is the probability that a nickel or a dime is chosen?

3. What is the probability that the chosen coin is worth more than 5 cents?

4. NUMBER CUBES Juan has two number cubes, each with faces numbered 1, 2, …6. What is the probability that he can roll the cubes so that the sum of the faces showing equals 11?

5. SKATEBOARDS Carlotta bought a new skateboard for which the probability of having a defective wheel is 0.015. What is the probability of not having a defective wheel?

6. CALCULATORS Jake's teacher had 6 calculators for 28 students to use. If the first students to use the calculators are chosen at random, what is the probability that Jake will get one?

7. VEHICLES The rental car company had 14 sedans and 8 minivans available to rent. If the next customer picks a vehicle at random, what is the probability that a minivan is chosen?

8. MUSIC Tina has 16 pop CDs, 6 classical, and 2 rock. Tina chooses a CD at random. What is the probability she does not choose a classical CD?

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9-1 Reading to Learn Mathematics

Simple Events

Pre-Activity ***Read the introduction at the top of page 370 in your textbook. Write your answers below.***

1. What fraction of the taffy is vanilla? Write in simplest form.

2. Suppose you take one piece of taffy from the box without looking. Are your chances of picking vanilla the same as picking root beer? Explain.

Reading the Lesson

Use the information from the introduction to answer Exercises 3–5.

3. How do you read P(cherry)?

4. $P(\text{cherry}) = \frac{6}{48}$; where does the 6 come from? Where does the 48 come from?

5. Probability can be written as a fraction, a decimal, or a percent. Write P(cherry) as a decimal.

6. If there is a 25% chance that something will happen, what is the chance that it will *not* happen? What are these two events called?

Helping You Remember

7. Write the equation $P(A) + P(\text{not } A) = 1$ in words. What does it mean with respect to event A?

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9-1 Enrichment

Coin-Tossing Experiments

If a coin is tossed 3 times, there are 8 possible outcomes. They are listed in the table below.

Number of Heads	0	1	2	3
Outcomes	TTT	HTT	HHT	HHH
		THT	THH	
		TTH	HTH	

Once all the outcomes are known, the probability of any event can be found. For example, the probability of getting 2 heads is $\frac{3}{8}$. Notice that this is the same as getting 1 tail.

1. A coin is tossed 4 times. Complete this chart to show the possible outcomes.

Number of Heads	0	1	2	3	4
Outcomes	TTTT				

2. What is the probability of getting all tails?

3. Now complete this table. Make charts like the one in Exercise 1 to help find the answers. Look for patterns in the numbers.

Number of Coin Tosses	2	3	4	5	6	7	8
Total Outcomes							
Probability of Getting All Tails							

4. What happens to the number of outcomes? the probability of all tails?

Lesson 9–1

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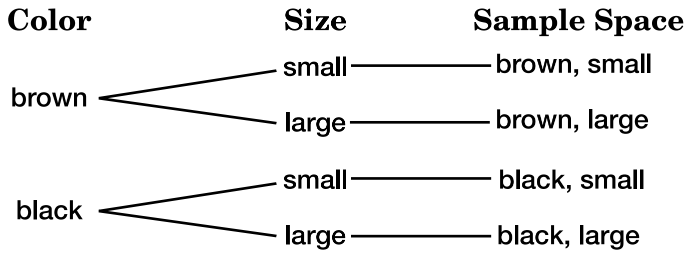
9-2 Study Guide and Intervention

Tree Diagrams

A game in which players of equal skill have an equal chance of winning is a **fair game**. A **tree diagram** is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

EXAMPLE 1 WATCHES **A certain type of watch comes in brown or black and in a small or large size. Find the number of color-size combinations that are possible.**

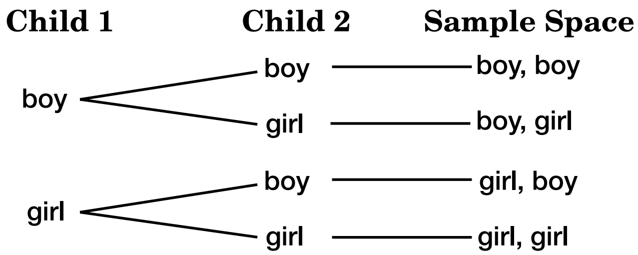
Make a tree diagram to show the sample space. Then give the total number of outcomes.



There are four different color and size combinations.

EXAMPLE 2 CHILDREN **The chance of having either a boy or a girl is 50%. What is the probability of the Smith's having two girls?**

Make a tree diagram to show the sample space. Then find the probability of having two girls.



The sample space contains 4 possible outcomes. Only 1 outcome has both children being girls. So, the probability of having two girls is $\frac{1}{4}$.

EXERCISES

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

1. choosing an outfit from a green shirt, blue shirt, or a red shirt, and black pants or blue pants

2. choosing a vowel from the word COUNTING and a consonant from the word PRIME

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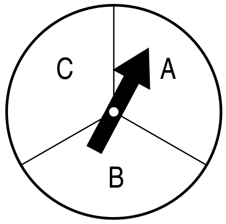
9-2 Practice: Skills

Tree Diagrams

The spinner at the right is spun twice.



1. Draw a tree diagram to represent the situation.

2. What is the probability of getting at least one A?

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

3. choosing a hamburger or hot dog and potato salad or macaroni salad

4. choosing a vowel from the word COMPUTER and a consonant from the word BOOK

5. choosing between the numbers 1, 2 or 3, and the colors blue, red, or green

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9-2 Practice: Word Problems

Tree Diagrams

1. GASOLINE Craig stops at a gas station to fill his gas tank. He must choose between full-service or self-service and between regular, midgrade, and premium gasoline. Draw a tree diagram showing the possible combinations of service and gasoline type. How many possible combinations are there?	**2. COINS** Judy tosses a coin 4 times. Draw a tree diagram showing the possible outcomes. What is the probability of getting at least 2 tails?
3. COINS In Exercise 2, what is the probability of getting 2 heads, then 2 tails?	**4. EQUIPMENT** The computer accessory that Joanne is considering selling at her store comes in white, beige, gray, or black and as an optical mouse, mechanical mouse, or trackball. How many combinations of color and model must she stock in order to have at least one of every possible combination?

9-2 Reading to Learn Mathematics

Tree Diagrams

Pre-Activity ***Complete the Mini Lab at the top of page 374 in your textbook. Write your answers below.***

1. Before you play, make a conjecture. Do you think that each player has an equal chance of winning? Explain.

2. Now, play the game. Who won? What was the final score?

3. Collect the data from the entire class. What is the combined score for Player 1 versus Player 2?

4. Do you want to change the conjecture you made in Exercise 1? Explain.

Reading the Lesson

5. How does a tree diagram resemble a tree?

6. How can you use a tree diagram to find the number of possible outcomes of an event?

7. How do you know the game played in Example 1 is fair?

Helping You Remember

8. Draw a tree diagram that shows a fair game that is different from the examples in your textbook. Can you think of a way to draw a tree diagram that shows a game that is *not* fair? Make sure you include a description if the game is not clear from your diagram.

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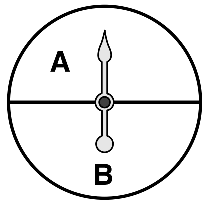
9-2 Enrichment

Probabilities and Regions

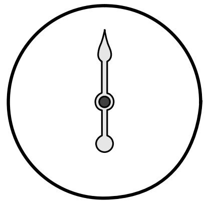
The spinner at the right can be used to indicate that the probability of landing in either of two regions is $\frac{1}{2}$.

$$P(A) = \frac{1}{2} \quad P(B) = \frac{1}{2}$$

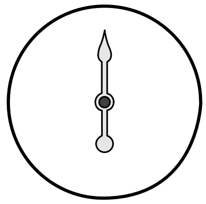


Read the description of each spinner. Using a protractor and ruler, divide each spinner into regions that show the indicated probability.

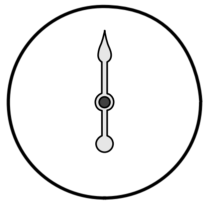
1. Two regions A and B: the probability of landing in region A is $\frac{3}{4}$. What is the probability of landing in region B?



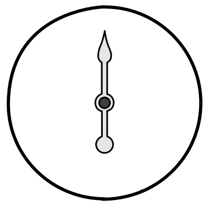
2. Three regions A, B, and C: the probability of landing in region A is $\frac{1}{2}$ and the probability of landing in region B is $\frac{1}{4}$. What is the probability of landing in region C?



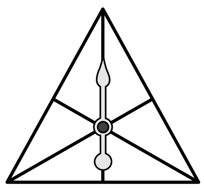
3. Three regions A, B, and C: the probability of landing in region A is $\frac{3}{8}$ and the probability of landing in region B is $\frac{1}{8}$. What is the probability of landing in region C?



4. Four regions A, B, C, and D: the probability of landing in region A is $\frac{1}{16}$, the probability of landing in region B is $\frac{1}{8}$, and the landing probability of in region C is $\frac{1}{4}$. What is the probability of landing in region D?



5. The spinner at the right is an equilateral triangle, divided into regions by line segments that divide the sides in half. Is the spinner divided into regions of equal probability?



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9-3 Study Guide and Intervention

The Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \times n$ ways. This is called the **Fundamental Counting Principle**.

EXAMPLE 1 CLOTHING **Andy has 5 shirts, 3 pairs of pants, and 6 pairs of socks. How many different outfits can Andy choose with a shirt, pair of pants, and pair of socks?**

$$\underbrace{5}_{\text{number of shirts}} \cdot \underbrace{3}_{\text{number of pants}} \cdot \underbrace{6}_{\text{number of socks}} = \underbrace{90}_{\text{total number of outfits}}$$

Andy can choose 90 different outfits.

EXERCISES

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. rolling two number cubes

2. tossing 3 coins

3. picking one consonant and one vowel

4. choosing one of 3 processor speeds, 2 sizes of memory, and 4 sizes of hard drive

5. choosing a 4-, 6-, or 8-cylinder engine and 2- or 4-wheel drive

6. rolling 2 number cubes and tossing 2 coins

7. choosing a color from 4 colors and a number from 4 to 10

Lesson 9–3

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9-3 Practice: Skills

The Fundamental Counting Principle

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. rolling two number cubes and tossing one coin

2. choosing rye or Bermuda grass and 3 different mixtures of fertilizer

3. making a sandwich with ham, turkey, or roast beef; Swiss or provolone cheese; and mustard or mayonaisse

4. tossing 4 coins

5. choosing from 3 sizes of distilled, filtered, or spring water

6. choosing from 3 flavors of juice and 3 sizes

7. choosing from 35 flavors of ice cream; one, two, or three scoops; and sugar or waffle cone

8. picking a day of the week and a date in the month of April

9. rolling 3 number cubes and tossing 2 coins

10. choosing a 4-letter password using only vowels

11. choosing a bicycle with or without shock absorbers; with or without lights; and 5 color choices

12. a license plate that has 3 numbers from 0 to 9 and 2 letters

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9-3 Practice: Word Problems

The Fundamental Counting Principle

1. SURFBOARD Jay owns 3 surfboards and 2 wetsuits. If he takes one surfboard and one wetsuit to the beach, how many different combinations can he choose?	**2. SHOPPING** John is trying to decide which bag of dog food to buy. The brand he wants comes in 4 flavors and 3 sizes. How many choices are there?
3. LOTTERY To purchase a lottery ticket, you must select 4 numbers from 0 to 9. How many possible lottery tickets can be chosen?	**4. RESTAURANTS** Miriam's favorite restaurant has 3 specials every day. Each special has 2 choices of vegetable and 3 choices of dessert. How many different meals could Miriam have?
5. ROUTES When Sunil goes to the building where he works, he can go through 4 different doors into the lobby. Then he can go to the seventh floor by taking 2 different elevators or 2 different stairways. How many different ways can Sunil get from outside the building to the seventh floor?	**6. STEREOS** Jailin went to her local stereo store. Given her budget and the available selection, she can choose between 2 CD players, 5 amplifiers, and 3 pairs of speakers. How many different stereos can Jailin purchase?
7. DESSERT For dessert you can choose apple, cherry, blueberry, or peach pie to eat, and milk or juice to drink. How many different combinations can you choose from?	**8. TESTS** John is taking a true or false quiz. There are six questions on the quiz. How many ways can the quiz be answered?

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9-3 Reading to Learn Mathematics

The Fundamental Counting Principle

Pre-Activity *Read the introduction at the top of page 378 in your textbook. Write your answers below.*

1. According to the table, how many sizes of juniors' jeans are there?

2. How many lengths are there?

3. Find the product of the two numbers you found in Exercises 1 and 2.

4. Draw a tree diagram to help you find the number of different size and length combinations. How does the number of outcomes compare to the product you found above?

Reading the Lesson

5. What operation is used in the Fundamental Counting Principle?

6. How is the information in a tree diagram different from the information provided by counting?

Helping You Remember

7. Write the Fundamental Counting Principle in your own words.

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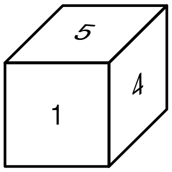
9-3 Enrichment

Curious Cubes

If a six-faced number cube is rolled any number of times, the theoretical probability of the number cube landing on any given face is $\frac{1}{6}$.

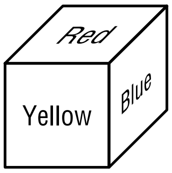
Each number cube below has six faces and has been rolled 100 times. The outcomes have been tallied and recorded in a frequency table. Based on the data in each frequency table, what can you say are probably on the unseen faces of each cube?

1.



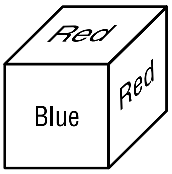
Outcome	Tally
1	15
2	14
3	18
4	16
5	19
6	18

2.



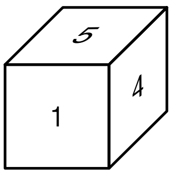
Outcome	Tally
blue	17
red	30
yellow	53

3.



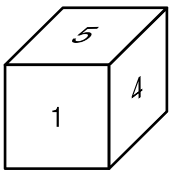
Outcome	Tally
red	30
blue	16
blank	54

4.



Outcome	Tally
1	34
4	32
5	34

5.



Outcome	Tally
1	14
5	13
4	18
2	16
blank	39

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9-4 Study Guide and Intervention

Permutations

The expression *n* **factorial** (*n*!) is the product of all counting numbers beginning with *n* and counting backward to 1. A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

EXAMPLE 1 **Find the value of 5!.**

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial
$= 120$ Simplify.

EXAMPLE 2 **Find the value of 4! · 2!.**

$4! \cdot 2! = 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1$ Definition of factorial
$= 48$ Simplify.

EXAMPLE 3 **BOOKS How many ways can 4 different books be arranged on a bookshelf?**

This is a permutation that can be written as 4!. Suppose the books are placed on the shelf from left to right.

There are **4** choices for the first book.
There are **3** choices that remain for the second book.
There are **2** choices that remain for the third book.
There is **1** choice that remains for the fourth book.

$4! = 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial
$= 24$ Simplify.

So, there are 24 ways to arrange 4 different books on a bookshelf.

EXERCISES

Find the value of each expression.

1. 3!

2. seven factorial

3. $6! \cdot 3!$

4. $9 \cdot 8 \cdot 7$

5. How many ways can you arrange the letters in the word GROUP?

6. How many different 4-digit numbers can be created if no digit can be repeated? Remember, a number cannot begin with 0.

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9-4 Practice: Skills

Permutations

Find the value of each expression.

1. 2!

2. 4!

3. $3! \cdot 5!$

4. nine factorial

5. $2! \cdot 8!$

6. $3! \cdot 2!$

7. $11 \cdot 10 \cdot 9$

8. 10!

9. $5! \cdot 2!$

10. $5 \cdot 4 \cdot 3 \cdot 2$

11. $8! \cdot 6!$

12. $6! \cdot 4!$

13. How many ways can you arrange the letters in the word PRIME?

14. How many ways can you arrange 8 different crates on a shelf if they are placed from left to right?

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9-4

Practice: Word Problems

Permutations

1. AREA CODES How many different 3-digit area codes can be created if no digit can be repeated?	**2. CARDS** Jason is dealt five playing cards. In how many different orders could Jason have been dealt the same hand?
3. PASSWORDS How many different 3-letter passwords are possible if no letter may be repeated?	**4. RACING** All 22 students in Amy's class are going to run the 100-meter dash. In how many ways can the students finish in first, second, and third place?
5. LETTERS How many ways can you arrange the letters in the word HISTORY?	**6. PARKING** The parking lot for a company has three parking spaces for compact cars. The company has 8 employees with compact cars. How many ways can the compact parking spaces be filled?
7. SERIAL NUMBERS How many different 6-digit serial numbers are available if no digit can be repeated?	**8. WINNERS** There are 156 ways for 2 cars to win first and second place in a race. How many cars are in the race?

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9-4 Reading to Learn Mathematics

Permutations

Pre-Activity ***Complete the Mini Lab at the top of page 381 in your textbook. Write your answers below.***

1. When you first started to make your list, how many choices did you have for your first class?

2. Once your first class was selected, how many choices did you have for the second class? Then, the third class?

3. Explain how you can use the Fundamental Counting Principle to find the number of arrangements.

Reading the Lesson

4. How do you write *five factorial* using symbols?

5. What are the factors of five factorial?

6. What is the value of five factorial?

7. In Example 4 on page 382, why are there only 7 choices for second place?

Helping You Remember

8. Look up the word *permute* in a dictionary. How does the meaning of this word relate to the concepts in this lesson, especially the concepts of permutations and factorials?

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9-4 Enrichment

Permutation Puzzles

When you change the order of a set of objects in a permutation by switching the places of two items next to one another, you *transpose* two items in the permutation. In the permutation at the left below, switch B and C to get the permutation at the right below.

A B C D → A C B D

For each arrangement at the left show how to switch two letters at a time to get the arrangement at the right. Show your switches in drawings.

1.

A	B
C	D

D	B
A	C

2.

A	B	C
D	E	F

B	F	C
A	D	E

3.

A	B	C
D	E	F
G	H	I

B	C	I
A	E	F
D	G	H

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9-5 Study Guide and Intervention

Combinations

Lesson 9–5

An arrangement, or listing, of objects in which order is *not* important is called a **combination**. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

EXAMPLE 1 **Jill was asked by her teacher to choose 3 topics from the 8 topics given to her. How many different three-topic groups could she choose?**

There are $8 \cdot 7 \cdot 6$ permutations of three-topic groups chosen from eight. There are 3! ways to arrange the groups.

$\frac{8 \cdot 7 \cdot 6}{3!} = \frac{336}{6} = 56$

So, there are 56 different three-topic groups.

EXAMPLES **Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.**

2 **On a quiz, you are allowed to answer any 4 out of the 6 questions. How many ways can you choose the questions?**

This is a combination because the order of the 4 questions is not important. So, there are $6 \cdot 5 \cdot 4 \cdot 3$ permutations of four questions chosen from six. There are 4! or $4 \cdot 3 \cdot 2 \cdot 1$ orders in which these questions can be chosen.

$\frac{6 \cdot 5 \cdot 4 \cdot 3}{4!} = \frac{360}{24} = 15$

So, there are 15 ways to choose the questions.

3 **Five different cars enter a parking lot with only 3 empty spaces. How many ways can these spaces be filled?**

This is a permutation because each arrangement of the same 3 cars counts as a distinct arrrangement. So, there are $5 \cdot 4 \cdot 3$ or 60 ways the spaces can be filled.

EXERCISES

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

1. How many ways can 4 people be chosen from a group of 11?

2. How many ways can 3 people sit in 4 chairs?

3. How many ways can 2 goldfish be chosen from a tank containing 15 goldfish?

9-5 Practice: Skills

Combinations

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

1. You are allowed to omit two out of 12 questions on a quiz. How many ways can you select the questions to omit?

2. Six students are to be chosen from a class of 18 to represent the class at a math contest. How many ways can the six students be chosen?

3. How many different 5-digit zip codes are possible if no digits are repeated?

4. In a race with six runners, how many ways can the runners finish first, second, or third?

5. How many ways can two names be chosen from 76 in a raffle if only one entry per person is allowed?

6. How many ways can six students be arranged in a lunch line?

7. A family has a bike rack that fits seven bikes but they only have five bikes. How many ways can the bikes fit in the bike rack?

8. How many ways can you select three sheriff deputies from eight candidates?

9. How many ways can four finalists be selected from 50 contestants?

10. How many 4-digit pin numbers are available if no number is repeated?

11. How many handshakes can occur between five people if everyone shakes hands?

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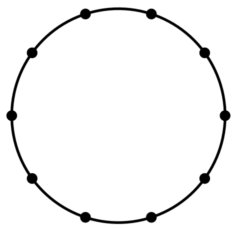
9-5

Practice: Word Problems

Combinations

1. SNACKS A vending machine can display six snacks. If there are eight different kinds of snacks available, how many different groups of six different snacks can be displayed?	**2. MUSIC** Each month, Jose purchases two CDs from a selection of 20 bestselling CDs. How many different pairs of CDs can Jose choose if he chooses two different CDs?
3. TESTS On a math test, you can choose any 20 out of 23 questions. How many different groups of 20 questions can you choose?	**4. RESTAURANTS** The dinner special at a local pizza parlor gives you the choice of two toppings from a selection of six toppings. How many different choices are possible if two different toppings are chosen?
5. TESTING In a science fair experiment, two units are selected for testing from every 500 units produced. How many ways can these two units be selected?	**6. MEETINGS** Linda's teacher divided the class into groups of five and required each member of a group to meet with every other member of that group. How many meetings will each group have?
7. BASEBALL A baseball coach has 13 players to fill nine positions. How many different teams could he put together?	**8. GEOMETRY** Ten points are marked on a circle. How many different triangles can be drawn between any three points? 

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9-5

Reading to Learn Mathematics

Combinations

Pre-Activity ***Read the introduction at the top of page 387 in your textbook. Write your answers below.***

1. Use the first letter of each name to list all of the permutations of co-captains. How many are there?

2. Cross out any arrangement that contains the same letters as another one in the list. How many are there now?

3. Explain the difference between the two lists above.

Reading the Lesson

4. How can you find the number of combinations of objects in a set?

5. Why might it be easier to calculate the number of combinations of a set of objects using a permutation rather than making a list?

For Exercises 6 and 7, refer to Example 2 on page 388 in your textbook.

6. In the diagram, how many points are there? How many line segments connect to any one point?

7. How does your answer to Exercise 6 above correspond to Example 2 in your book?

Helping You Remember

8. Work with a partner. Take turns thinking of situations in which a selection from a group must be made, where order is or is not important. Tell each other which situations are permutations and which are combinations. Solve each problem and show your work.

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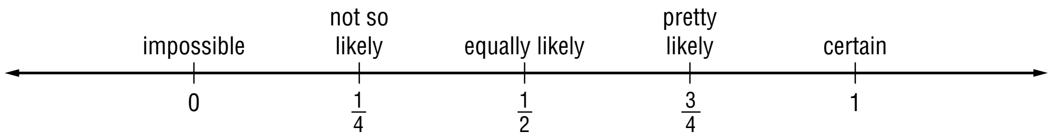
9-5 Enrichment

From Impossible to Certain Events

A probability is often expressed as a fraction. As you know, an event that is impossible is given a probability of 0 and an event that is certain is given a probability of 1. Events that are neither impossible nor certain are given a probability somewhere between 0 and 1. The probability line below shows relative probabilities.



Determine the probability of an event by considering its place on the diagram above.

1. Medical research will find a cure for all diseases.

2. There will be a personal computer in each home by the year 2010.

3. One day, people will live in space or under the sea.

4. Wildlife will disappear as Earth's human population increases.

5. There will be a fifty-first state in the United States.

6. The sun will rise tomorrow morning.

7. Most electricity will be generated by nuclear power by the year 2010.

8. The fuel efficiency of automobiles will increase as the supply of gasoline decreases.

9. Astronauts will land on Mars.

10. The percent of high school students who graduate and enter college will increase.

11. Global warming problems will be solved.

12. All people in the United States will exercise regularly within the near future.

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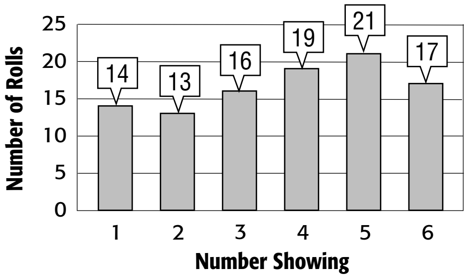
9-6 Study Guide and Intervention

Theoretical and Experimental Probability

Experimental probability is found using frequencies obtained in an experiment or game. **Theoretical probability** is the expected probability of an event occurring.

EXAMPLE 1 **The graph shows the results of an experiment in which a number cube was rolled 100 times. Find the experimental probability of rolling a 3 for this experiment.**



$$P(3) = \frac{\text{number of times 3 occurs}}{\text{number of possible outcomes}}$$

$$= \frac{16}{100} \text{ or } \frac{4}{25}$$

The experimental probability of rolling a 3 is $\frac{4}{25}$, which is close to its theoretical probability of $\frac{1}{6}$.

EXAMPLES

2 **In a telephone poll, 225 people were asked for whom they planned to vote in the race for mayor. What is the experimental probability of Juarez being elected?**

Candidate	Number of People
Juarez	75
Davis	67
Abramson	83

Of the 225 people polled, 75 planned to vote for Juarez.

So, the experimental probability is $\frac{75}{225}$ or $\frac{1}{3}$.

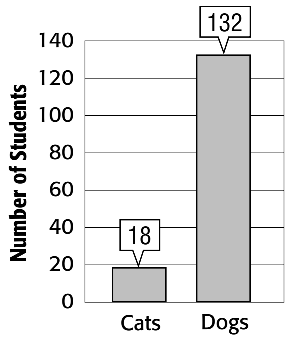
3 **Suppose 5,700 people vote in the election. How many can be expected to vote for Juarez?**

$\frac{1}{3} \cdot 5{,}700 = 1{,}900$

About 1,900 will vote for Juarez.

EXERCISES

For Exercises 1–3, use the graph of a survey of 150 students asked whether they prefer cats or dogs.



1. What is the probability of a student preferring dogs?
2. Suppose 100 students were surveyed. How many can be expected to prefer dogs?
3. Suppose 300 students were surveyed. How many can be expected to prefer cats?

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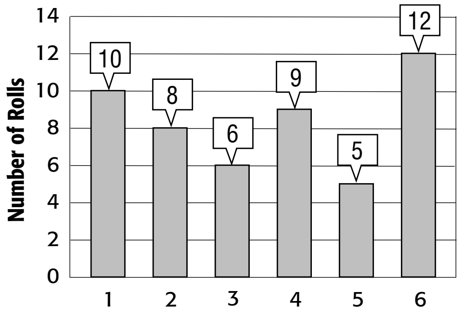
9-6 Practice: Skills

Theoretical and Experimental Probability

For Exercises 1–5, a number cube is rolled 50 times and the results are shown in the graph below.



1. Find the experimental probability of rolling a 2.

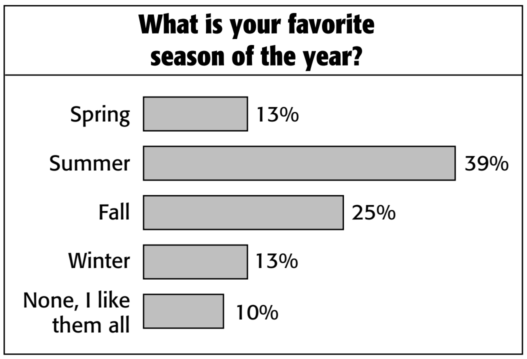
2. What is the theoretical probability of rolling a 2?

3. Find the experimental probability of *not* rolling a 2.

4. What is the theoretical probability of *not* rolling a 2?

5. Find the experimental probability of rolling a 1.

For Exercises 6–9, use the results of the survey at the right.



6. What is the probability that a person's favorite season is fall? Write the probability as a fraction.

7. Out of 300 people, how many would you expect to say that fall is their favorite season?

8. Out of 20 people, how many would you expect to say that they like all the seasons?

9. Out of 650 people, how many more would you expect to say that they like summer than say that they like winter?

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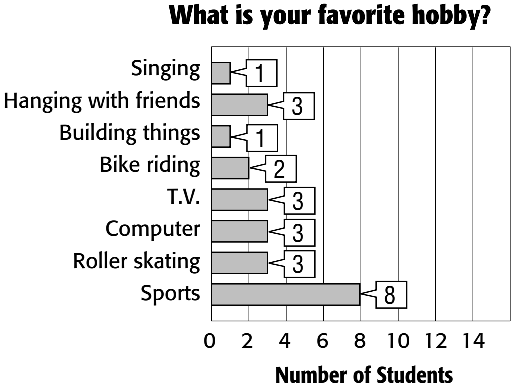
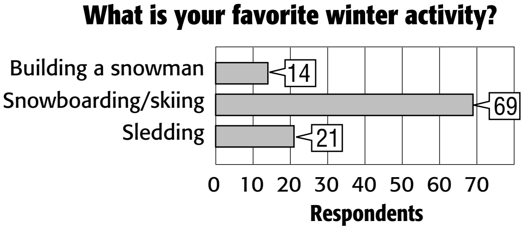
9-6 Practice: Word Problems

Theoretical and Experimental Probability

HOBBIES For Exercises 1–3, use the graph of a survey of 24 seventh grade students asked to name their favorite hobby.



TELEVISION For Exercises 5 and 6, use the graph of a survey with 104 responses in which respondents were asked about their favorite winter activities.



1. What is the probability that a student's favorite hobby is roller skating?

2. Suppose 200 seventh grade students were surveyed. How many can be expected to say that roller skating is their favorite hobby?

3. Suppose 60 seventh grade students were surveyed. How many can be expected to say that bike riding is their favorite hobby?

4. **MARBLES** A bag contains 5 blue, 4 red, 9 white, and 6 green marbles. If a marble is drawn at random and replaced 100 times, how many times would you expect to draw a green marble?

5. What is the probability that someone's favorite winter activity is building a snowman? Write the probability as a fraction.

6. If 500 people had responded, how many would have been expected to list sledding as their favorite winter activity? Round to the nearest whole person.

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9-6 Reading to Learn Mathematics

Theoretical and Experimental Probability

Pre-Activity ***Complete the Mini Lab at the top of page 393 in your textbook. Write your answers below.***

1. How many times did you roll a sum of 7? What is the probability of rolling a sum of 7?

2. How does your result compare to the results of other groups? Explain.

3. What is the expected probability of rolling a sum of 7?

4. How does your result compare to the expected probability of rolling a sum of 7? Explain any differences.

Reading the Lesson

5. Look up the word *experimental* in a dictionary. Write the meaning for the word as used in the lesson.

6. How does theoretical probability differ from experimental probability?

7. Complete the sentence: Experimental probability can be based on ____________________ and can be used to make predictions about future events.

Helping You Remember

8. Work with a partner. Design an experiment that you can use to express the experimental probability of an event. Compare your findings with those of others in your class.

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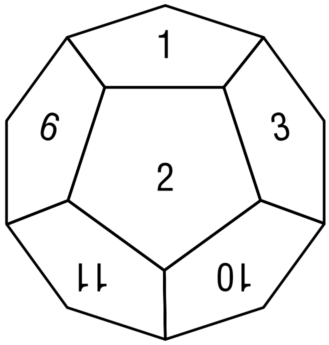
9-6 Enrichment

Rolling a Dodecahedron



A **dodecahedron** is a solid. It has twelve faces, and each face is a pentagon.

At the right, you see a dodecahedron whose faces are marked with the integers from 1 through 12. You can roll this dodecahedron just as you roll a number cube. With the dodecahedron, however, there are *twelve* equally likely outcomes.

Refer to the dodecahedron shown at the right. Find the probability of each event.

1. *P*(5)

2. *P*(odd)

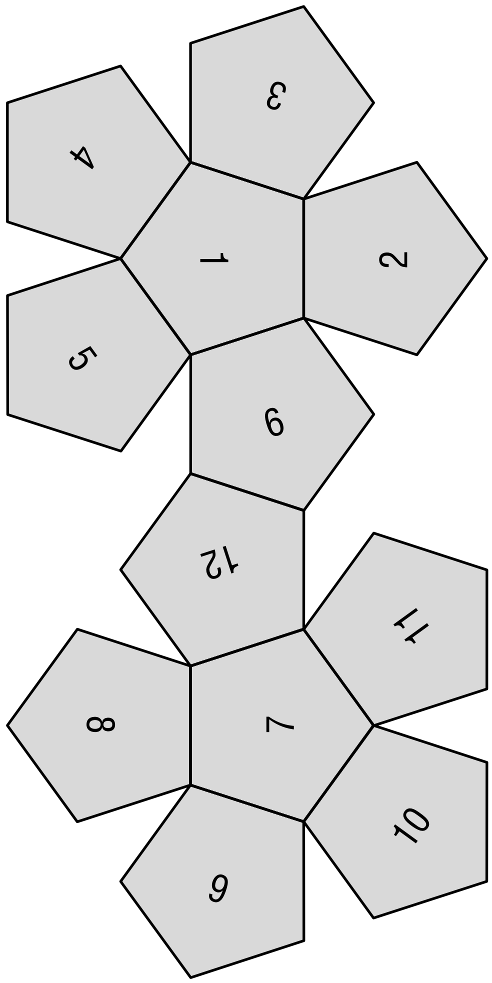
3. *P*(prime)

4. *P*(divisible by 5)

5. *P*(less than 4)

6. *P*(fraction)

You can make your own dodecahedron by cutting out the pattern at the right. Fold along each of the solid lines. Then use tape to join the faces together so that your dodecahedron looks like the one shown above.



7. Roll your dodecahedron 100 times. Record your results on a separate sheet of paper, using a table like this.

Outcome	Tally	Frequency
1		
2		

8. Use your results from Exercise 7. Find the experimental probability for each of the events described in Exercises 1–6.

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9-7 Study Guide and Intervention

Independent and Dependent Events

A **compound event** consists of two or more simple events. If the outcome of one event does not affect the outcome of a second event, the events are called **independent events**. The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event. If the outcome of one event affects the outcome of a second event, the events are called **dependent events**.

EXAMPLE 1 **A coin is tossed and a number cube is rolled. Find the probability of tossing tails and rolling a 5.**

$P(\text{tails}) = \frac{1}{2}$ $\quad\quad P(5) = \frac{1}{6}$

$P(\text{tails and } 5) = \frac{1}{2} \cdot \frac{1}{6} \text{ or } \frac{1}{12}$

So, the probability of tossing tails and rolling a 5 is $\frac{1}{12}$.

EXAMPLE 2 MARBLES **A bag contains 7 blue, 3 green, and 3 red marbles. If Agnes randomly draws two marbles from the bag, one after another, what is the probability of drawing a green and then a blue marble?**

$P(\text{green}) = \frac{3}{13}$ 13 marbles, 3 are green

$P(\text{blue}) = \frac{7}{12}$ 12 marbles after 1 green marble is removed, 7 are blue

$P(\text{green, then blue}) = \frac{3}{13} \cdot \frac{7}{12} = \frac{7}{52}$

So, the probability that Agnes will draw a green, then a blue marble is $\frac{7}{52}$.

EXERCISES

1. Find the probability of rolling a 2 and then an even number on two consecutive rolls of a number cube.

2. A penny and a dime are tossed. What is the probability that the penny lands on heads and the dime lands on tails?

3. Lazlo's sock drawer contains 8 blue and 5 black socks. If he randomly pulls out two socks, what is the probability that he picks two blue socks?

Lesson 9–7

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9-7 Practice: Skills

Independent and Dependent Events

1. Four coins are tossed. What is the probability of tossing all heads?

2. One letter is randomly selected from the word PRIME and one letter is randomly selected from the word MATH. What is the probability that both letters selected are vowels?

3. A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of getting a jack and then an eight?

For Exercises 4–6, use the information below.

A standard deck of playing cards contains 52 cards in four suits of 13 cards each. Two suits are red and two suits are black. Find each probability. Assume the first card is replaced before the second card is drawn.

4. *P*(black, queen)

5. *P*(black, diamond)

6. *P*(jack, queen)

7. What is the probability of spinning a number greater than 5 on a spinner numbered 1 to 8 and tossing a tail on a coin?

8. Two cards are chosen at random from a standard deck of cards with replacement. What is the probability of getting 2 aces?

9. A CD rack has 8 classical CDs, 5 pop CDs, and 3 rock CDs. Two CDs are chosen without replacement. What is the probability of choosing a rock CD then a classical CD?

10. A jar holds 15 red pencils and 10 blue pencils. What is the probability of drawing two red pencils from the jar?

9-7 Practice: Word Problems

Independent and Dependent Events

1. SAFETY Eighty percent of all California drivers wear seat belts. If three drivers are pulled over, what is the probability that all would be wearing their seat belts? Write as a percent to the nearest tenth.	**2. VEGETABLES** A nationwide survey showed that 65% of all children in the United States dislike eating vegetables. If three children are chosen at random, what is the probability that all three dislike eating vegetables? Write as a percent to the nearest tenth.
3. QUALITY In a shipment of 50 calculators, 4 are defective. Two calculators are randomly selected and tested. What is the probability that both are defective if the first one is not replaced after being tested?	**4. MARBLES** A bag contains 6 green marbles, 2 blue marbles, and 3 white marbles. Gwen draws one marble from the jar, and then Jeff draws one marble from the remaining marbles. What is the probability that Gwen draws a blue marble and Jeff draws a white marble?
5. DEMONSTRATION Ms. Morris needs two students to help her with a demonstration for her class of 12 girls and 14 boys. She randomly chooses two students. What is the probability that she chooses two girls?	**6. SURVEY** Ruben surveyed his class and found that 4 out of 22 students walk to school. If two of the 22 students are selected at random, what is the probability that both walk to school?

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9-7 Reading to Learn Mathematics

Independent and Dependent Events

Pre-Activity ***Read the introduction at the top of page 398 in your textbook. Write your answers below.***

1. What is the probability of Omar being in the second heat?

2. What is the probability of Omar being in lane 3?

3. Multiply your answers in Exercises 1 and 2 above. What does this number mean? Explain.

Reading the Lesson

Use the introduction to the lesson to answer Exercises 4–6.

4. Is choosing a number from jar 1 a simple event or a compound event? Explain.

5. Is choosing a number from jar 1 and a number from jar 2 a simple event or a compound event?

6. Choosing a number from jar 1 and choosing a number from jar 2 are independent events. Why are they called independent?

7. How can you find the probability of two independent events?

Helping You Remember

8. Is the formula $P(A) \cdot P(B \text{ following } A)$ the formula for finding the probability of independent events or dependent events? Explain.

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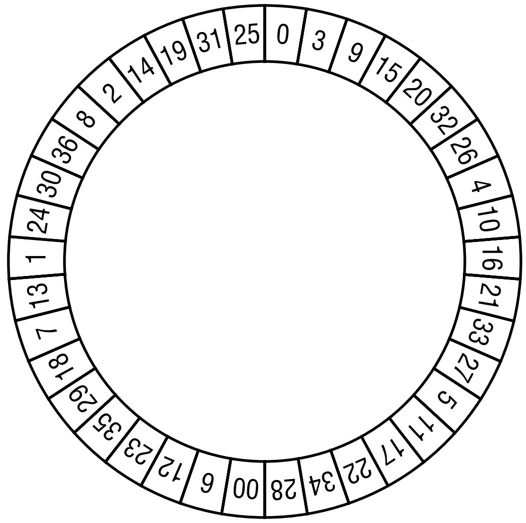
9-7 Enrichment

Independent Events



The game of roulette is played by dropping a ball into a spinning, bowl-shaped wheel. When the wheel stops spinning, the ball will come to rest in any of 38 locations.

On a roulette wheel, the eighteen even numbers from 2 through 36 are colored red and the eighteen odd numbers from 1 through 35 are colored black. The numbers 0 and 00 are colored green.

To find the probability of two independent events, the results of two spins, find the probability of each event first.

$$P(\text{red}) = \frac{18}{38} \text{ or } \frac{9}{19}$$

$$P(\text{black}) = \frac{18}{38} \text{ or } \frac{9}{19}$$

Then multiply.

$$P(\text{red, then black}) = \frac{9}{19} \times \frac{9}{19} \text{ or } \frac{81}{361}$$

Find each probability.

1. black, then black

2. prime number, then a composite number

3. a number containing at least one 0, then a number containing at least one 2

4. red, then black

5. the numbers representing your age, month of birth, and then day of birth

Lesson 9–7